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Supplementary materials for

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Proof S1 Proof of Lemma 3

Proof Given any $0 < \upsilon < 1$, the following can be obtained:

$$V_3(t) = \rho \int_0^L \frac{\alpha}{\sqrt{\upsilon}} x y_x(x,t) \sqrt{\upsilon} y_t(x,t) \mathrm{d}x.$$

Then, we obtain

$$\begin{aligned} |V_3(t)| \le \rho \int_0^L \left| \frac{\alpha}{\sqrt{\upsilon}} x y_x(x,t) \right| \left| \sqrt{\upsilon} y_t(x,t) \right| \mathrm{d}x \le \frac{\rho}{2} \int_0^L \left(\left| \frac{\alpha}{\sqrt{\upsilon}} x y_x(x,t) \right|^2 + \left| \sqrt{\upsilon} y_t(x,t) \right|^2 \right) \mathrm{d}x \\ \le \frac{\rho \alpha^2 L^2}{2\upsilon} \int_0^L |y_x(x,t)|^2 \mathrm{d}x + \frac{\rho \upsilon}{2} \int_0^L |y_t(x,t)|^2 \mathrm{d}x. \end{aligned} \tag{S1}$$

Hence, if $\alpha < \sqrt{\frac{T}{\rho L^2}}$ holds, $\frac{\rho \upsilon}{2} < \frac{\rho}{2}$ and $\frac{\rho \alpha^2 L^2}{2\upsilon} < \frac{T}{2}$ are satisfied with $0 < \sqrt{\upsilon} < 1$. For instance, one can choose $\upsilon = \frac{\rho \alpha^2 L^2}{T}$. Then, using the condition in Lemma 3, $\upsilon < 1$ holds. Hence, it follows from inequality (S1) and the definition of V_1 that $|V_3(t)| < \upsilon V_1(t)$ is satisfied.

Similarly, if $\alpha < \sqrt{\frac{2E_I}{\rho L^4}}$ holds, then $\frac{\rho \upsilon}{2} < \frac{\rho}{2}$ and $\frac{\rho \alpha^2 L^4}{4\upsilon} < \frac{E_I}{2}$ are satisfied with $0 < \upsilon < 1$. For instance, one can choose $\upsilon = \frac{\rho \alpha^2 L^4}{2E_I}$. Then, using the condition in Lemma 3, $\upsilon < 1$ holds. Hence, it follows from inequality (S1) and the definition of V_1 that $|V_3(t)| < \upsilon V_1(t)$. The proof is completed.

Proof S2 Proof of Theorem 2

Proof The positiveness of the Lyapunov function (23) is guaranteed by inequality (26). First, we prove the boundedness of the Lyapunov function. Then, the convergence of the Lyapunov function (23) is discussed.

Taking the derivation of the Lyapunov function (23), we obtain

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \tag{S2}$$

where

$$\dot{V}_1(t) = E_I \int_0^L y_{xx}(x,t) y_{xxt}(x,t) \mathrm{d}x + \rho \int_0^L y_t(x,t) y_{tt}(x,t) \mathrm{d}x + T \int_0^L y_x(x,t) y_{xt}(x,t) \mathrm{d}x, \tag{S3}$$

$$\dot{V}_2(t) = M \phi(t) \phi_t(t) \ln \frac{2 l_0^2}{l_0^2 - y_x^2(L,t)} + M \phi^2(t) \frac{y_x(L,t) y_{xt}(L,t)}{l_0^2 - y_x^2(L,t)}, \tag{S4}$$

$$\begin{aligned} \dot{V}_3(t) = & \alpha \rho \int_0^L x y_{xt}(x,t) y_t(x,t) \mathrm{d}x + \alpha \rho \int_0^L x y_x(x,t) y_{tt}(x,t) \mathrm{d}x = \alpha \rho \int_0^L x y_{xt}(x,t) y_t(x,t) \mathrm{d}x \\ & + \alpha \rho \int_0^L x y_x(x,t) (-E_I y_{xxxx}(x,t) + T y_{xx}(x,t) + f(x,y)) \mathrm{d}x, \end{aligned} \tag{S5}$$

$$\dot{V}_4(t) = \frac{1}{\beta}\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\dot{\hat{\boldsymbol{W}}}(t) = \frac{1}{\beta}\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\Big[-\beta\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)}\phi(t)\boldsymbol{\varphi}(\boldsymbol{d}(t)) - k_4\hat{\boldsymbol{W}}(t)\Big]. \tag{S6}$$

Integrating the integral terms by parts, we further obtain

$$\dot{V}_1(t) = (-E_I y_{xxx}(L,t) + T y_x(L,t)) y_t(L,t) + \int_0^L y_t(x,t) f(x,t)\mathrm{d}x, \tag{S7}$$

$$\dot{V}_2(t) = M\phi(t)\phi_t(t)\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)} + M\phi^2(t)\frac{y_x(L,t)y_{xt}(L,t)}{l_0^2-y_x^2(L,t)}, \tag{S8}$$

$$\begin{aligned}\dot{V}_3(t) =& \big(-E_I y_{xxx}(L,t) + T y_x(L,t)\big)\alpha L y_x(L,t) - \frac{\alpha T L}{2} y_x^2(L,t) + \frac{\alpha\rho L}{2} y_t^2(L,t) \\ &- \frac{3\alpha E_I}{2}\int_0^L y_{xx}^2(x,t)\mathrm{d}x - \frac{\alpha T}{2}\int_0^L y_x^2(x,t)\mathrm{d}x - \frac{\alpha\rho}{2}\int_0^L y_t^2(x,t)\mathrm{d}x + \alpha\int_0^L x y_x(x,t) f(x,t)\mathrm{d}x,\end{aligned} \tag{S9}$$

$$\dot{V}_4(t) = -\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)}\phi(t)\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(\boldsymbol{d}(t)) - k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\hat{\boldsymbol{W}}(t). \tag{S10}$$

Using boundary condition (3) and converting controller (17), we obtain

$$\begin{aligned}&-E_I y_{xxx}(L,t) + T y_x(L,t) \\ =& -k_1\phi(t) - k_3 y_{xt}(L,t) - E_I y_{xxx}(L,t) + T y_x(L,t) - \Bigg(k_2\phi(t) - E_I y_{xxx}(L,t) + T y_x(L,t) \\ &+ M\phi(t)\frac{y_x(L,t)y_{xt}(L,t)}{l_0^2-y_x^2(L,t)}\Bigg)\left(\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)}\right)^{-1} + \hat{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(\boldsymbol{d}(t)) - M y_{tt}(L,t) - g(\boldsymbol{d}(t)).\end{aligned} \tag{S11}$$

Eliminating the term $-E_I y_{xxx}(L,t) + T y_x(L,t)$ on both sides gives

$$\begin{aligned}&\left(k_2\phi(t) - E_I y_{xxx}(L,t) + T y_x(L,t) + M\phi(t)\frac{y_x(L,t)y_{xt}(L,t)}{l_0^2-y_x^2(L,t)}\right)\left(\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)}\right)^{-1} \\ =& -k_1\phi(t) - k_3 y_{xt}(L,t) + \hat{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(\boldsymbol{d}(t)) - M y_{tt}(L,t) - g(\boldsymbol{d}(t)).\end{aligned} \tag{S12}$$

Then we have

$$\begin{aligned}&k_2\phi(t) - E_I y_{xxx}(L,t) + T y_x(L,t) + M\phi(t)\frac{y_x(L,t)y_{xt}(L,t)}{l_0^2-y_x^2(L,t)} \\ =& \Big[-k_1\phi(t) - k_3 y_{xt}(L,t) + \tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(\boldsymbol{d}(t)) + \epsilon_{\boldsymbol{W}} - M y_{tt}(L,t)\Big]\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)}.\end{aligned} \tag{S13}$$

Consequently, we derive

$$\begin{aligned}&-E_I y_{xxx}(L,t) + T y_x(L,t) \\ =& -k_1\phi(t)\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)} - M\phi_t(t)\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)} - k_2\phi(t) - M\phi(t)\frac{y_x(L,t)y_{xt}(L,t)}{l_0^2-y_x^2(L,t)} \\ &+ \tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(\boldsymbol{d}(t))\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)} + \epsilon_{\boldsymbol{W}}\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)}.\end{aligned} \tag{S14}$$

Then, combining Eqs. (3), (12), and (S14), the derivative of $V(t)$ is shown as follows:

$$\begin{aligned}\dot{V}(t) \leq& \int_0^L y_t(x,t) f(x,t)\mathrm{d}x - k_1\phi^2(t)\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)} - k_2\phi^2(t) + \tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(\boldsymbol{d}(t))\phi(t)\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)} \\ &+ \epsilon_{\boldsymbol{W}}\phi(t)\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)} - \frac{\alpha T L}{2} y_x^2(L,t) + \frac{\alpha\rho L}{2} y_t^2(L,t) - \frac{3\alpha E_I}{2}\int_0^L y_{xx}^2(x,t)\mathrm{d}x - \frac{\alpha T}{2}\int_0^L y_x^2(x,t)\mathrm{d}x \\ &- \frac{\alpha\rho}{2}\int_0^L y_t^2(x,t)\mathrm{d}x + \alpha\int_0^L x y_x(x,t) f(x,t)\mathrm{d}x - \left(\ln\frac{2l_0^2}{l_0^2-y_x^2(L,t)}\right)\phi(t)\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{\varphi}(\boldsymbol{d}(t)) - k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\hat{\boldsymbol{W}}(t),\end{aligned} \tag{S15}$$

where

$$\phi^2(t) = y_t^2(L,t) + \alpha^2 L^2 y_x^2(L,t) + 2\alpha L y_t(L,t) y_x(L,t).$$

Applying Young's inequality with $\delta_1 > 0$, $\delta_2 > 0$, and $\delta_3 > 0$, one has

$$\int_0^L y_t(x,t) f(x,t)\mathrm{d}x \le \frac{\delta_1}{2}\int_0^L y_t^2(x,t)\mathrm{d}x + \frac{1}{2\delta_1}\int_0^L f^2(x,t)\mathrm{d}x, \tag{S16}$$

$$-2k_2\alpha L y_t(L,t) y_x(L,t) \le k_2\alpha L\delta_2 y_t^2(L,t) + \frac{k_2\alpha L}{\delta_2} y_x^2(L,t), \tag{S17}$$

$$\alpha\int_0^L x y_x(x,t) f(x,t)\mathrm{d}x \le \frac{\alpha\delta_3 L}{2}\int_0^L y_x^2(x,t)\mathrm{d}x + \frac{\alpha L}{2\delta_3}\int_0^L f^2(x,t)\mathrm{d}x. \tag{S18}$$

Consider the weight term $-k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\hat{\boldsymbol{W}}(t)$ in inequality (S15). Because $\tilde{\boldsymbol{W}}(t) = \hat{\boldsymbol{W}}(t) - \boldsymbol{W}^*$, we can obtain

$$\begin{aligned}
&-k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\hat{\boldsymbol{W}}(t) = -k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)(\tilde{\boldsymbol{W}}(t) + \boldsymbol{W}^*) = -k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\tilde{\boldsymbol{W}}(t) - k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\boldsymbol{W}^*\\
&\le -k_4\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\tilde{\boldsymbol{W}}(t) + k_4\left[\frac{\delta_4}{2}\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\tilde{\boldsymbol{W}}(t) + \frac{1}{2\delta_4}\left(\boldsymbol{W}^*\right)^{\mathrm{T}}\boldsymbol{W}^*\right]\\
&= -\frac{2k_4 - k_4\delta_4}{2}\tilde{\boldsymbol{W}}^{\mathrm{T}}(t)\tilde{\boldsymbol{W}}(t) + \frac{k_4}{2\delta_4}\left(\boldsymbol{W}^*\right)^{\mathrm{T}}\boldsymbol{W}^*,
\end{aligned} \tag{S19}$$

where $\delta_4 > 0$.

Substituting inequalities (S16)–(S18) into inequality (S15) leads to

$$\begin{aligned}
\dot{V}(t) \le &\left(\frac{\delta_1}{2} - \frac{\alpha\rho}{2}\right)\int_0^L y_t^2(x,t)\mathrm{d}x - \frac{3\alpha E_I}{2}\int_0^L y_{xx}^2(x,t)\mathrm{d}x - \left(\frac{\alpha T}{2} - \frac{\alpha L\delta_3}{2}\right)\int_0^L y_x^2(x,t)\mathrm{d}x\\
&- \left(k_2 - k_2\alpha L\delta_2 - \frac{\alpha\rho L}{2}\right) y_t^2(L,t) - \left(-\frac{k_2\alpha L}{\delta_2} + k_2\alpha^2 L^2 + \frac{\alpha T L}{2}\right) y_x^2(L,t) - \frac{2k_1}{M}V_2(t)\\
&- \frac{2\beta k_4 - \beta k_4\delta_4}{2}V_4(t) + \left(\frac{1}{2\delta_1} + \frac{\alpha L}{2\delta_3}\right)\bar{f}^2 + \frac{k_4}{2\delta_4}\left(\boldsymbol{W}^*\right)^{\mathrm{T}}\boldsymbol{W}^* + \epsilon_{\boldsymbol{W}}\phi(t)\ln\frac{2l_0^2}{l_0^2 - y_x^2(L,t)}.
\end{aligned} \tag{S20}$$

For convenience, it can be expressed as

$$\begin{aligned}
\dot{V}(t) \le &-\lambda_1\int_0^L y_t^2(x,t)\mathrm{d}x - \lambda_2\int_0^L y_{xx}^2(x,t)\mathrm{d}x - \lambda_3\int_0^L y_x^2(x,t)\mathrm{d}x - \lambda_4 y_t^2(L,t) - \lambda_5 y_x^2(L,t)\\
&- \frac{2k_1}{M}V_2(t) - \frac{2\beta k_4 - \beta k_4\delta_4}{2}V_4(t) + \left(\frac{1}{2\delta_1} + \frac{\alpha L}{2\delta_3}\right)\bar{f}^2 + \frac{k_4}{2\delta_4}\left(\boldsymbol{W}^*\right)^{\mathrm{T}}\boldsymbol{W}^* + (\ln 2)\epsilon_{\boldsymbol{W}},
\end{aligned} \tag{S21}$$

where

$$\begin{cases}
\lambda_1 := -\frac{\delta_1}{2} + \frac{\alpha\rho}{2},\\
\lambda_2 := \frac{3\alpha E_I}{2},\\
\lambda_3 := \frac{\alpha T}{2} - \frac{\alpha L\delta_3}{2},\\
\lambda_4 := k_2 - k_2\alpha L\delta_2 - \frac{\alpha\rho L}{2},\\
\lambda_5 := -\frac{k_2\alpha L}{\delta_2} + k_2\alpha^2 L^2 + \frac{\alpha T L}{2}.
\end{cases} \tag{S22}$$

Then, inequality (S21) can be written as

$$\dot{V}(t) \le -\varphi_1 V_1(t) - \varphi_2 V_2(t) - \varphi_3 V_4(t) + \epsilon, \tag{S23}$$

where

$$\begin{cases}
\varphi_1 = \min\left\{\frac{2\lambda_1}{\rho}, \frac{2\lambda_2}{E_I}, \frac{2\lambda_3}{T}\right\},\\
\varphi_2 = \frac{2k_1}{M} + \min\left\{\frac{2\lambda_4}{(\ln 2)M}, \frac{2\lambda_5}{(\ln 2)\alpha^2 L^2 M}\right\},\\
\varphi_3 = \frac{2\beta k_4 - \beta k_4\delta_4}{2},\\
\epsilon = \left(\frac{1}{2\delta_2} + \frac{\alpha L}{2\delta_3}\right)\bar{f}^2 + \frac{k_4}{2\delta_4}\left(\boldsymbol{W}^*\right)^{\mathrm{T}}\boldsymbol{W}^* + (\ln 2)\epsilon_{\boldsymbol{W}}.
\end{cases} \tag{S24}$$

We require the parameters λ_1, λ_2, λ_3, λ_4, λ_5, φ_2, and φ_3 to be positive. Then we choose $\delta_1 = \delta_3 = \delta_4 = 1$ and $\delta_2 = 0.0098$ to simplify the calculation, which leads to the constraints on these parameters shown as

$$\begin{cases} 0 < \delta_1 < \alpha\rho, \\ 0 < \delta_3 < \frac{T}{L}, \\ k_1 > 0, \\ k_2 \left(\alpha^2 L^2 - \frac{\alpha L}{\delta_2}\right) > -\frac{\alpha T L}{2}, \\ k_2 \left(1 - \alpha L \delta_2\right) > \frac{\alpha \rho L}{2}, \\ \beta k_4 (2 - \delta_4) > 0. \end{cases} \tag{S25}$$

Applying inequality (26) into inequality (S23) yields

$$\dot{V}(t) \leq -\varphi V(t) + \epsilon, \tag{S26}$$

where

$$\begin{cases} \varphi = \dfrac{\min\{\varphi_1, \varphi_2, \varphi_3\}}{1 + \upsilon}, \\ \epsilon = \left(\dfrac{1}{2\delta_2} + \dfrac{\alpha L}{2\delta_3}\right) \bar{f}^2 + \dfrac{k_4}{2\delta_4} \left(\boldsymbol{W}^*\right)^{\mathrm{T}} \boldsymbol{W}^* + (\ln 2)\epsilon_{\boldsymbol{W}}. \end{cases} \tag{S27}$$

Multiplying $\mathrm{e}^{\varphi t}$ on both sides of inequality (S21), we have

$$\dot{V}(t)\mathrm{e}^{\varphi t} + \varphi V(t)\mathrm{e}^{\varphi t} \leq \epsilon \mathrm{e}^{\varphi t}.$$

Taking the integral through time on $[0, t]$ gives

$$V(t)\mathrm{e}^{\varphi t} - V(0) \leq \frac{\epsilon}{\varphi}\mathrm{e}^{\varphi t},$$

$$V(t) \leq \frac{\epsilon}{\varphi} + V(0)\mathrm{e}^{-\varphi t},$$

which implies that the Lyapunov function $V(t)$ is bounded and that the stability of the marine riser system with controller can be guaranteed by the proposed strategy.

Next, we prove the uniform boundedness of the state of the system. Using Lemma 2, for $x \in [0, L]$, we have

$$\left(\frac{9E_I}{8L^3} + \frac{T}{2L}\right) y^2(x,t) \leq \frac{T}{2}\int_0^L y_x^2(x,t)\mathrm{d}x + \frac{E_I}{2}\int_0^L y_{xx}^2(x,t)\mathrm{d}x \leq V_1(t), \tag{S28}$$

based on inequality (26), which implies

$$y^2(x,t) \leq \left(\frac{9E_I}{8L^3} + \frac{T}{2L}\right)^{-1} V_1(t) \leq \frac{8L^3}{(1-\upsilon)(9E_I + 4L^2 T)} V(t) \leq \frac{8L^3}{(1-\upsilon)(9E_I + 4L^2 T)} \left(\frac{\epsilon}{\varphi} + V(0)\mathrm{e}^{-\varphi t}\right). \tag{S29}$$

Using inequality (S29) and the fact that $0 < \mathrm{e}^{-\varphi t} < 1$, the state $y(x,t)$ is uniformly bounded, that is,

$$|y(x,t)| \leq \sqrt{\frac{8L^3}{(1-\upsilon)(9E_I + 4L^2 T)} \left(\frac{\epsilon}{\varphi} + V(0)\right)}, \tag{S30}$$

for any $(x,t) \in [0, L] \times [0, +\infty)$.

Besides, one has that $V_2(t)$ approaches infinity as $|y_x(L,t)|$ tends to l_0. Thus, if $-l_0 < y_x(L,0) < l_0$, we can deduce that $-l_0 < y_x(L,t) < l_0$. The proof is completed.