



Supplementary materials for

Shahid HUSSAIN, Li-xin SONG, Shabbir AHMAD, et al., 2019. A new auxiliary information based cumulative sum median control chart for location monitoring. *Front Inform Technol Electron Eng*, 20(4):554-570. <https://doi.org/10.1631/FITEE.1700428>

Table S1 Mean and standard deviations of median estimators

Est	Mean
M_1	M_y
M_2	$\frac{0.25M_y}{M_x^2 f_x^2(M_x)} \left[\lambda \left\{ 1 - \phi_{xy} \frac{M_x f_x(M_x)}{M_y f_x(M_x)} \right\} + 4M_x^2 f_x^2(M_x) \right]$
M_3	$\frac{0.25M_y}{M_x^2 f_x^2(M_x)} \left[\lambda \Delta_{1x} \left\{ A_{1x} - \phi_{xy} \frac{M_x f_x(M_x)}{M_y f_x(M_x)} \right\} + 4M_x^2 f_x^2(M_x) \right]$
M_4	$\frac{0.25M_y}{M_x^2 f_x^2(M_x)} \left[\lambda \Delta_{2x} \left\{ A_{2x} - \phi_{xy} \frac{M_x f_x(M_x)}{M_y f_x(M_x)} \right\} + 4M_x^2 f_x^2(M_x) \right]$
M_5	$\frac{0.25M_y}{M_x^2 f_x^2(M_x)} \left[\lambda \Delta_{3x} \left\{ A_{3x} - \phi_{xy} \frac{M_x f_x(M_x)}{M_y f_x(M_x)} \right\} + 4M_x^2 f_x^2(M_x) \right]$
M_6	$\frac{0.25M_y}{M_x^2 f_x^2(M_x)} \left[\lambda \Delta_{4x} \left\{ A_{4x} - \phi_{xy} \frac{M_x f_x(M_x)}{M_y f_x(M_x)} \right\} + 4M_x^2 f_x^2(M_x) \right]$
M_7	$\frac{0.125(1 - \phi_{zx})^{-1}}{M_y f_y^2(M_y)} \left[\lambda \frac{M_y f_y(M_y)}{M_x f_x(M_x)} (\phi_{xy} - \phi_{yz} \phi_{zx}) + \lambda \frac{M_y f_y(M_y)}{M_z f_z(M_z)} (\phi_{yz} - \phi_{xy} \phi_{zx}) \right. \\ \left. - \lambda(\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy} \phi_{yz} \phi_{zx}) + 8M_y^2 f_y^2(M_y)(1 - \phi_{zx}^2) \right]$
M_8	$\frac{0.125(1 - \phi_{zx})^{-1}}{M_y f_y^2(M_y)} \left[\lambda \Delta_{1x} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} (\phi_{xy} - \phi_{yz} \phi_{zx}) + \lambda \Delta_{1z} \frac{M_y f_y(M_y)}{M_z f_z(M_z)} (\phi_{yz} - \phi_{xy} \phi_{zx}) \right. \\ \left. - \lambda(\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy} \phi_{yz} \phi_{zx}) + 8M_y^2 f_y^2(M_y)(1 - \phi_{zx}^2) \right]$
M_9	$\frac{0.125(1 - \phi_{zx})^{-1}}{M_y f_y^2(M_y)} \left[\lambda \Delta_{2x} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} (\phi_{xy} - \phi_{yz} \phi_{zx}) + \lambda \Delta_{2z} \frac{M_y f_y(M_y)}{M_z f_z(M_z)} (\phi_{yz} - \phi_{xy} \phi_{zx}) \right. \\ \left. - \lambda(\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy} \phi_{yz} \phi_{zx}) + 8M_y^2 f_y^2(M_y)(1 - \phi_{zx}^2) \right]$
M_{10}	$\frac{0.125(1 - \phi_{zx})^{-1}}{M_y f_y^2(M_y)} \left[\lambda \Delta_{3x} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} (\phi_{xy} - \phi_{yz} \phi_{zx})^{-1} + \lambda \Delta_{3z} \frac{M_y f_y(M_y)}{M_z f_z(M_z)} (\phi_{yz} - \phi_{xy} \phi_{zx}) \right. \\ \left. - \lambda(\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy} \phi_{yz} \phi_{zx}) + 8M_y^2 f_y^2(M_y)(1 - \phi_{zx}^2) \right]$
M_{11}	$\frac{0.125(1 - \phi_{zx})^{-1}}{M_y f_y^2(M_y)} \left[\lambda \Delta_{4x} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} (\phi_{xy} - \phi_{yz} \phi_{zx}) + \lambda \Delta_{4z} \frac{M_y f_y(M_y)}{M_z f_z(M_z)} (\phi_{yz} - \phi_{xy} \phi_{zx}) \right. \\ \left. - \lambda(\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy} \phi_{yz} \phi_{zx}) + 8M_y^2 f_y^2(M_y)(1 - \phi_{zx}^2) \right]$

To be continued

Table S1

Est	Standard deviation
M_1	$0.5\sqrt{\lambda f_y^2(M_y)}$, where $\lambda = \frac{N-n}{Nn}$
M_2	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left[1 + \frac{M_y^2 f_y^2(M_y)}{M_x^2 f_x^2(M_x)} \right] \left\{ 1 - 2\phi_{xy} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} \right\}}$
M_3	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left[1 + \Delta_{1x} \frac{M_y^2 f_y^2(M_y)}{M_x^2 f_x^2(M_x)} \right] \left\{ \Delta_{1x} - 2\phi_{xy} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} \right\}}$
M_4	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left[1 + \Delta_{2x} \frac{M_y^2 f_y^2(M_y)}{M_x^2 f_x^2(M_x)} \right] \left\{ \Delta_{2x} - 2\phi_{xy} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} \right\}}$
M_5	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left[1 + \Delta_{3x} \frac{M_y^2 f_y^2(M_y)}{M_x^2 f_x^2(M_x)} \right] \left\{ \Delta_{3x} - 2\phi_{xy} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} \right\}}$
M_6	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left[1 + \Delta_{4x} \frac{M_y^2 f_y^2(M_y)}{M_x^2 f_x^2(M_x)} \right] \left\{ \Delta_{4x} - 2\phi_{xy} \frac{M_y f_y(M_y)}{M_x f_x(M_x)} \right\}}$
M_7	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left(1 - \frac{\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy}\phi_{yz}\phi_{zx}}{1 - \phi_{zx}^2} \right)}$
M_8	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left(1 - \frac{\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy}\phi_{yz}\phi_{zx}}{1 - \phi_{zx}^2} \right)}$
M_9	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left(1 - \frac{\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy}\phi_{yz}\phi_{zx}}{1 - \phi_{zx}^2} \right)}$
M_{10}	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left(1 - \frac{\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy}\phi_{yz}\phi_{zx}}{1 - \phi_{zx}^2} \right)}$
M_{11}	$\sqrt{\frac{0.25\lambda}{f_y^2(M_y)} \left(1 - \frac{\phi_{xy}^2 + \phi_{yz}^2 - 2\phi_{xy}\phi_{yz}\phi_{zx}}{1 - \phi_{zx}^2} \right)}$

Table S2 Average run length values of CUSUM median and mean charts for the process following the normal distribution

$\rho_{xy} = 0.50, \rho_{yz} = 0.40, \rho_{zx} = 0.70, \text{ and } n = 10$											
τ	M_1	T_1	M_2	T_2	M_3	T_3	M_4	T_4	M_5	T_5	M_6
0	199.97	200.16	200.53	199.95	199.97	200.30	199.88	200.00	200.21	200.05	200.39
0.05	97.65	88.42	108.62	89.03	106.78	88.05	95.72	77.94	108.10	88.49	93.46
0.10	50.66	42.01	61.66	44.43	60.38	43.38	50.14	36.24	61.05	44.08	48.18
0.15	30.14	23.73	36.69	24.97	35.97	23.74	29.14	19.82	36.44	24.48	27.31
0.20	19.51	14.94	24.86	15.92	23.82	15.49	18.64	12.26	24.46	15.84	17.81
0.25	13.55	10.64	17.44	11.38	16.74	10.81	12.94	8.70	17.21	11.20	12.04
0.40	6.72	5.34	8.50	5.64	8.22	5.54	6.52	4.53	8.41	5.62	6.00
0.50	4.88	3.99	6.16	4.32	6.01	4.17	4.74	3.45	6.11	4.26	4.51
0.60	3.83	3.20	4.92	3.43	4.78	3.35	3.83	2.83	4.85	3.40	3.57
0.75	3.00	2.52	3.74	2.71	3.65	2.66	2.93	2.22	3.66	2.69	2.78
1.00	2.22	1.90	2.70	2.05	2.65	2.01	2.19	1.72	2.69	2.02	2.08
1.25	1.79	1.53	2.19	1.68	2.12	1.65	1.78	1.33	2.17	1.67	1.69
1.50	1.49	1.23	1.87	1.39	1.82	1.35	1.50	1.10	1.85	1.38	1.37
2.00	1.08	1.01	1.41	1.05	1.36	1.04	1.08	1.00	1.39	1.04	1.04
3.00	1.00	1.00	1.01	1.00	1.01	1.00	1.00	1.00	1.01	1.00	1.00
τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
0	199.98	200.06	200.01	199.97	200.07	199.76	200.02	200.39	199.98	199.85	200.07
0.05	79.12	112.22	77.76	82.97	77.54	83.47	77.92	93.46	79.12	83.77	78.50
0.10	36.66	66.11	36.08	39.75	34.87	40.34	35.17	48.18	36.66	40.58	35.26
0.15	20.40	40.83	19.64	21.73	18.90	21.65	19.00	27.31	20.40	21.62	19.22
0.20	12.62	27.01	12.22	13.57	11.81	13.63	11.84	17.81	12.62	13.71	11.87
0.25	8.82	19.53	8.63	9.59	8.46	9.61	8.44	12.04	8.82	9.60	8.45
0.40	4.56	9.04	4.51	4.95	4.40	4.90	4.40	6.00	4.56	4.88	4.39
0.50	3.48	6.66	3.44	3.74	3.38	3.73	3.36	4.51	3.48	3.70	3.36
0.60	2.83	5.18	2.78	3.01	2.72	2.99	2.71	3.57	2.83	2.96	2.70
0.75	2.24	3.86	2.22	2.41	2.19	2.39	2.17	2.78	2.24	2.35	2.16
1.00	1.71	2.79	1.70	1.84	1.69	1.81	1.68	2.08	1.71	1.79	1.66
1.25	1.33	2.21	1.32	1.46	1.30	1.43	1.29	1.69	1.33	1.41	1.27
1.50	1.09	1.88	1.09	1.18	1.07	1.16	1.07	1.37	1.09	1.15	1.07
2.00	1.00	1.38	1.00	1.00	1.00	1.00	1.00	1.04	1.00	1.00	1.00
3.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

To be continued

Table S3 Average run length values of CUSUM median and mean charts for the process following Student's t distribution

$\rho_{xy} = 0.50, \rho_{yz} = 0.50, \rho_{zx} = 0.10, \text{ and } n = 10$											
τ	M_1	T_1	M_2	T_2	M_3	T_3	M_4	T_4	M_5	T_5	M_6
0	200.13	199.95	199.92	199.97	199.97	200.08	200.25	200.00	199.92	199.95	199.92
0.05	87.69	91.29	97.59	89.25	95.35	87.52	81.02	77.16	97.17	88.52	82.86
0.10	41.62	43.83	51.26	44.01	50.12	42.85	37.29	35.65	51.03	43.71	39.09
0.15	23.42	25.00	29.56	25.34	28.33	24.62	21.33	19.21	29.39	25.26	22.11
0.20	14.89	15.82	18.97	15.94	18.45	15.32	13.28	12.24	18.93	15.76	13.7
0.25	10.49	10.80	13.35	11.58	12.77	11.17	9.40	8.66	13.24	11.55	9.77
0.40	5.17	5.33	6.52	5.72	6.32	5.60	4.82	4.38	6.50	5.68	4.93
0.50	3.89	4.06	4.96	4.27	4.77	4.21	3.61	3.36	4.94	4.25	3.70
0.60	3.17	3.25	3.97	3.46	3.80	3.37	2.92	2.76	3.94	3.46	3.01
0.75	2.48	2.53	3.04	2.72	2.95	2.64	2.33	2.18	3.03	2.71	2.37
1.00	1.89	1.94	2.27	2.07	2.21	2.01	1.78	1.67	2.27	2.06	1.81
1.25	1.54	1.55	1.86	1.70	1.82	1.64	1.40	1.29	1.86	1.69	1.44
1.50	1.21	1.24	1.59	1.39	1.54	1.34	1.12	1.07	1.58	1.39	1.15
2.00	1.01	1.01	1.14	1.05	1.12	1.04	1.00	1.00	1.14	1.04	1.01
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
0	199.99	199.84	199.4	199.82	199.85	199.99	200.08	199.84	199.99	199.94	199.85
0.05	83.23	92.71	104.45	75.59	67.13	76.36	67.72	75.60	67.19	76.42	68.59
0.10	39.30	47.10	54.53	34.07	28.38	34.34	28.97	34.03	28.39	34.49	29.19
0.15	21.93	26.19	31.84	18.14	15.09	18.46	15.11	18.14	15.06	18.58	15.14
0.20	13.73	17.29	20.55	11.72	9.54	11.73	9.58	11.70	9.55	11.73	9.62
0.25	9.59	11.76	14.31	8.28	6.88	8.30	6.85	8.28	6.89	8.30	6.83
0.40	4.90	5.93	6.88	4.33	3.71	4.29	3.64	4.34	3.71	4.27	3.61
0.50	3.67	4.41	5.07	3.31	2.90	3.27	2.86	3.32	2.91	3.25	2.80
0.60	2.98	3.50	4.02	2.73	2.38	2.70	2.33	2.73	2.37	2.68	2.31
0.75	2.36	2.76	3.08	2.17	1.95	2.15	1.91	2.17	1.96	2.14	1.89
1.00	1.80	2.06	2.27	1.66	1.45	1.65	1.42	1.67	1.46	1.64	1.38
1.25	1.43	1.68	1.87	1.29	1.12	1.28	1.10	1.29	1.12	1.27	1.09
1.50	1.14	1.37	1.56	1.06	1.01	1.06	1.01	1.06	1.02	1.05	1.01
2.00	1.00	1.04	1.11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

To be continued

Table S4 Average run length values of CUSUM median and mean charts for the process following the log-normal distribution

$\rho_{xy} = 0.90, \rho_{yz} = 0.60, \rho_{zx} = 0.30, \text{ and } n = 10$											
τ	M_1	T_1	M_2	T_2	M_3	T_3	M_4	T_4	M_5	T_5	M_6
0	200.26	200.02	200.46	200.12	200.3	200.38	200.2	200.04	200.38	197.38	200.27
0.05	137.09	126.01	107.48	58.47	109.27	61.09	137.78	127.63	107.4	57.43	139.68
0.10	97.56	75.37	56.04	18.54	58.14	19.23	100.27	72.68	56.06	18.58	101.44
0.15	64.89	43.52	28.95	9.20	30.27	9.42	66.03	41.36	28.95	9.19	67.52
0.20	43.54	25.58	15.00	5.91	16.48	6.06	44.8	23.92	14.95	5.92	45.8
0.25	28.33	16.66	7.93	4.45	8.30	4.47	28.32	15.78	7.97	4.39	29.1
0.40	7.35	7.37	1.75	2.60	1.78	2.61	7.27	6.96	1.75	2.56	7.47
0.50	3.73	5.29	1.19	2.11	1.19	2.11	3.51	5.02	1.19	2.11	3.71
0.60	2.34	4.13	1.06	1.82	1.05	1.85	2.25	3.97	1.05	1.83	2.37
0.75	1.58	3.19	1.01	1.37	1.01	1.39	1.50	3.07	1.01	1.35	1.55
1.00	1.03	2.36	1.00	1.06	1.00	1.06	1.01	2.28	1.00	1.06	1.03
1.25	1.00	1.93	1.00	1.01	1.00	1.01	1.00	1.90	1.00	1.01	1.00
1.50	1.00	1.76	1.00	1.00	1.00	1.00	1.00	1.70	1.00	1.00	1.00
2.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00	1.02	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
0	199.95	199.48	200.09	200.11	200.2	199.76	200.04	199.95	200.34	200.05	200.04
0.05	128.99	131.07	102.33	101.27	43.63	98.96	48.44	101.36	43.53	99.15	48.74
0.10	76.05	83.43	50.67	52.35	12.53	50.08	12.97	52.59	12.51	49.9	13.07
0.15	43.25	50.87	25.44	27.45	6.48	26.08	6.61	27.62	6.49	26.04	6.62
0.20	25.81	32.45	14.88	15.09	4.46	13.97	4.33	15.12	4.47	13.98	4.31
0.25	16.68	20.17	9.98	8.08	3.40	7.48	3.22	8.12	3.40	7.48	3.22
0.40	7.27	4.60	4.95	1.74	2.09	1.67	2.00	1.73	2.09	1.67	2.00
0.50	5.27	2.33	3.70	1.07	1.81	1.05	1.64	1.07	1.81	1.05	1.64
0.60	4.14	1.53	3.01	1.00	1.41	1.00	1.24	1.00	1.42	1.00	1.24
0.75	3.19	1.06	2.37	1.00	1.06	1.00	1.04	1.00	1.06	1.00	1.03
1.00	2.35	1.00	1.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.92	1.00	1.59	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.77	1.00	1.12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

To be continued

Table S5 Performance measures of CUSUM median control charts for $\rho_{xy}=-0.20$, $\rho_{yz}=-0.10$, and $\rho_{zx}=-0.10$

n	Model	Measure	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}
5	Normal	EQL	4.894	7.854	7.623	5.945	7.774	4.906	4.871	4.871	3.923	4.871	4.843
		RARL	1.363	2.152	2.089	1.632	2.130	1.361	1.354	1.354	1.000	1.354	1.349
		PCI	1.248	2.002	1.943	1.515	1.982	1.251	1.242	1.242	1.000	1.242	1.235
	Student's t	EQL	4.268	6.737	6.554	4.631	6.698	4.238	4.246	4.246	4.239	4.246	4.235
		RARL	1.009	1.593	1.548	1.097	1.584	1.000	1.002	1.002	1.001	1.002	1.000
		PCI	1.008	1.591	1.548	1.093	1.582	1.001	1.003	1.003	1.001	1.003	1.000
	Log-normal	EQL	3.913	4.161	4.144	3.922	4.160	3.926	3.901	3.901	3.902	3.901	3.901
		RARL	1.005	1.087	1.081	1.008	1.087	1.010	1.000	1.000	1.000	1.000	1.000
		PCI	1.003	1.067	1.062	1.006	1.066	1.006	1.000	1.000	1.000	1.000	1.000
10	Normal	EQL	3.649	5.063	4.926	4.018	5.015	3.650	3.629	3.628	3.625	3.629	3.624
		RARL	1.010	1.463	1.423	1.144	1.449	1.011	1.002	1.001	1.000	1.002	1.000
		PCI	1.007	1.397	1.359	1.109	1.384	1.007	1.001	1.001	1.000	1.001	1.000
	Student's t	EQL	3.392	4.216	4.130	3.479	4.199	3.375	3.378	3.378	3.376	3.379	3.377
		RARL	1.010	1.374	1.341	1.056	1.368	1.000	1.001	1.001	1.000	1.001	1.000
		PCI	1.005	1.249	1.224	1.031	1.244	1.000	1.001	1.001	1.000	1.001	1.000
	Log-normal	EQL	3.241	3.289	3.285	3.249	3.288	3.249	3.242	3.243	3.242	3.242	3.242
		RARL	1.000	1.045	1.041	1.006	1.044	1.006	1.000	1.000	1.000	1.000	1.000
		PCI	1.000	1.015	1.014	1.002	1.015	1.002	1.000	1.000	1.000	1.000	1.000
15	Normal	EQL	3.408	4.330	4.232	3.651	4.297	3.419	3.401	3.401	3.398	3.400	3.397
		RARL	1.008	1.396	1.361	1.128	1.384	1.013	1.001	1.001	1.000	1.001	1.000
		PCI	1.003	1.275	1.246	1.075	1.265	1.006	1.001	1.001	1.000	1.001	1.000
	Student's t	EQL	3.233	3.729	3.676	3.298	3.719	3.235	3.230	3.230	3.229	3.231	3.228
		RARL	1.003	1.311	1.284	1.052	1.306	1.006	1.001	1.001	1.000	1.001	1.000
		PCI	1.001	1.155	1.139	1.021	1.152	1.002	1.001	1.001	1.000	1.001	1.000
	Log-normal	EQL	3.147	3.165	3.164	3.146	3.165	3.147	3.144	3.144	3.144	3.144	3.144
		RARL	1.004	1.034	1.031	1.003	1.033	1.004	1.000	1.000	1.000	1.000	1.000
		PCI	1.001	1.007	1.006	1.001	1.007	1.001	1.000	1.000	1.000	1.000	1.000

Table S6 Performance measures of CUSUM median control charts for $\rho_{xy}=0.50$, $\rho_{yz}=0.50$, and $\rho_{zx}=0.10$

n	Model	Measure	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}
5	Normal	EQL	4.894	6.631	6.423	5.024	6.563	4.541	5.495	4.345	4.291	4.349	4.249
		RARL	1.166	1.555	1.507	1.186	1.540	1.076	1.312	1.018	1.008	1.020	1.000
		PCI	1.152	1.561	1.512	1.182	1.545	1.069	1.293	1.023	1.010	1.024	1.000
	Student's t	EQL	4.262	5.671	5.481	4.077	5.635	4.092	4.740	3.880	3.845	3.881	3.821
		RARL	1.146	1.515	1.465	1.083	1.506	1.091	1.286	1.015	1.006	1.015	1.000
		PCI	1.116	1.484	1.434	1.067	1.475	1.071	1.241	1.015	1.006	1.016	1.000
	Log-normal	EQL	3.913	3.841	3.837	3.903	3.842	3.922	4.055	3.667	3.664	3.666	3.665
		RARL	1.119	1.077	1.075	1.115	1.077	1.123	1.181	1.002	1.000	1.002	1.000
		PCI	1.068	1.048	1.047	1.065	1.049	1.070	1.107	1.001	1.000	1.001	1.000
10	Normal	EQL	3.649	4.288	4.181	3.639	4.251	3.527	3.872	3.428	3.419	3.430	3.408
		RARL	1.125	1.370	1.333	1.115	1.357	1.065	1.230	1.008	1.005	1.010	1.000
		PCI	1.071	1.258	1.227	1.068	1.247	1.035	1.136	1.006	1.003	1.007	1.000
	Student's t	EQL	3.391	3.746	3.682	3.318	3.732	3.338	3.521	3.258	3.252	3.258	3.248
		RARL	1.104	1.297	1.266	1.052	1.291	1.067	1.187	1.006	1.002	1.006	1.000
		PCI	1.044	1.153	1.134	1.022	1.149	1.028	1.084	1.003	1.001	1.003	1.000
	Log-normal	EQL	3.241	3.206	3.207	3.245	3.206	3.248	3.283	3.181	3.180	3.181	3.180
		RARL	1.072	1.035	1.035	1.074	1.034	1.079	1.118	1.001	1.000	1.002	1.000
		PCI	1.019	1.008	1.009	1.021	1.008	1.022	1.032	1.000	1.000	1.000	1.000
15	Normal	EQL	3.408	3.843	3.780	3.426	3.821	3.341	3.544	3.289	3.282	3.291	3.276
		RARL	1.092	1.312	1.282	1.097	1.302	1.046	1.173	1.007	1.003	1.008	1.000
		PCI	1.040	1.173	1.154	1.046	1.166	1.020	1.082	1.004	1.002	1.004	1.000
	Student's t	EQL	3.230	3.465	3.432	3.208	3.459	3.210	3.313	3.170	3.166	3.170	3.164
		RARL	1.066	1.241	1.219	1.044	1.237	1.047	1.138	1.005	1.002	1.005	1.000
		PCI	1.021	1.095	1.085	1.014	1.093	1.014	1.047	1.002	1.001	1.002	1.000
	Log-normal	EQL	3.147	3.123	3.123	3.144	3.123	3.146	3.166	3.110	3.109	3.109	3.109
		RARL	1.058	1.027	1.026	1.054	1.027	1.057	1.089	1.001	1.000	1.001	1.000
		PCI	1.012	1.005	1.004	1.011	1.004	1.012	1.018	1.000	1.000	1.000	1.000

Table S7 Performance measures of CUSUM median control charts for $\rho_{xy}=0.60$, $\rho_{yz}=0.60$, and $\rho_{zx}=0.60$

n	Model	Measure	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}
5	Normal	EQL	4.894	6.118	5.922	4.652	6.051	4.410	5.191	4.242	4.200	4.244	4.162
		RARL	1.195	1.464	1.418	1.121	1.448	1.069	1.268	1.016	1.007	1.017	1.000
		PCI	1.176	1.470	1.423	1.118	1.454	1.060	1.247	1.019	1.009	1.020	1.000
	Student's t	EQL	4.265	5.200	5.014	3.922	5.170	4.024	4.492	3.821	3.788	3.825	3.767
		RARL	1.169	1.414	1.365	1.053	1.406	1.090	1.239	1.014	1.005	1.015	1.000
		PCI	1.132	1.380	1.331	1.041	1.372	1.068	1.192	1.014	1.006	1.015	1.000
	Log-normal	EQL	3.913	3.726	3.725	3.898	3.725	3.920	4.002	3.624	3.621	3.625	3.621
		RARL	1.144	1.046	1.046	1.137	1.045	1.147	1.184	1.002	1.000	1.002	1.000
		PCI	1.080	1.029	1.029	1.076	1.029	1.082	1.105	1.001	1.000	1.001	1.000
10	Normal	EQL	3.649	4.021	3.932	3.514	3.989	3.479	3.756	3.399	3.388	3.399	3.380
		RARL	1.144	1.289	1.255	1.071	1.276	1.057	1.195	1.008	1.003	1.008	1.000
		PCI	1.079	1.189	1.163	1.040	1.180	1.029	1.111	1.006	1.002	1.006	1.000
	Student's t	EQL	3.391	3.584	3.535	3.276	3.574	3.313	3.455	3.241	3.235	3.243	3.231
		RARL	1.120	1.229	1.201	1.035	1.224	1.065	1.162	1.006	1.002	1.007	1.000
		PCI	1.049	1.109	1.094	1.014	1.106	1.025	1.069	1.003	1.001	1.004	1.000
	Log-normal	EQL	3.241	3.175	3.176	3.243	3.175	3.248	3.263	3.170	3.168	3.169	3.168
		RARL	1.088	1.013	1.013	1.087	1.013	1.094	1.114	1.002	1.000	1.001	1.000
		PCI	1.023	1.002	1.002	1.023	1.002	1.025	1.030	1.000	1.000	1.000	1.000
15	Normal	EQL	3.408	3.683	3.633	3.356	3.667	3.316	3.480	3.270	3.264	3.270	3.260
		RARL	1.106	1.250	1.224	1.067	1.242	1.042	1.149	1.005	1.002	1.005	1.000
		PCI	1.046	1.130	1.114	1.029	1.125	1.017	1.067	1.003	1.001	1.003	1.000
	Student's t	EQL	3.230	3.378	3.348	3.184	3.371	3.200	3.273	3.159	3.155	3.159	3.154
		RARL	1.077	1.191	1.170	1.031	1.187	1.049	1.117	1.003	1.001	1.003	1.000
		PCI	1.024	1.071	1.061	1.009	1.069	1.015	1.038	1.002	1.000	1.001	1.000
	Log-normal	EQL	3.147	3.107	3.107	3.143	3.107	3.146	3.160	3.103	3.103	3.103	3.103
		RARL	1.071	1.011	1.011	1.065	1.011	1.069	1.092	1.001	1.000	1.001	1.000
		PCI	1.014	1.001	1.001	1.013	1.001	1.014	1.018	1.000	1.000	1.000	1.000

Table S9 Contaminated average run length values of CUSUM median and mean charts for the process following the normal distribution

$\rho_{xy} = 0.50, \rho_{yz} = 0.40, \rho_{zx} = 0.70, \text{ and } n = 10$												
Scenario	τ	M_1	T_1	M_2	T_2	M_3	T_3	M_7	M_4	M_5	T_5	M_6
II	0	93.0	77.2	105.4	81	103.9	79.8	91.8	68.2	105	80.5	90.1
	0.10	32.3	24.8	38.6	25.7	37.3	24.8	30.4	20.1	38.1	25.3	28.7
	0.15	20.7	15.8	26.1	16.6	25.1	15.9	19.8	13.0	25.8	16.4	18.5
	0.20	14.9	11.2	18.7	11.9	18.2	11.5	14.0	9.2	18.5	11.7	13.1
	0.25	11.1	8.4	14.1	8.9	13.7	8.7	10.5	7.1	13.9	8.9	9.9
III	0	188.6	191.6	185.8	177.4	184.1	175.4	179.3	169.5	185.4	176.2	174.6
	0.10	51.9	42.7	60.5	44.2	58.9	43.2	48.7	35.3	60.0	43.8	47.3
	0.15	31.7	25.1	37.1	25.8	36.2	24.9	28.9	20.2	37.0	25.4	27.5
	0.20	20.5	16.0	25.5	16.3	24.4	15.8	19.2	13.1	25.0	16.2	18.0
	0.25	14.6	11.4	18.0	12.0	17.2	11.6	13.5	9.3	17.7	11.8	12.9
IV	0	97.7	73.0	113.9	72.4	112.2	71.1	100.8	60.4	113.6	71.8	98.9
	0.10	33.0	24.5	41.5	25.0	40.3	24.1	32.3	19.1	41.0	24.7	30.3
	0.15	21.5	15.9	27.9	16.1	27.1	15.7	21.1	13.0	27.6	15.9	19.8
	0.20	15.4	11.1	19.7	11.8	18.8	11.3	14.9	9.1	19.5	11.7	13.9
	0.25	11.2	8.4	14.7	8.9	14.1	8.7	11.1	7.0	14.4	8.9	10.2
Scenario	τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
II	0	69.4	92.0	68.9	88.7	67.2	88.7	67.5	88.7	67.3	89	67.7
	0.10	20.9	29.2	19.7	27.4	18.9	27.5	19.0	27.4	18.9	27.7	19.3
	0.15	13.3	19.2	12.8	18.0	12.5	18.0	12.6	18.0	12.5	18.1	12.6
	0.20	9.3	13.6	9.2	12.9	8.9	13.0	8.9	12.9	8.9	13.0	8.9
	0.25	7.3	10.4	6.9	9.7	6.9	9.6	6.8	9.6	6.9	9.7	6.8
III	0	164.5	179.2	166.5	179.4	163.7	179.4	163.8	179.4	163.8	180	165.1
	0.10	36.0	46.3	34.6	44.4	33.7	44.5	33.8	44.4	33.7	44.6	34.0
	0.15	20.6	29.1	20.1	27.0	19.5	27.1	19.6	26.9	19.5	27.4	19.7
	0.20	13.3	19.0	12.9	17.7	12.4	17.7	12.5	17.7	12.4	17.8	12.6
	0.25	9.6	13.4	9.3	12.5	9.1	12.5	9.1	12.5	9.1	12.6	9.0
IV	0	61.4	101.3	60.1	98.4	58.5	97.8	58.4	98.3	58.5	98.4	58.7
	0.10	19.7	31.4	19.2	29.7	18.6	29.7	18.7	29.7	18.6	29.9	18.7
	0.15	13.2	20.4	12.6	19.3	12.2	19.4	12.2	19.3	12.2	19.4	12.2
	0.20	9.4	14.5	9.1	13.4	8.8	13.4	8.8	13.4	8.8	13.6	8.9
	0.25	7.1	10.7	7.1	10.0	6.8	10.0	6.8	10.0	6.8	10.0	6.8

To be continued

Table S9

$\rho_{xy} = 0.30, \rho_{yz} = 0.30, \rho_{zx} = 0.30, \text{ and } n = 15$												
Scenario	τ	M_1	T_1	M_2	T_2	M_3	T_3	M_7	M_4	M_5	T_5	M_6
II	0	88.1	67.3	105.3	80.7	104	79.2	93.6	67.8	104.9	80.2	88.1
	0.10	26.0	18.2	37.9	24.5	36.7	23.5	28.9	18.1	37.4	24.2	25.2
	0.15	16.1	11.5	25.3	15.5	24.7	15.0	18.8	12.0	25.2	15.4	16.3
	0.20	11.5	8.3	18.0	11.1	17.3	11.0	13.1	8.5	17.8	11.0	11.2
	0.25	8.7	6.3	13.4	8.6	12.9	8.2	10.1	6.5	13.2	8.5	8.5
III	0	192.6	188.9	190.4	184.5	190.2	185.5	186.3	175.6	190	184.9	183.5
	0.10	44.7	34.1	59.4	41.6	58.1	40.4	48.1	33.3	59.2	41.3	42.1
	0.15	25.5	18.3	37.3	24.7	36.0	24.0	28.0	18.2	36.8	24.4	24
	0.20	16.3	11.7	25.0	16.0	24.3	15.4	18.5	12.0	24.7	15.8	15.7
	0.25	11.6	8.4	17.4	11.3	16.8	11.0	12.9	8.7	17.3	11.2	11.2
IV	0	94.1	64.1	116.7	74.8	116.1	73.3	106.5	61.4	116.1	74.3	99.0
	0.10	26.8	18.1	41.0	23.6	39.8	22.8	31.4	17.7	40.5	23.4	27.6
	0.15	16.7	11.5	27.2	15.3	26.3	15.1	19.9	11.8	26.9	15.1	17.4
	0.20	11.9	8.2	18.9	11.2	18.5	10.7	14.1	8.5	18.7	11.1	11.8
	0.25	9.0	6.4	14	8.5	13.5	8.2	10.4	6.6	13.8	8.3	9.0
Scenario	τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
II	0	63.5	87.5	67.7	82.1	58.9	82.3	59.1	82.1	59.0	82.5	59.2
	0.10	16.5	26.1	18.9	23.8	15.8	23.9	15.9	23.8	15.9	23.9	15.9
	0.15	10.8	16.5	12.1	15.2	10.1	15.3	10.1	15.2	10.1	15.2	10.1
	0.20	7.7	11.7	8.8	10.7	7.3	10.7	7.3	10.6	7.3	10.7	7.3
	0.25	6.1	9.2	6.7	8.2	5.7	8.2	5.6	8.2	5.7	8.2	5.7
III	0	172.4	175.3	170.1	173.5	163.6	173.2	163.8	173.3	163.8	173.6	163.5
	0.10	30.6	45.1	35.1	41.2	28.9	41.2	28.9	41.1	28.9	41.4	29.0
	0.15	16.5	25.4	19.6	23.3	15.9	23.4	16.0	23.3	16.0	23.5	16.0
	0.20	10.7	16.6	12.5	15.1	10.4	15.1	10.4	15.1	10.4	15.0	10.4
	0.25	7.7	11.5	8.9	10.6	7.5	10.6	7.5	10.6	7.5	10.6	7.4
IV	0	57.0	97.7	61.0	93.0	53.2	93.2	53.3	92.9	53.3	93.2	53.2
	0.10	16.1	28.5	18.4	25.8	15.3	25.8	15.4	25.7	15.4	25.9	15.4
	0.15	10.6	17.9	12	16.2	10.1	16.2	10.1	16.2	10.1	16.2	10.1
	0.20	7.7	12.7	8.7	11.6	7.3	11.6	7.3	11.6	7.3	11.6	7.3
	0.25	6.0	9.5	6.7	8.6	5.7	8.7	5.6	8.6	5.7	8.7	5.6

Table S10 Contaminated average run length values of CUSUM median and mean charts for the process following Student's t distribution

$\rho_{xy} = 0.75, \rho_{yz} = 0.10, \rho_{zx} = 0.10, \text{ and } n = 10$												
Scenario	τ	M_1	T_1	M_2	T_2	M_3	T_3	M_7	M_4	M_5	T_5	M_6
II	0	33.8	34.4	32.3	27.5	31.9	27.3	29.9	27.9	32.2	27.5	31.6
	0.10	16.2	16.6	15.2	11.8	14.9	11.7	13.6	12.3	15.5	11.7	15.8
	0.15	12.3	12.4	11.1	8.6	11.0	8.4	9.7	8.8	11	8.6	11
	0.20	9.6	9.7	8.7	6.6	8.4	6.4	7.1	6.5	8.6	6.5	8.7
	0.25	7.6	7.8	7.0	5.3	6.9	5.2	5.9	5.2	7.0	5.3	6.8
III	0	52.7	55.0	53.2	52.6	53.1	52.4	51.5	49.6	53.2	52.7	50.5
	0.10	23.2	23.7	21.8	18.2	21.2	17.5	19.3	18.3	21.4	17.6	21.6
	0.15	16.5	17.0	15.1	12.1	14.3	11.9	13.1	11.9	14.9	12.2	15
	0.20	11.9	12.6	11.5	8.6	11.0	8.3	9.5	8.6	11.2	8.5	11.2
	0.25	9.4	10.0	8.7	6.8	8.5	6.5	7.3	6.7	8.7	6.7	8.6
IV	0	34.9	33.2	35.3	26.9	35.0	26.4	32.7	25.9	35.2	26.9	33.8
	0.10	16.8	16.4	16.3	11.8	15.7	11.4	14.7	11.7	16.0	11.7	14.3
	0.15	12.7	12.2	11.7	8.6	11.4	8.3	10.1	8.7	11.9	8.5	11.6
	0.20	9.7	9.6	9.0	6.6	8.8	6.4	7.9	6.5	9.0	6.5	9.0
	0.25	7.8	7.8	7.1	5.3	7.0	5.2	6.2	5.2	7.1	5.3	6.9
Scenario	τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
II	0	31.7	30	26.5	29.8	26.1	30.2	26.9	29.8	26	30.4	27.1
	0.10	15.4	12.9	10.8	12.9	10.7	13	10.8	12.8	10.6	13.1	10.9
	0.15	11.4	9.4	7.7	9.4	7.6	9.5	7.6	9.4	7.7	9.5	7.7
	0.20	8.7	7.2	5.9	7.2	5.8	7.3	5.9	7.2	5.9	7.3	5.9
	0.25	6.8	5.8	4.7	5.7	4.7	5.8	4.7	5.8	4.7	5.8	4.7
III	0	50.2	51.1	51.2	51.2	50.7	51.8	52	51.2	50.5	52.3	52.6
	0.10	22.3	18.5	16.3	18.5	16.3	19	16.5	18.5	16.1	19.3	17
	0.15	15.3	12.7	10.8	12.6	10.8	12.7	10.7	12.5	10.8	13	10.9
	0.20	10.8	9.3	7.7	9.3	7.7	9.3	7.9	9.3	7.7	9.4	7.9
	0.25	8.7	7.1	6	7.1	6	7.1	5.9	7.1	5.9	7.1	6
IV	0	29.3	32.8	24.7	32.5	24.6	33.1	25.2	32.5	24.6	33.2	25.5
	0.10	14.9	13.8	10.5	13.8	10.5	13.9	10.6	13.8	10.5	13.8	10.6
	0.15	11.2	9.8	7.7	9.8	7.7	9.9	7.7	9.8	7.7	10	7.7
	0.20	8.6	7.7	5.8	7.6	5.9	7.6	5.9	7.6	5.9	7.7	5.8
	0.25	6.8	6.1	4.8	6	4.7	6.1	4.7	6	4.7	6.1	4.7

To be continued

Table S10

$\rho_{xy} = 0.30, \rho_{yz} = 0.30, \rho_{zx} = 0.30, \text{ and } n = 15$												
Scenario	τ	M_1	T_1	M_2	T_2	M_3	T_3	M_7	M_4	M_5	T_5	M_6
II	0	31.3	31.0	36.9	34.4	36.4	34.0	32.9	30.2	36.7	34.2	32.5
	0.10	14.0	13.6	18.4	16.7	17.9	16.4	14.1	13.2	18.4	16.6	13.9
	0.15	10.4	9.9	14.0	12.5	13.7	12.4	10.6	9.6	13.9	12.5	10.4
	0.20	7.8	7.6	11.2	10.0	10.7	9.7	8.2	7.5	11.1	9.9	7.7
	0.25	6.3	6.0	8.8	7.9	8.6	7.7	6.6	5.9	8.9	7.9	6.3
III	0	52.6	52.9	54.0	54.4	53.9	54.3	54.6	52.0	53.9	54.4	54.8
	0.10	20.1	20.0	25.0	24.3	24.7	24.0	20.9	19.3	24.6	24.1	19.9
	0.15	13.8	13.9	17.8	16.6	17.5	16.3	14.2	13.4	17.8	16.6	13.5
	0.20	10.1	10.1	13.7	12.3	13.3	12.3	10.3	9.4	13.5	12.4	10.0
	0.25	7.6	7.6	10.8	9.7	10.4	9.7	7.9	7.4	10.8	9.7	7.6
IV	0	32.6	30.1	39.6	33.1	39.2	32.6	35.9	28.5	39.5	32.9	35.2
	0.10	14.5	13.3	19.6	16.6	19.0	16.2	15	12.7	19.4	16.6	14.8
	0.15	10.6	9.8	14.5	12.6	14.3	12.2	11.2	9.5	14.5	12.4	10.9
	0.20	8.0	7.5	11.4	9.8	11.1	9.6	8.4	7.4	11.2	9.8	8.0
	0.25	6.4	6.0	9.2	7.8	8.9	7.6	6.8	5.9	9.1	7.8	6.5
Scenario	τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
II	0	29.9	32.1	31.9	31.8	30.2	32.1	30.5	31.8	30.2	32.3	30.7
	0.10	13.0	14.3	13.9	13.6	12.6	13.6	12.5	13.6	12.6	13.7	12.5
	0.15	9.6	10.4	10.4	9.8	9.0	9.8	9.0	9.8	9.0	9.9	9.1
	0.20	7.4	7.8	7.8	7.5	6.9	7.5	6.9	7.5	6.9	7.5	6.9
	0.25	5.8	6.4	6.4	6.1	5.6	6.1	5.6	6.1	5.6	6.1	5.6
III	0	50.3	51.5	51.8	52.7	51.8	53.3	52.4	52.7	51.9	53.6	52.6
	0.10	19.9	20.0	20.3	19.2	18.4	19.2	18.5	19.2	18.5	19.3	18.5
	0.15	13.3	13.8	14.2	13.2	12.4	13.2	12.5	13.2	12.5	13.3	12.6
	0.20	9.4	10.1	10.5	9.6	9.1	9.6	9.2	9.6	9.2	9.7	9.2
	0.25	7.3	7.8	7.9	7.4	7.0	7.5	7.0	7.4	7.0	7.4	7.0
IV	0	28.1	34.7	30.1	34.5	28.5	34.7	28.7	34.5	28.5	34.9	28.8
	0.10	12.7	15	13.7	14.3	12.3	14.4	12.3	14.4	12.3	14.6	12.3
	0.15	9.4	10.9	10.3	10.3	8.9	10.3	9.0	10.3	9.0	10.4	9.0
	0.20	7.3	8.2	7.8	7.8	6.8	7.9	6.9	7.8	6.8	7.9	6.9
	0.25	5.9	6.6	6.4	6.4	5.5	6.4	5.5	6.4	5.5	6.3	5.5

Table S11 Contaminated average run length values of CUSUM median and mean charts for the process following the log-normal distribution

$\rho_{xy} = 0.90, \rho_{yz} = 0.90, \rho_{zx} = 0.90, \text{ and } n = 10$												
Scenario	τ	M_1	T_1	M_2	T_2	M_3	T_3	M_7	M_4	M_5	T_5	M_6
II	0	82.2	119.2	31.3	45.5	31.9	47.2	83.8	118.1	31.3	45.0	87.3
	0.10	41.2	41.0	14.7	8.9	15.1	9.1	40.4	38.9	14.7	8.9	42.2
	0.15	29.3	25.0	10.0	5.9	10.3	6.1	28.6	23.1	10.0	5.9	30.4
	0.20	20.4	16.2	6.6	4.4	6.8	4.5	20.4	15.1	6.6	4.4	21.0
	0.25	14.9	11.7	4.3	3.5	4.6	3.6	14.1	11.1	4.3	3.5	14.8
III	0	202.8	181.2	224.2	175	224.7	174	217.5	181.9	224	172.5	219
	0.10	101.4	73.7	61.0	18.6	63.2	19.2	106	71.3	61.0	18.5	106.8
	0.15	70.7	41.8	31.5	9.4	33.2	9.6	70.7	40.7	31.6	9.4	72.2
	0.20	45.9	25.3	15.9	6.0	16.9	6.1	47.2	23.8	15.9	6.0	48.5
	0.25	31.0	16.5	8.3	4.4	8.8	4.5	30.2	15.7	8.3	4.4	31.1
IV	0	77.4	112.6	30.4	44.7	30.9	46.5	77.6	111.8	30.4	44.2	81.7
	0.10	39.3	40.9	14.6	9.0	14.9	9.2	38.5	38.2	14.6	9.0	40.0
	0.15	28.2	25.1	10.0	6.0	10.2	6.2	27.4	23.3	10.0	6.0	28.7
	0.20	19.9	16.3	6.5	4.5	6.8	4.5	19.9	15.2	6.5	4.4	20.7
	0.25	14.3	11.7	4.4	3.5	4.6	3.6	13.9	11.2	4.4	3.5	14.4
Scenario	τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
II	0	119.7	83.3	120.6	28.0	40.0	27.0	41.8	28.1	40.0	27.0	41.8
	0.10	41.4	42.7	44.4	13.3	8.0	12.9	7.9	13.4	8.0	12.9	7.9
	0.15	24.2	30.2	26.4	9.0	5.3	8.6	5.3	9.0	5.3	8.6	5.3
	0.20	16.3	21.5	17.2	6.1	4.0	5.8	3.9	6.2	4.0	5.8	3.9
	0.25	11.6	14.8	12.3	4.3	3.3	4.0	3.1	4.3	3.3	4.0	3.1
III	0	182.5	194.5	185.5	225.5	171.8	222.3	171.5	225.5	172.6	222.5	171.5
	0.10	73.7	102.5	75.2	58.3	16.6	55.9	17.5	58.4	16.5	55.9	17.6
	0.15	42.6	68.1	45.9	29.4	8.0	27.9	8.1	29.4	8.1	27.9	8.2
	0.20	25.4	47.0	27.5	15.0	5.3	14.3	5.2	15.0	5.3	14.3	5.2
	0.25	16.5	30.8	17.9	8.4	4.0	7.6	3.9	8.3	4.0	7.6	3.9
IV	0	113.7	78.6	114.7	27.7	40.1	26.8	41.7	27.8	40	26.7	41.8
	0.10	40.6	40.7	43.9	13.4	8.1	13.0	8.0	13.4	8.1	13.0	8.0
	0.15	24.5	29.1	27.0	9.1	5.3	8.7	5.3	9.1	5.3	8.7	5.3
	0.20	16.2	20.9	17.0	6.2	4.0	5.9	3.9	6.2	4.0	5.9	3.9
	0.25	11.8	14.8	12.4	4.3	3.3	4.0	3.2	4.3	3.3	4.0	3.2

To be continued

Table S11

$\rho_{xy} = 0.50, \rho_{yz} = 0.40, \rho_{zx} = 0.70, \text{ and } n = 15$												
Scenario	τ	M_1	T_1	M_2	T_2	M_3	T_3	M_7	M_4	M_5	T_5	M_6
II	0	101.1	63.8	85.6	63.8	86	63.7	102.1	63.8	85.6	63.8	101.7
	0.10	27.3	32.8	22.2	29.3	22.2	29.2	26.5	31.6	22.2	29.3	27.1
	0.15	16.0	22.1	13.6	19.6	13.5	19.7	15.6	22.7	13.6	19.6	16
	0.20	10.9	15.8	9.5	12.8	9.4	12.9	10.6	15.5	9.5	12.9	10.8
	0.25	8.1	10.9	7.3	8.9	7.2	8.9	7.9	10.6	7.3	8.9	8.0
III	0	185.4	206.9	180.2	228.0	181.1	228.1	180.4	214.3	180.2	228.2	180.4
	0.10	52.7	86.9	41.9	80.0	41.9	80.3	51.6	87.7	41.9	80.1	52.2
	0.15	27.9	56.7	21.9	46.7	21.9	47.3	27.2	56.5	21.9	46.7	27.7
	0.20	16.1	34.5	13.9	27.5	13.8	27.5	16.1	34.4	13.9	27.5	16.4
	0.25	11.1	21.7	9.6	16.0	9.5	16.3	10.6	21.6	9.6	16.0	10.8
IV	0	96.2	62.5	82.2	62.1	82.7	61.9	95.8	62.8	82.1	62.1	96.2
	0.10	27.3	32.6	22.3	28.9	22.2	28.8	26.9	31.6	22.2	28.9	27.5
	0.15	16.1	22.1	13.7	19.6	13.6	19.7	15.6	22.7	13.8	19.6	15.9
	0.20	10.9	15.9	9.5	12.9	9.5	13.0	10.7	15.4	9.5	13.0	10.8
	0.25	8.1	10.8	7.3	8.7	7.2	8.8	8.0	10.7	7.3	8.8	8.1
Scenario	τ	T_6	M_7	T_7	M_8	T_8	M_9	T_9	M_{10}	T_{10}	M_{11}	T_{11}
II	0	64	96.6	62.9	93.4	59.0	94.5	59.1	93.3	59.1	94.5	59.1
	0.10	31.9	23.4	30.3	22.2	28.5	22.5	28.4	22.2	28.5	22.5	28.4
	0.15	22.8	13.6	21.4	13.1	19.5	13.2	19.4	13.1	19.5	13.2	19.5
	0.20	15.6	9.3	14.7	9.0	13.5	9.1	13.5	9.0	13.5	9.1	13.4
	0.25	10.8	7.1	10.0	6.8	9.0	6.9	9.0	6.8	9.0	6.9	9.0
III	0	213.9	182.2	212.3	182.3	211.9	183.1	211.8	182.2	212	183.1	211.8
	0.10	87.8	46.4	83.8	44.0	79.8	45.1	79.9	43.9	79.8	45.1	79.9
	0.15	56.4	24.0	53.0	23.1	50.0	23.3	49.9	23.0	50.1	23.3	50.0
	0.20	34.5	14.0	32.5	13.2	30.3	13.6	30.3	13.2	30.3	13.6	30.3
	0.25	21.8	9.5	19.7	9.2	18.0	9.2	18.1	9.2	18.1	9.3	18.0
IV	0	63.1	91.9	61.8	89.3	58.0	90.4	58.1	89.2	58.1	90.4	58.1
	0.10	31.7	23.4	30.3	22.4	28.4	22.7	28.3	22.4	28.4	22.7	28.4
	0.15	22.9	13.8	21.5	13.1	19.6	13.2	19.6	13.1	19.6	13.2	19.6
	0.20	15.6	9.4	14.7	9.1	13.5	9.2	13.5	9.1	13.5	9.1	13.5
	0.25	10.8	7.1	10.0	6.8	9.1	6.9	9.1	6.8	9.1	6.9	9.1

Table S12 Contaminated average run length values of CUSUM median charts for $\rho_{xy}=0.20, \rho_{yz}=0.10, \rho_{zx}=0.70$, and $n=5$

Model	Scenario II					Scenario III					Scenario IV					
	$\tau=0.0$	0.10	0.15	0.20	0.25	0.0	0.10	0.15	0.20	0.25	0.0	0.10	0.15	0.20	0.25	
Normal	M_1	110.7	49.9	34.6	25.5	19.4	189.4	75.6	49.8	34.0	25.4	113.3	50.7	35.3	26.1	19.6
	M_2	135.8	70.6	54.7	40.8	33.5	190.2	97.4	72.5	53.8	40.6	142.1	73.5	55.8	42.3	34.1
	M_3	133.3	69.3	52.8	39.9	31.9	190.4	96.1	71.1	52.4	39.3	140.0	71.5	54.6	40.6	33.0
	M_4	120.6	56.3	40.8	31.0	23.9	181.3	80.6	56.8	41.2	30.6	125.2	58.2	42.7	32.0	24.3
	M_5	135.0	70.2	54.1	40.7	32.9	190.3	96.9	72.0	53.4	40.1	141.2	72.7	55.8	41.5	33.8
	M_6	110.6	48.4	33.9	25.3	19.4	174.3	70.6	48.5	34.9	24.9	114.2	50.0	35.1	26.3	19.9
	M_7	110.2	49.2	34.7	26.0	18.6	178.8	72.8	48.5	33.7	24.5	113.4	51.0	36.0	26.4	19.4
	M_8	110.2	49.2	34.7	26.0	18.6	178.7	72.9	48.5	33.7	24.5	113.4	51.0	36.0	26.4	19.4
	M_9	44.8	24.7	19.3	15.4	12.1	62.2	33.7	24.7	19.3	14.6	45.9	25.3	19.9	15.7	12.3
	M_{10}	110.1	49.2	34.7	25.9	18.7	178.7	72.8	48.6	33.7	24.6	113.4	51.0	35.9	26.4	19.4
	M_{11}	110.2	49.4	34.8	26.0	18.6	178.0	73.0	48.8	33.7	24.7	113.5	51.2	36.0	26.4	19.5
Student's t	M_1	39.4	23.7	18.2	15.1	12.4	54.2	30.7	23.8	18.3	14.8	39.8	24.0	18.5	15.2	12.4
	M_2	43.8	28.6	24.2	20.1	17.5	54.5	34.2	29.1	24.4	20.2	45.0	29.2	24.9	20.3	17.8
	M_3	43.7	28.1	23.6	19.8	16.9	54.5	34.2	28.6	23.8	19.7	44.7	28.8	24.4	20.0	17.4
	M_4	40.6	23.7	19.3	15.8	14.2	54.0	28.8	23.3	19.7	14.9	41.9	24.5	19.8	16.5	13.3
	M_5	43.8	28.5	23.9	20.0	17.4	54.4	34.4	28.9	24.3	19.9	44.9	29.3	24.7	20.3	17.8
	M_6	39.2	23.4	18.5	14.8	12.1	53.4	29.7	22.8	18.0	14.2	40.0	24.1	18.8	15.2	12.4
	M_7	39.4	23.3	17.8	14.6	12.2	53.5	29.5	22.4	17.9	14.7	40.2	23.9	18.4	14.9	12.4
	M_8	39.5	23.3	17.8	14.5	12.2	53.6	29.5	22.4	17.9	14.7	40.2	23.9	18.3	14.8	12.4
	M_9	39.8	23.4	18.0	14.6	12.2	54.0	29.5	22.6	18.2	14.7	40.4	23.9	18.5	14.9	12.4
	M_{10}	39.5	23.3	17.8	14.5	12.2	53.6	29.5	22.4	17.9	14.6	40.2	23.8	18.3	14.8	12.4
	M_{11}	39.8	23.5	18.0	14.6	12.3	54.0	29.5	22.6	18.1	14.7	40.4	24.0	18.4	14.9	12.4
Log-normal	M_1	152.9	76.4	52.9	35.2	24.9	191.8	109.7	79.3	53.7	37.6	145.3	75.6	52.1	35.6	25.2
	M_2	136.3	66.8	45.2	32.2	23.7	189.2	99.0	68.1	46.8	33.3	132.9	65.5	45.4	32.4	23.7
	M_3	136.9	67.3	45.5	32.2	23.5	189.8	99.8	68.2	46.9	33.4	133.2	65.9	45.6	32.6	23.6
	M_4	143.4	73.8	50.4	34.8	24.4	183.9	108.7	77.7	52.1	36.0	137.5	72.9	50.2	35.1	24.4
	M_5	136.3	66.8	45.2	32.2	23.7	189.0	99.0	68.1	46.7	33.2	132.9	65.6	45.4	32.4	23.7
	M_6	145.1	74.2	50.6	35.3	24.8	185.1	108.6	78.4	52.6	36.6	138.6	72.9	50.7	35.3	24.7
	M_7	146.9	75.5	49.7	34.1	23.7	190.4	110.1	75.8	51.5	35.6	139.5	73.5	49.5	33.5	24.3
	M_8	149.1	77.4	50.8	34.9	24.0	192.7	110.6	76.7	52.7	36.3	141.8	76.6	50.3	34.3	24.3
	M_9	149.4	77.9	51.3	35.5	24.4	192.6	110.5	77.4	52.9	36.8	142.0	76.9	50.7	34.9	24.6
	M_{10}	149.0	77.4	50.7	34.9	24.0	192.9	110.6	76.7	52.6	36.3	141.7	76.6	50.2	34.3	24.3
	M_{11}	149.5	77.9	51.3	35.5	24.4	192.5	110.5	77.4	52.9	36.9	142.0	76.9	50.7	34.9	24.6

Table S13 Average run length values of CUSUM median charts under a contaminated environment for $\rho_{xy}=0.90$, $\rho_{yz}=0.90$, $\rho_{zx}=0.90$, and $n=10$

Model	Scenario II					Scenario III					Scenario IV					
	$\tau=0.0$	0.10	0.15	0.20	0.25	0.0	0.10	0.15	0.20	0.25	0.0	0.10	0.15	0.20	0.25	
Normal	M_1	93	32.3	20.7	14.9	11.1	188.6	51.9	31.7	20.5	14.6	97.7	33	21.5	15.4	11.2
	M_2	58.1	16.2	10.7	7.9	6.1	190.3	31.4	16.4	10.5	7.6	72.6	18.4	11.7	8.1	6.3
	M_3	56.7	15.7	10.3	7.5	5.8	188.4	30.3	16	10.1	7.3	71.1	17.7	11.3	7.8	6
	M_4	56.9	15.1	9.8	7.1	5.5	175.2	28.3	15.1	9.7	7	68.1	16.9	10.8	7.5	5.6
	M_5	57.7	16.1	10.5	7.7	6	189.3	31.1	16.3	10.4	7.5	72.1	18.2	11.6	8	6.1
	M_6	77.7	23.4	14.8	10.4	7.9	177	39.6	22.8	14.3	10.2	88.8	24.8	15.9	11	8.3
	M_7	93.9	32.3	21.7	15.7	11.6	179.1	52	32.7	21.7	15.1	103.2	35.2	23.4	16.1	12
	M_8	46.2	12.5	8.1	5.9	4.6	182	22.7	12.3	7.9	5.8	58.3	13.7	8.8	6.2	4.8
	M_9	46.1	12.6	8.1	5.9	4.6	182	23	12.4	7.9	5.8	58.5	13.8	8.8	6.1	4.8
	M_{10}	46.3	12.5	8.1	5.9	4.6	182.1	22.7	12.3	7.9	5.8	58.4	13.7	8.8	6.2	4.8
	M_{11}	46.3	12.6	8.1	5.9	4.6	181.2	23.4	12.5	7.9	5.8	58.5	13.9	8.8	6.1	4.7
Student's t	M_1	33.8	16.2	12.3	9.6	7.6	52.7	23.2	16.5	11.9	9.4	34.9	16.8	12.7	9.7	7.8
	M_2	25.5	10.7	7.8	5.9	4.8	52.6	16.2	10.5	7.3	5.9	29.2	11.6	8.1	6	4.9
	M_3	25.1	10.4	7.5	5.7	4.6	52.8	15.9	10.1	7.2	5.7	28.7	11.4	7.9	6	4.8
	M_4	26.1	10.7	7.7	5.9	4.8	50.5	16.4	10.6	7.6	5.8	28.7	11.8	8.3	6.1	5
	M_5	25.4	10.6	7.6	5.9	4.7	52.7	16.3	10.4	7.3	5.9	29.2	11.5	8.1	6.1	4.9
	M_6	31.3	14	10.4	7.8	6.5	49.7	20.1	13.5	10.3	7.8	33.3	13.7	10.9	8.3	6.6
	M_7	34.7	16.9	12.6	10	8	51.6	23.4	16.8	12.4	9.7	36.7	17.5	13.3	10.4	8.3
	M_8	22.4	8.6	6	4.7	3.8	52.3	13.3	8.5	6	4.7	25.7	9.2	6.4	4.8	3.9
	M_9	22.6	8.6	6	4.6	3.8	53.2	13.6	8.6	6.1	4.6	26.5	9.4	6.6	4.9	3.9
	M_{10}	22.3	8.5	6	4.7	3.8	52.1	13.3	8.4	6.1	4.6	25.6	9.2	6.4	4.8	3.9
	M_{11}	22.9	8.8	6.1	4.6	3.8	54.4	13.6	8.7	6.1	4.6	27	9.6	6.6	4.9	3.9
Log-normal	M_1	119.2	41	25	16.2	11.7	181.2	73.7	41.8	25.3	16.5	112.6	40.9	25.1	16.3	11.7
	M_2	45.5	8.9	5.9	4.4	3.5	175	18.6	9.4	6	4.4	44.7	9	6	4.5	3.5
	M_3	47.2	9.1	6.1	4.5	3.6	174	19.2	9.6	6.1	4.5	46.5	9.2	6.2	4.5	3.6
	M_4	118.1	38.9	23.1	15.1	11.1	181.9	71.3	40.7	23.8	15.7	111.8	38.2	23.3	15.2	11.2
	M_5	45	8.9	5.9	4.4	3.5	172.5	18.5	9.4	6	4.4	44.2	9	6	4.4	3.5
	M_6	119.7	41.4	24.2	16.3	11.6	182.5	73.7	42.6	25.4	16.5	113.7	40.6	24.5	16.2	11.8
	M_7	120.6	44.4	26.4	17.2	12.3	185.5	75.2	45.9	27.5	17.9	114.7	43.9	27	17	12.4
	M_8	40	8	5.3	4	3.3	171.8	16.6	8	5.3	4	40.1	8.1	5.3	4	3.3
	M_9	41.8	7.9	5.3	3.9	3.1	171.5	17.5	8.1	5.2	3.9	41.7	8	5.3	3.9	3.2
	M_{10}	40	8	5.3	4	3.3	172.6	16.5	8.1	5.3	4	40	8.1	5.3	4	3.3
	M_{11}	41.8	7.9	5.3	3.9	3.1	171.5	17.6	8.2	5.2	3.9	41.8	8	5.3	3.9	3.2

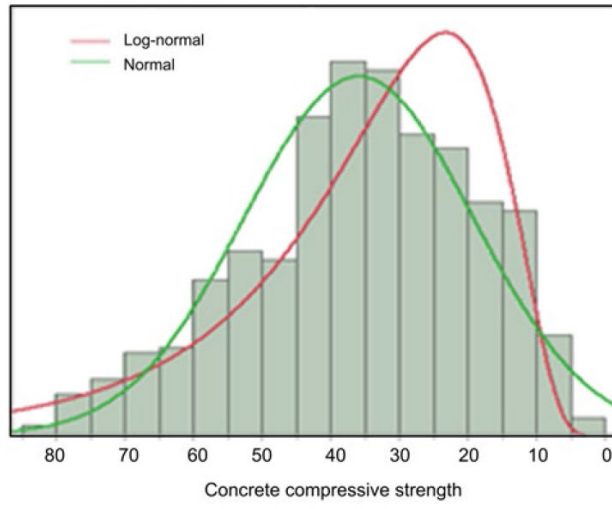


Fig. S1 Density plot of the compressive strength process