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Supplementary materials for

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1 Proof of Proposition 1

The power of the incident signal into the unit $U_{n,m}$ can be expressed as

$$P_{n,m}^{\mathrm{in}} = \frac{G_{\mathrm{T}}P_{\mathrm{T}}}{4\pi (r_{n,m}^{\mathrm{T}})^2} F_{\mathrm{NCR}}(\theta_{n,m}^{\mathrm{T}}, \phi_{n,m}^{\mathrm{T}}) d_x d_y. \tag{S1}$$

According to the Friis transfer formula, the received signal is $P_{\mathrm{r}} = P_{\mathrm{t}}G_{\mathrm{t}}G_{\mathrm{r}}\lambda^2/(4\pi d)^2 L$. Considering the path loss $L = 1$, the incident gain is

$$G_{n,m}^{\mathrm{in}} = F_{\mathrm{NCR}}(\theta_{n,m}^{\mathrm{T}}, \phi_{n,m}^{\mathrm{T}}) \frac{4\pi d_x d_y}{\lambda^2}. \tag{S2}$$

The receiving power for unit $U_{n,m}$ in NCR is

$$P_{n,m}^{(1)} = \left|\exp(\mathrm{j}\beta_{n,m})\right|^2 P_{n,m}^{\mathrm{in}}. \tag{S3}$$

The corresponding electric field of the incident signal into $U_{n,m}$ is

$$E_{n,m}^{(1)} = \sqrt{\frac{2Z_0 P_{n,m}^{(1)}}{d_x d_y}} \exp\left(-\frac{\mathrm{j}2\pi}{\lambda} r_{n,m}^{\mathrm{T}}\right). \tag{S4}$$

The total electric field of the received signal is the superposition of the electric fields reflected by all unit cells toward the receiver, which can be written as

$$E^{(1)} = \sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}} \sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}} E_{n,m}^{(1)}. \tag{S5}$$

The receiving power at the NCR from the transmitter is $P^{(1)} = |E^{(1)}|^2 d_x d_y/(2Z_0)$. Under far-field assumption, $\theta_{n,m}^{\mathrm{T}} \to \theta_{\mathrm{T}}$ and $\phi_{n,m}^{\mathrm{T}} \to \phi_{\mathrm{T}}$. Thus, we have

$$P^{(1)} = P_{\mathrm{T}} \frac{G_{\mathrm{T}} F_{\mathrm{NCR}}(\theta_{\mathrm{T}}, \phi_{\mathrm{T}}) d_x d_y}{4\pi d_{\mathrm{T}}^2} \left| \sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}} \sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}} \exp\left(\mathrm{j}\beta_{n,m} - 2\pi \frac{r_{n,m}^{\mathrm{T}}}{\lambda}\right) \right|^2. \tag{S6}$$

The position of the transmitter can be expressed as

$$(x_{\mathrm{T}}, y_{\mathrm{T}}, z_{\mathrm{T}}) = (d_{\mathrm{T}} \sin\theta_{\mathrm{T}} \cos\phi_{\mathrm{T}}, d_{\mathrm{T}} \sin\theta_{\mathrm{T}} \sin\phi_{\mathrm{T}}, d_{\mathrm{T}} \cos\theta_{\mathrm{T}}). \tag{S7}$$

Thus, $r_{n,m}^{\mathrm{T}}$ can be written as follows:

$$\begin{aligned} r_{n,m}^{\mathrm{T}} &= \sqrt{\left(d_{\mathrm{T}} \sin\theta_{\mathrm{T}} \cos\phi_{\mathrm{T}} - \left(m - \frac{1}{2}\right) d_x\right)^2 + \left(d_{\mathrm{T}} \sin\theta_{\mathrm{T}} \sin\phi_{\mathrm{T}} - \left(n - \frac{1}{2}\right) d_y\right)^2 + (d_{\mathrm{T}} \cos\theta_{\mathrm{T}})^2} \\ &\approx d_{\mathrm{T}} - \sin\theta_{\mathrm{T}} \cos\phi_{\mathrm{T}} \left(m - \frac{1}{2}\right) d_x - \sin\theta_{\mathrm{T}} \sin\phi_{\mathrm{T}} \left(n - \frac{1}{2}\right) d_y. \end{aligned} \tag{S8}$$

To divide M_{NCR} and N_{NCR}, we let $\lambda\beta_{n,m}/(2\pi) = \epsilon_1^{\mathrm{NCR}}(m-1/2)d_x + \epsilon_2^{\mathrm{NCR}}(n-1/2)d_y$. Then, the phase part in Eq. (S6) can be written as

$$\begin{aligned}
&\sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}} \sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}} \exp\left(\mathrm{j}\beta_{n,m} - 2\pi\frac{r_{n,m}^{\mathrm{T}}}{\lambda}\right) \\
= &\sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}} \exp\left(\mathrm{j}\frac{2\pi}{\lambda}\left(m-\frac{1}{2}\right)(\sin\theta_{\mathrm{T}}\cos\phi_{\mathrm{T}}+\epsilon_1^{\mathrm{NCR}})d_x\right) \sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}} \exp\left(\mathrm{j}\frac{2\pi}{\lambda}\left(n-\frac{1}{2}\right)(\sin\theta_{\mathrm{T}}\sin\phi_{\mathrm{T}}+\epsilon_2^{\mathrm{NCR}})d_y\right) \\
= &M_{\mathrm{NCR}}N_{\mathrm{NCR}} \frac{\mathrm{sinc}\left(\frac{M_{\mathrm{NCR}}\pi}{\lambda}(\sin\theta_{\mathrm{T}}\cos\phi_{\mathrm{T}}+\epsilon_1^{\mathrm{NCR}})d_x\right)}{\mathrm{sinc}\left(\frac{\pi}{\lambda}(\sin\theta_{\mathrm{T}}\cos\phi_{\mathrm{T}}+\epsilon_1^{\mathrm{NCR}})d_x\right)} \frac{\mathrm{sinc}\left(\frac{N_{\mathrm{NCR}}\pi}{\lambda}(\sin\theta_{\mathrm{T}}\sin\phi_{\mathrm{T}}+\epsilon_2^{\mathrm{NCR}})d_y\right)}{\mathrm{sinc}\left(\frac{\pi}{\lambda}(\sin\theta_{\mathrm{T}}\sin\phi_{\mathrm{T}}+\epsilon_2^{\mathrm{NCR}})d_y\right)}.
\end{aligned} \tag{S9}$$

Obviously, we can achieve the maximum received power

$$P_{\max}^{(1)} = P_{\mathrm{T}}\frac{G_{\mathrm{T}}M_{\mathrm{NCR}}^2N_{\mathrm{NCR}}^2d_xd_yF_{\mathrm{NCR}}(\theta_{\mathrm{T}},\phi_{\mathrm{T}})}{4\pi d_{\mathrm{T}}^2}, \tag{S10}$$

when $\sin\theta_{\mathrm{T}}\cos\phi_{\mathrm{T}} + \epsilon_1^{\mathrm{NCR}} = 0$ and $\sin\theta_{\mathrm{T}}\sin\phi_{\mathrm{T}} + \epsilon_2^{\mathrm{NCR}} = 0$. Correspondingly, the beamforming coefficient at unit $U_{n,m}$ is

$$\begin{aligned}
\beta_{n,m} =&\frac{2\pi}{\lambda}\left(\epsilon_1^{\mathrm{NCR}}\left(m-\frac{1}{2}\right)d_x + \epsilon_2^{\mathrm{NCR}}\left(n-\frac{1}{2}\right)d_y\right) \\
=&-\frac{2\pi}{\lambda}\left(\left(m-\frac{1}{2}\right)d_x\sin\theta_{\mathrm{T}}\cos\phi_{\mathrm{T}} + \left(n-\frac{1}{2}\right)d_y\sin\theta_{\mathrm{T}}\sin\phi_{\mathrm{T}}\right).
\end{aligned} \tag{S11}$$

2 Proof of Proposition 2

Given transmission coefficients $\mathrm{vec}([q_{n,m}\exp(\mathrm{j}\alpha_{n,m})]_{n=1:N_{\mathrm{NCR}}}^{m=1:M_{\mathrm{NCR}}})$, the transmitting power at $U_{n,m}$ is

$$P_{n,m}^{(2)} = P_{n,m}^{\mathrm{R}}q_{n,m} = q_{n,m}|\exp(\mathrm{j}\alpha_{n,m})|^2\bar{P}_{n,m}^{(1)}, \tag{S12}$$

where $\bar{P}_{n,m}^{(1)}$ is the re-assigned received power at the NCR and $\sum_{n=1-N_{\mathrm{NCR}}}^{N_{\mathrm{NCR}}}\sum_{m=1-M_{\mathrm{NCR}}}^{M_{\mathrm{NCR}}}\bar{P}_{n,m}^{(1)} = P_{\max}^{(1)}$. For $U_{n,m}$, the received power at the receiver is

$$P_{n,m}^{\mathrm{NCR}} = P_{n,m}^{(2)}\frac{G_{\mathrm{NCR}}G_{\mathrm{R}}\lambda^2}{16\pi^2(r_{n,m}^{\mathrm{R}})^2}F_{\mathrm{NCR}}(\theta_{n,m}^{\mathrm{R}},\phi_{n,m}^{\mathrm{R}}). \tag{S13}$$

The corresponding electric field is

$$E_{n,m}^{(2)} = \sqrt{\frac{8\pi Z_0P_{n,m}^{\mathrm{R}}}{G_{\mathrm{R}}\lambda^2}}\exp\left(\frac{-\mathrm{j}2\pi}{\lambda}r_{n,m}^{\mathrm{R}}\right). \tag{S14}$$

The total electric field at the receiver is

$$E^{(2)} = \sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}}\sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}}E_{n,m}^{(2)}. \tag{S15}$$

Similar to the transmitter–NCR link, the received power at the receiver is

$$P_{\mathrm{NCR}} = \frac{G_{\mathrm{NCR}}G_{\mathrm{R}}\lambda^2F_{\mathrm{NCR}}(\theta_{\mathrm{R}},\phi_{\mathrm{R}})}{16\pi^2d_{\mathrm{R}}^2}\left|\sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}}\sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}}\sqrt{q_{n,m}\bar{P}_{n,m}^{(1)}}\cdot\exp\left(\mathrm{j}\left(\alpha_{n,m}-\frac{2\pi}{\lambda}r_{n,m}^{\mathrm{R}}\right)\right)\right|^2. \tag{S16}$$

Given uniform power distribution assumption $\bar{P}_{n,m}^{(1)} = P_{\max}^{(1)}/(M_{\mathrm{NCR}}N_{\mathrm{NCR}})$ and unified magnitude at the NCR $q_{n,m} = B$ ($\forall n$ and $\forall m$), the received power at the receiver can be further simplified as

$$P_{\mathrm{NCR}} = P_{\mathrm{T}}\frac{G_{\mathrm{T}}G_{\mathrm{R}}G_{\mathrm{NCR}}M_{\mathrm{NCR}}^2N_{\mathrm{NCR}}^2d_xd_y\lambda^2B}{64\pi^3d_{\mathrm{T}}^2d_{\mathrm{R}}^2}F_{\mathrm{NCR}}(\theta_{\mathrm{T}},\phi_{\mathrm{T}})F_{\mathrm{NCR}}(\theta_{\mathrm{R}},\phi_{\mathrm{R}})\left|\sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}}\sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}}\exp\left(\mathrm{j}\left(\alpha_{n,m}-\frac{2\pi}{\lambda}r_{n,m}^{\mathrm{R}}\right)\right)\right|^2. \tag{S17}$$

As Eq. (S9), we let $\lambda\alpha_{n,m}/(2\pi) = \epsilon_3^{\mathrm{NCR}}(m-1/2)d_x + \epsilon_4^{\mathrm{NCR}}(n-1/2)d_y$. Then, the phase part in Eq. (S17) can be written as

$$\begin{aligned}&\sum_{m=1-\frac{M_{\mathrm{NCR}}}{2}}^{\frac{M_{\mathrm{NCR}}}{2}}\sum_{n=1-\frac{N_{\mathrm{NCR}}}{2}}^{\frac{N_{\mathrm{NCR}}}{2}}\exp\left(\mathrm{j}\alpha_{n,m}-2\pi\frac{r_{n,m}^{\mathrm{R}}}{\lambda}\right)\\ =&M_{\mathrm{NCR}}N_{\mathrm{NCR}}\frac{\mathrm{sinc}\left(\frac{M_{\mathrm{NCR}}\pi}{\lambda}(\sin\theta_{\mathrm{R}}\cos\phi_{\mathrm{R}}+\epsilon_3^{\mathrm{NCR}})d_x\right)}{\mathrm{sinc}\left(\frac{\pi}{\lambda}(\sin\theta_{\mathrm{R}}\cos\phi_{\mathrm{R}}+\epsilon_3^{\mathrm{NCR}})d_x\right)}\frac{\mathrm{sinc}\left(\frac{N_{\mathrm{NCR}}\pi}{\lambda}(\sin\theta_{\mathrm{R}}\sin\phi_{\mathrm{R}}+\epsilon_4^{\mathrm{NCR}})d_y\right)}{\mathrm{sinc}\left(\frac{\pi}{\lambda}(\sin\theta_{\mathrm{R}}\sin\phi_{\mathrm{R}}+\epsilon_4^{\mathrm{NCR}})d_y\right)}.\end{aligned} \tag{S18}$$

Obviously, we can achieve the maximum received power

$$P_{\mathrm{NCR}}^{\max} = P_{\mathrm{T}}\frac{G_{\mathrm{T}}G_{\mathrm{R}}G_{\mathrm{NCR}}M_{\mathrm{NCR}}^3N_{\mathrm{NCR}}^3d_xd_y\lambda^2B}{64\pi^3d_{\mathrm{T}}^2d_{\mathrm{R}}^2}F_{\mathrm{NCR}}(\theta_{\mathrm{T}},\phi_{\mathrm{T}})F_{\mathrm{NCR}}(\theta_{\mathrm{R}},\phi_{\mathrm{R}}), \tag{S19}$$

when $\sin\theta_{\mathrm{R}}\cos\phi_{\mathrm{R}} + \epsilon_3^{\mathrm{NCR}} = 0$ and $\sin\theta_{\mathrm{R}}\sin\phi_{\mathrm{R}} + \epsilon_4^{\mathrm{NCR}} = 0$. Correspondingly, the transmission coefficient at unit $U_{n,m}$ is

$$\begin{aligned}\alpha_{n,m}&=\frac{2\pi}{\lambda}\left(\epsilon_3^{\mathrm{NCR}}\left(m-\frac{1}{2}\right)d_x+\epsilon_3^{\mathrm{NCR}}\left(n-\frac{1}{2}\right)d_y\right)\\ &=-\frac{2\pi}{\lambda}\left(\left(m-\frac{1}{2}\right)d_x\sin\theta_{\mathrm{R}}\cos\phi_{\mathrm{R}}+\left(n-\frac{1}{2}\right)d_y\sin\theta_{\mathrm{R}}\sin\phi_{\mathrm{R}}\right).\end{aligned} \tag{S20}$$