



## Supplementary materials for

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### 1 Supplement to the operation principle

Suppose that  $r$  is the distance away from the center of the homogeneous cylindrical lens (HCL), the theoretical analysis of the proposed crossover can be further deepened using the Green function approach (Gunderson, 1972). Excited by a TE<sub>10</sub> mode from the air-filled substrate-integrated waveguide (SIW), only the  $z$  component of the E-field exists within the parallel waveguide. It can be analyzed that such a case is equivalent to an infinitely long cylindrical lens with a line source. Using a line source excitation at  $r'$  and  $\phi'$ , the Green function for the lens is the solution to the wave equation in two dimensions. By dividing the area into three zones, the solution is written as

$$G(r, \phi | r', \phi') = \sum_n \begin{cases} A_n J_n(k_1 r) \cos[n(\phi - \phi')], & r \leq a, \\ [B_n H_n(k_0 r) + \varepsilon_n J_n(k_0 r) H_n(k_0 r')] \cos[n(\phi - \phi')], & a < r < r', \\ [B_n H_n(k_0 r) + \varepsilon_n J_n(k_0 r') H_n(k_0 r)] \cos[n(\phi - \phi')], & r' \leq r, \end{cases} \quad (\text{S1})$$

where  $J_n$  represents the Bessel function of the first kind,  $H_n$  represents the Hankel function,  $\phi$  represents the rotation angle in the cylindrical coordinate system,  $n$  is the order of derivative,  $\varepsilon_n$  is the dielectric constant, and  $k_0$  and  $k_1$  are the first and second zeros of the Bessel function, respectively.  $A_n$  and  $B_n$  can be solved by matching the electric and magnetic fields at  $r=a$ . The E-field can be solved according to

$$E_z(r, \phi | r', \phi') = \int_c \left[ E_z(r_0, \phi') \frac{\partial G(r, \phi | r', \phi')}{\partial n} - G(r, \phi | r', \phi') \frac{\partial E_z(r_0, \phi')}{\partial n} \right] d\phi', \quad (\text{S2})$$

where  $c$  represents the integral path. By expressing this source field in cylindrical coordinates and integration along  $r'=r_0$ , the electric field may be written as

$$E_z(r, \phi | r', \phi') = K \sum_n \varepsilon_n (-1)^n \cos(n\phi) \exp\left(jn \frac{\pi}{2}\right) \left[ \frac{\partial P_n(r_0)}{\partial r_0} I_1 + P_n(r_0) \left( j\beta_{10} I_1 - \frac{\pi}{\omega} I_2 \right) \right], \quad (\text{S3})$$

where

$$K = \left( \frac{2}{\pi k_0 r} \right)^{1/2} D \exp\left[-j \left( k_0 r - \frac{\pi}{4} \right)\right], \quad (\text{S4})$$

$$P_n(r_0) = C_n H_n(k_0 r_0) + J_n(k_0 r_0), \quad (\text{S5})$$

$$C_n = \frac{k_0 J'_n(k_0 a) J_n(k_1 a) - k_1 J_n(k_0 a) J'_n(k_1 a)}{k_1 H_n(k_0 a) J'_n(k_1 a) - k_0 H'_n(k_0 a) J_n(k_1 a)}, \quad (\text{S6})$$

$$I_1 = \int_{-\omega/(2r_0)}^{\omega/(2r_0)} \cos\left(\frac{\pi r_0 \alpha}{\omega}\right) \cos(n\alpha) d\alpha = \frac{2\omega \pi r_0}{(\pi r_0 - n\omega)(\pi r_0 + n\omega)} \cos\left(\frac{n\omega}{2r_0}\right), \quad (\text{S7})$$

$$I_2 = \int_{-\omega/(2r_0)}^{\omega/(2r_0)} \alpha \sin\left(\frac{\pi r_0 \alpha}{\omega}\right) \cos(n\alpha) d\alpha \quad (\text{S8})$$

$$= \frac{\omega^2}{(\pi r_0)^2 - (n\omega)^2} \left[ \frac{n}{\omega r_0} \sin\left(\frac{n\omega}{2r_0}\right) - 2 \frac{(\pi r_0)^2 + (n\omega)^2}{\omega^2} \cos\left(\frac{n\omega}{2r_0}\right) \right].$$

## 2 Supplement to the design process

Next, parametric study is carried out. Since the designed two-channel crossover is symmetric, we have  $S_{21}=S_{41}$ . Here, only  $S_{11}$  and  $S_{21}$  related to different parameters are investigated. In Figs. S1a–S1h, Figs. S1a–S1d are about  $S_{11}$  while Figs. S1e–S1h plot  $S_{21}$ . It can be seen that the bandwidth ( $S_{11} < -15$  dB) decreases with an increasing  $\epsilon_{r2}$ . As can be seen in Fig. S1b, the thickness  $h$  of the substrate has no impact on the reflection loss, as expected. However, the reflection coefficient is highly sensitive to the diameter of the HCL (Fig. S1c), which is selected as  $a=2.0$  mm. The width of the open air-filled SIW influences mainly the feasible bandwidth, as shown in Fig. S1d. For the isolation counterparts, the change in  $\epsilon_{r2}$  barely affects the bandwidth of  $S_{21}$ , but would shift the frequency range. Similarly, in Fig. S1f, the thickness  $h$  of the substrate does not affect the coupling between ports 1 and 2. In Fig. S1g, a maximum bandwidth of  $S_{21}$  can be achieved with  $a=2.5$  mm. However, the reflection coefficient deteriorates greatly in this case. Therefore, to balance  $S_{21}$  and  $S_{11}$ ,  $a=2.0$  mm is selected. Considering  $S_{21}$  in Fig. S1h, the results would shift related to different  $w$ 's. Similarly,  $w=7.2$  mm is chosen to balance  $S_{21}$  and  $S_{11}$ .

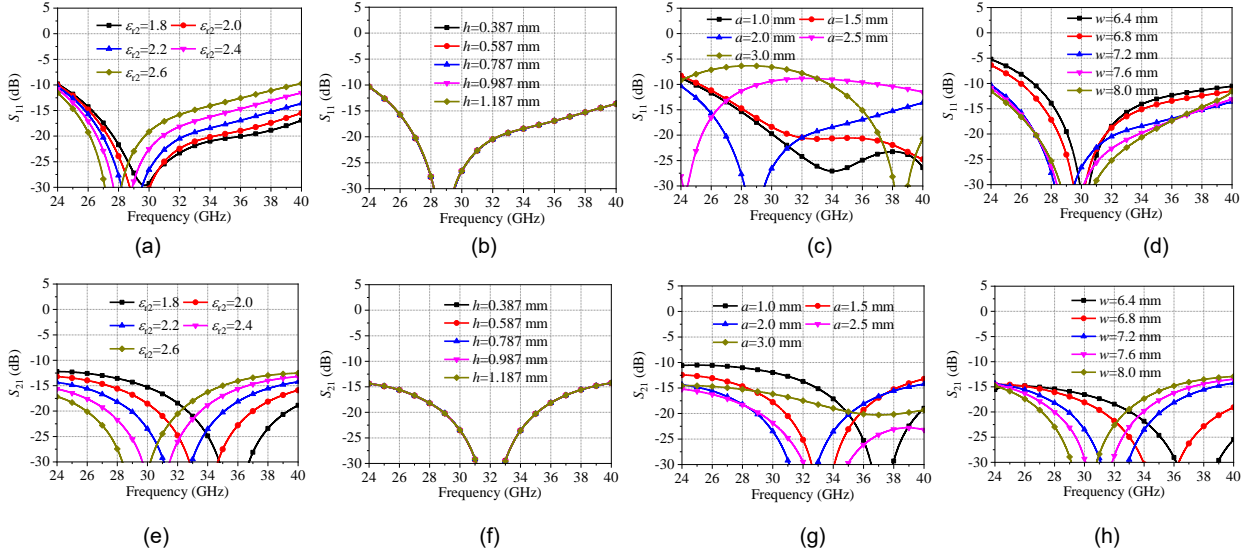


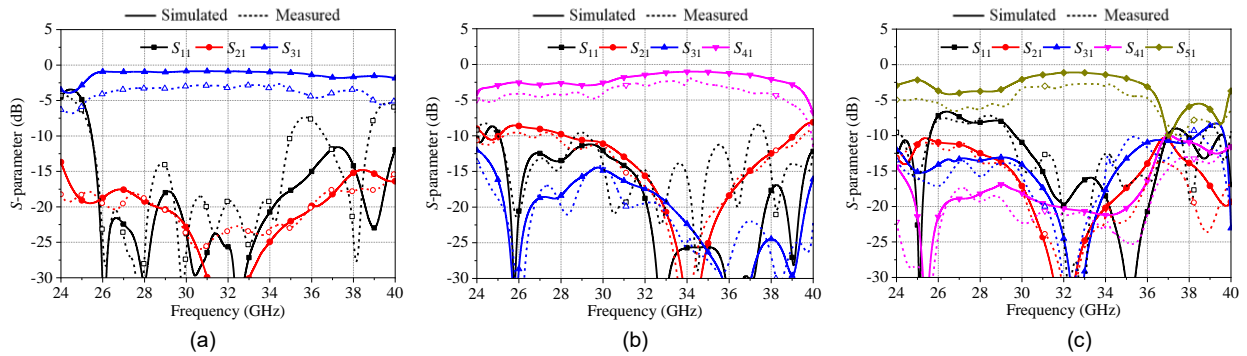
Fig. S1  $S_{11}$  with different  $\epsilon_{r2}$  (a),  $h$  (b),  $a$  (c), and  $w$  (d) and  $S_{21}$  with different  $\epsilon_{r2}$  (e),  $h$  (f),  $a$  (g), and  $w$  (h)

## 3 Supplement to the experiment

The results of the fabricated three-channel air-filled SIW crossover are plotted in Fig. S2b. A fractional bandwidth (FBW) of 14% is observed in the simulation when the reflection and isolation coefficients are lower

than  $-15$  dB. The measured  $S_{21}$  and  $S_{31}$  agree well with the simulated counterparts, while the measured  $S_{11}$  is deteriorated to about  $-10$  dB. The minimum insertion losses in the simulation and measurement are 1.0 dB and 2.5 dB, respectively.

Similar to the above, the simulated four-channel air-filled SIW crossover has an FBW of 10%, and its reflection and isolation coefficients are below  $-15$  dB from 31.1 to 34.4 GHz. The measured isolation coefficients are in a satisfactory agreement with the simulation results. The bandwidth of the measured  $S_{11}$  is reduced to 31.3–33.3 GHz. The simulated and measured maximum insertion losses are 1.1 dB and 3.1 dB, respectively.



**Fig. S2** Simulated and measured  $S$ -parameters of two-channel (a), three-channel (b), and four-channel (c) SIW crossovers

## References

Gunderson L, 1972. An electromagnetic analysis of a cylindrical homogeneous lens. *IEEE Trans Antenn Propag*, 20(4): 476-479. <https://doi.org/10.1109/TAP.1972.1140224>