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# Supplementary materials for

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## 1 Proof of Theorem 1

**Proof** For the system (Eqs. (10) and (11)), we construct the discontinuous Lyapunov–Krasovskii functional (LKF) as follows:

$$\begin{aligned}\mathcal{V}(t) =&\eta_1^{\mathrm{T}}(t)P\eta_1(t)+\int_{t-\eta_M}^{t} z^{\mathrm{T}}(s)Qz(s)\mathrm{d}s+\int_{-\eta_M}^{0}\int_{t+s}^{t}\dot{z}^{\mathrm{T}}(\theta)R\dot{z}(\theta)\mathrm{d}\theta\mathrm{d}s+(\eta_M-\tau(t))\int_{t-\bar{\tau}(t)}^{t}\eta_2^{\mathrm{T}}(s)S\eta_2(s)\mathrm{d}s\\&+\tau(t)\nu_k^{\mathrm{T}}(t-\tau(t))G_2\nu_k(t-\tau(t))+h\int_{t-\tau(t)}^{t}\dot{z}^{\mathrm{T}}(s)G_3\dot{z}(s)\mathrm{d}s+d^{\mathrm{T}}(t)G_1d(t)\end{aligned}$$

for $t\in\mathcal{F}_{k,i}$ ($i=0,\ 1,\ \ldots,\ \mathcal{J}_k,\ \forall k\in\mathbb{N}$), where $P>0$, $Q>0$, $R>0$, $S>0$, $G_i>0$ ($i=1,2,3$), $\bar{\tau}(t)=\tau(t)-\tau_{t_k+i}$, $\eta_1(t)=\mathrm{col}\{z(t),\int_{t-\eta_M}^{t}z(s)\mathrm{d}s\}$, and $\eta_2(s)=\mathrm{col}\{\dot{z}(s),\ z(s-\tau(s)),\ z(s-\bar{\tau}(s))\}$.

(1) We show that $\mathcal{V}(t)$ satisfies

$$\mathbb{E}\{\mathcal{V}(t^-)-\mathcal{V}(t)\}>0,\ i=1,2,\ldots,\mathcal{J}_k, \tag{S1}$$

$$\mathbb{E}\{\mathcal{V}(t^-)-\mathcal{V}(t)-\omega d^{\mathrm{T}}(t^-)Wd(t^-)\}>0,\ i=0, \tag{S2}$$

for $t\in\{f_{t_k+i}\}$ ($\forall k\in\mathbb{N}$).

For $t=f_{t_k+i}$ ($i=1,2,\ldots,\mathcal{J}_k,\forall k\in\mathbb{N}$), one has

$$\mathbb{E}\{\mathcal{V}(t^-)-\mathcal{V}(t)\}\geq\xi_1^{\mathrm{T}}(t)\varTheta_1\xi_1(t),$$

where

$$\xi_1(t)=\begin{bmatrix} z((t_k+i-1)h)-z(t_kh)\\ z((t_k+i)h)-z(t_kh)\end{bmatrix}\ \text{and}\ \varTheta_1=\begin{bmatrix} G_3+(h+\tau_{t_k+i})G_2 & -G_3\\ -G_3 & G_3-\tau_{t_k+i}T_2\end{bmatrix}.$$

For $t=f_{t_k}$ ($\forall k\in\mathbb{N}$), one has

$$\begin{aligned}&\mathbb{E}\{\mathcal{V}(t^-)-\mathcal{V}(t)-\omega d^{\mathrm{T}}(t^-)Wd(t^-)\}\\=&d^{\mathrm{T}}(t^-)G_1d(t^-)-\omega d^{\mathrm{T}}(t^-)Wd(t^-)+h\int_{(t_k-1)h}^{t_kh}\dot{z}^{\mathrm{T}}(s)G_3\dot{z}(s)\mathrm{d}s-d^{\mathrm{T}}(t)G_1d(t)\\&+(h+\tau_{t_k})(z((t_k-1)h)-z(t_{k-1}h))^{\mathrm{T}}G_2(z((t_k-1)h)-z(t_{k-1}h)).\end{aligned} \tag{S3}$$

It can be derived from Eqs. (11) and (S3) that

$$\mathbb{E}\{\mathcal{V}(t^-)-\mathcal{V}(t)-\omega d^{\mathrm{T}}(t^-)Wd(t^-)\}\geq\xi_2^{\mathrm{T}}(t)\varTheta_2\xi_2(t),$$

where

$$\xi_2(t)=\begin{bmatrix} d(t^-)\\ z(t_{k-1}h)-z(t_kh)\\ z((t_k-1)h)-z(t_{k-1}h)\end{bmatrix}\ \text{and}\ \varTheta_2=\begin{bmatrix} \varPi_3^{(1,1)} & \varPi_3^{(1,2)}\\ \star & G_3+(h+\tau_{t_k})G_2\end{bmatrix}.$$

Since $\varTheta_1$ and $\varTheta_2$ are convex on $\tau_{t_k+i} \in [0, \tau_M]$ $(i = 0, 1, \ldots, \mathcal{J}_k, \forall k \in \mathbb{N})$, conditions (S1) and (S2) can be ensured when condition (16) is satisfied.

(2) We prove that

$$\mathbb{E}\left\{\mathcal{L}\mathcal{V}(t) - d^{\mathrm{T}}(t)Wd(t)\right\} < 0 \tag{S4}$$

holds for $t \in \mathcal{F}_{k,i}$ $(i = 0,\, 1,\, \ldots,\, \mathcal{J}_k, \forall k \in \mathbb{N})$, where the infinitesimal operator $\mathcal{L}$ is defined as

$$\mathcal{L}\mathcal{V}(t) = \lim_{\Delta t \to 0^+} \Delta t^{-1}\{\mathbb{E}\{\mathcal{V}(t+\Delta t) \mid t\} - \mathcal{V}(t)\}. \tag{S5}$$

Using Eq. (S5), one obtains

$$\begin{aligned}
&\mathcal{L}\mathcal{V}(t) - d^{\mathrm{T}}(t)Wd(t)\\
=&\vartheta^{\mathrm{T}}(t)(\varPsi_0 + \tau(t)\varPsi_1 + \tau_{t_k+i}\varPsi_2)\vartheta(t) - \int_{t-\bar{\tau}(t)}^{t} \dot{z}^{\mathrm{T}}(s)R_s\dot{z}(s)\mathrm{d}s - \int_{t-\eta_M}^{t-\bar{\tau}(t)} \dot{z}^{\mathrm{T}}(s)R\dot{z}(s)\mathrm{d}s,
\end{aligned} \tag{S6}$$

where

$$\begin{aligned}
&\vartheta(t) = \mathrm{col}\{\vartheta_1(t), \vartheta_2(t)\},\\
&\vartheta_1(t) = \mathrm{col}\{z(t), \dot{z}(t), z(t-\tau(t)),\ \nu_k(t-\tau(t)), z(t-\eta_M)\},\\
&\vartheta_2(t) = \mathrm{col}\left\{z(t-\bar{\tau}(t)), \int_{t-\bar{\tau}(t)}^{t} \frac{z(s)}{\bar{\tau}(t)}\mathrm{d}s,\ \int_{t-\tau(t)}^{t-\bar{\tau}(t)} \frac{z(s)}{\tau_{t_k+i}}\mathrm{d}s, \int_{t-\eta_M}^{t-\tau(t)} \frac{z(s)}{\eta_M - \tau(t)}\mathrm{d}s, d(t)\right\}.
\end{aligned}$$

Using Corollary 2 in Zhang et al. (2018) to the integral terms in Eq. (S6) yields

$$\begin{aligned}
&-\int_{t-\bar{\tau}(t)}^{t} \dot{z}^{\mathrm{T}}(s)R_s\dot{z}(s)\mathrm{d}s - \int_{t-\eta_M}^{t-\bar{\tau}(t)} \dot{z}^{\mathrm{T}}(s)R\dot{z}(s)\mathrm{d}s\\
\leq&\vartheta^{\mathrm{T}}(t)\left[\tau_{t_k+i}U_2\,\mathrm{diag}\{R^{-1}, (3R)^{-1}\}U_2^{\mathrm{T}} + \mathrm{Sym}\{U_1\varGamma\varphi_7 + U_2\varGamma\varphi_8 + U_3\varGamma\varphi_{10}\}\right.\\
&\left. + (\tau(t) - \tau_{t_k+i})U_1\,\mathrm{diag}\{R_s^{-1}, (3R_s)^{-1}\}U_1^{\mathrm{T}} + (\eta_M - \tau(t))U_3\,\mathrm{diag}\{R^{-1}, (3R)^{-1}\}U_3^{\mathrm{T}}\right]\vartheta(t).
\end{aligned} \tag{S7}$$

For system (10) with $h(t) = 0$ , there exist two arbitrary matrices $X_1$ and $X_2$ such that

$$\sum_{i=1}^{3}\sum_{j=1}^{3} \varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)(\epsilon_1^{\mathrm{T}}X_1 + \epsilon_2^{\mathrm{T}}X_2)\left[A_i\epsilon_1 + BK_j(\varepsilon_3 - \varepsilon_4 + (1-\beta)\varepsilon_{10}) - \varepsilon_2\right]\vartheta(t) = 0. \tag{S8}$$

It is derived from condition (12) that

$$\vartheta^{\mathrm{T}}(t)\left[\epsilon_4^{\mathrm{T}}\bar{T}\epsilon_4 - \sigma^2(\epsilon_3 - \epsilon_4)^{\mathrm{T}}\bar{T}(\epsilon_3 - \epsilon_4)\right]\vartheta(t) \leq 0. \tag{S9}$$

From conditions (S6)–(S8), one can further derive that

$$\mathcal{L}\mathcal{V}(t) - d^{\mathrm{T}}(t)Wd(t) \leq \sum_{i=1}^{3}\sum_{j=1}^{3} \varsigma_i(t)\varsigma_j\left(t-\tau(t)\right)\vartheta^{\mathrm{T}}(t)\mathcal{E}_{ij}(t)\vartheta(t) < 0$$

holds if and only if there exists a constant $\rho > 0$ such that the following inequality is valid for $T = \rho\bar{T}$ and all nonzero $\vartheta(t)$:

$$\mathcal{L}\mathcal{V}(t) - d^{\mathrm{T}}(t)Wd(t) \leq \sum_{i=1}^{3}\sum_{j=1}^{3} \varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\bar{\mathcal{E}}_{ij}(t)\vartheta(t) < 0 \tag{S10}$$

for $t \in \mathcal{F}_{k,i}$ $(i = 0,\, 1,\, \ldots,\, \mathcal{J}_k, \forall k \in \mathbb{N})$, where

$$\begin{aligned}
\bar{\mathcal{E}}_{ij}(t) =&\mathcal{E}_{ij}(t) - \epsilon_4^{\mathrm{T}}T\epsilon_4 + \sigma^2(\epsilon_3 - \epsilon_4)^{\mathrm{T}}T(\epsilon_3 - \epsilon_4),\\
\mathcal{E}_{ij}(t) =&\varPsi_0 + \eta_M U_3\,\mathrm{diag}\left\{R^{-1}, (3R)^{-1}\right\}U_3^{\mathrm{T}} + \tau(t)(\varPsi_1 + U_1\,\mathrm{diag}\{R_s^{-1}, (3R_s)^{-1}\}U_1^{\mathrm{T}}\\
&- U_3\,\mathrm{diag}\{R^{-1}, (3R)^{-1}\}U_3^{\mathrm{T}}) + \tau_{t_k+i}(\varPsi_2 + U_2\,\mathrm{diag}\{R^{-1}, (3R)^{-1}\}U_2^{\mathrm{T}} - U_1\,\mathrm{diag}\{R_s^{-1}, (3R_s)^{-1}\}U_1^{\mathrm{T}})\\
&+ \mathrm{Sym}\{(\epsilon_1^{\mathrm{T}}X_1 + \epsilon_2^{\mathrm{T}}X_2)(A_i\epsilon_1 + BK_j\mathcal{E} - \varepsilon_2)\} + \mathrm{Sym}\{U_1\varGamma\varphi_7 + U_2\varGamma\varphi_8 + U_3\varGamma\varphi_{10}\}.
\end{aligned}$$

Furthermore, inequality (S10) is satisfied if there exists a matrix $H<0$ such that

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\bar{\mathcal{E}}_{ij}(t)\vartheta(t)-S(t)<0, \tag{S11}$$

where

$$S(t)=\Sigma^{\mathrm{T}}(t)H\Sigma(t)\text{ and }\Sigma(t)=\begin{bmatrix}\varsigma_1(t)\varphi_{11}\vartheta(t)\\ \varsigma_2(t)\varphi_{11}\vartheta(t)\\ \varsigma_3(t)\varphi_{11}\vartheta(t)\\ \varsigma_1(t-\tau(t))\varphi_{11}\vartheta(t)\\ \varsigma_2(t-\tau(t))\varphi_{11}\vartheta(t)\\ \varsigma_3(t-\tau(t))\varphi_{11}\vartheta(t)\end{bmatrix}.$$

The left side of inequality (S11) can be rewritten as follows:

$$\begin{aligned}&\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\bar{\mathcal{E}}_{ij}(t)\vartheta(t)-S(t)\\ =&\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\left(\bar{\mathcal{E}}_{ij}(t)-\varphi_{11}^{\mathrm{T}}(H^{(i,j+3)}+H^{(j+3,i)})\varphi_{11}\right)\vartheta(t)\\ &-\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\varphi_{11}^{\mathrm{T}}\left(H^{(i,j)}+H^{(j,i)}+H^{(i+3,j+3)}+H^{(j+3,i+3)}\right)\varphi_{11}\vartheta(t)\\ &-\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)(\varsigma_j(t)-\varsigma_j(t-\tau(t)))\vartheta^{\mathrm{T}}(t)\varphi_{11}^{\mathrm{T}}\left(H^{(i,j)}+H^{(j,i)}\right)\varphi_{11}\vartheta(t)\\ &+\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}(\varsigma_i(t)-\varsigma_i(t-\tau(t)))\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\varphi_{11}^{\mathrm{T}}\left(H^{(i+3,j+3)}+H^{(j+3,i+3)}\right)\varphi_{11}\vartheta(t).\end{aligned} \tag{S12}$$

For arbitrary symmetric matrices $M_i$ and $N_i$ $(i=1,2,3)$, the following equations hold:

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)(\varsigma_j(t)-\varsigma_j(t-\tau(t)))N_i=0, \tag{S13}$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3}(\varsigma_i(t)-\varsigma_i(t-\tau(t)))\varsigma_j(t-\tau(t))M_j=0. \tag{S14}$$

If conditions (18) and (19) are satisfied, using Lemma 2, we can derive from Eqs. (S12)–(S14) that

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\bar{\mathcal{E}}_{ij}(t)\vartheta(t)-S(t)<\sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\tilde{\mathcal{E}}_{ij}(t)\vartheta(t), \tag{S15}$$

where $t\in\mathcal{F}_{k,i}$ $(i=0,\,1,\,\ldots,\,\mathcal{J}_k,\,\forall k\in\mathbb{N})$ and

$$\begin{aligned}\tilde{\mathcal{E}}_{ij}(t)&=\bar{\Psi}_{ij}+\tau(t)\bar{\Psi}_1+\tau_{t_k+i}\bar{\Psi}_2,\\ \bar{\Psi}_{ij}&=\Psi_{ij}+\eta_M U_3\operatorname{diag}\{R^{-1},(3R)^{-1}\}U_3^{\mathrm{T}},\\ \bar{\Psi}_1&=\Psi_1+U_1\operatorname{diag}\{R_s^{-1},(3R_s)^{-1}\}U_1^{\mathrm{T}}-U_3\operatorname{diag}\{R^{-1},(3R)^{-1}\}U_3^{\mathrm{T}},\\ \bar{\Psi}_2&=\Psi_2+U_2\operatorname{diag}\{R^{-1},(3R)^{-1}\}U_2^{\mathrm{T}}-U_1\operatorname{diag}\{R_s^{-1},(3R_s)^{-1}\}U_1^{\mathrm{T}}.\end{aligned}$$

Using Lemma 7 in Zhang and Han (2014) and Lemma 2 in Kim (2016) in relation to $\tilde{\mathcal{E}}_{ij}(t)<0$ related to $\tau(t)\in[\tau_{t_k+i},\eta_M]$ for the cases that $\bar{\Psi}_1\geq 0$ and $\bar{\Psi}_1<0$, respectively, one has that $\tilde{\mathcal{E}}_{ij}(t)<0$ can be ensured by

$$\bar{\Psi}_{ij} + (\bar{\Psi}_1 + \bar{\Psi}_2)\tau_{t_k+i} < 0, \tag{S16}$$
$$\bar{\Psi}_{ij} + \eta_M \bar{\Psi}_1 + \tau_{t_k+i}\Psi_2 < 0. \tag{S17}$$

Using the same methods in relation to conditions (S16) and (S17) that are related to $\tau_{t_k+i}$ and the Schur complement, we obtain

$$\sum_{i=1}^{3}\sum_{j=1}^{3} \varsigma_i(t)\varsigma_j(t-\tau(t))\vartheta^{\mathrm{T}}(t)\tilde{\mathcal{E}}_{ij}(t)\vartheta(t) < 0 \tag{S18}$$

if condition (17) is satisfied. Combining conditions (S6), (S10), (S11), (S15), and (S18) yields $\mathbb{E}\{\mathcal{L}\mathcal{V}(t) - d^{\mathrm{T}}(t)Wd(t)\} < 0$ for $t \in \mathcal{F}_{k,i}$ ($i = 0, 1, \ldots, \mathcal{J}_k, \forall k \in \mathbb{N}$).

(3) We prove that the system (Eqs. (10) and (11)) is exponentially mean-square stable with $H_\infty$ performance. Accordingly, we construct an LKF $\tilde{\mathcal{V}}(t) = \mathrm{e}^{\upsilon_1 t}\mathcal{V}(t)$. Then, calculating $\mathbb{E}\{\mathcal{L}\tilde{\mathcal{V}}(t)\}$ and using condition (S4), we have

$$\mathbb{E}\{\mathcal{L}\tilde{\mathcal{V}}(t)\} = \upsilon_1\mathrm{e}^{\upsilon_1 t}\mathbb{E}\{\mathcal{V}(t)\} + \mathrm{e}^{\upsilon_1 t}\mathbb{E}\{\mathcal{L}\mathcal{V}(t)\} \leq \upsilon_1\mathrm{e}^{\upsilon_1 t}\mathbb{E}\{\mathcal{V}(t)\} + \mathrm{e}^{\upsilon_1 t}\mathbb{E}\{d^{\mathrm{T}}(t)Wd(t)\}. \tag{S19}$$

Integrating condition (S19) on the interval $[t_0h, t)$ and using conditions (S1) and (S2) and $t_{k+1}h - t_kh \leq \varkappa$, one has

$$\mathbb{E}\{\mathcal{V}(t) - \mathrm{e}^{\upsilon_1(t_0h-t)}\mathcal{V}(t_0h)\} \leq \int_{t_0h}^{t} \upsilon_1\mathrm{e}^{\upsilon_1(s-t)}\mathbb{E}\{\mathcal{V}(s)\}\mathrm{d}s + \omega\mathbb{E}\{d^{\mathrm{T}}(f_{t_k})Wd(f_{t_k})\}. \tag{S20}$$

There exists a constant $\alpha$ for a sufficiently small $\upsilon_1 > 0$ such that

$$\mathbb{E}\{\mathcal{V}(t) - \omega d^{\mathrm{T}}(f_{t_k})Wd(f_{t_k})\} \leq \alpha\mathrm{e}^{-\upsilon_1(t-t_0h)}\mathbb{E}\{\|z(t_0h)\|^2 + \|d(t_0h)\|^2\}. \tag{S21}$$

It is clear from $\mathcal{V}(t)$ and $d^{\mathrm{T}}(t)G_1d(t) < \mathcal{V}(t)$ that

$$\lambda_{\min}(P)\mathbb{E}\{\|z(t)\|^2\} \leq \mathbb{E}\{\mathcal{V}(t) - d^{\mathrm{T}}(f_{t_k})G_1d(f_{t_k})\} \tag{S22}$$

and

$$\lambda_{\min}(G_1 - \omega W)\mathbb{E}\{\|d^{\mathrm{T}}(t)\|^2\} \leq \mathbb{E}\{\mathcal{V}(t) - \omega d^{\mathrm{T}}(f_{t_k})Wd(f_{t_k})\} \tag{S23}$$

hold for $t \in \mathcal{F}_{k,i}$ ($i = 0, 1, \ldots, \mathcal{J}_k, \forall k \in \mathbb{N}$). Since $G_1 - \omega W > 0$ and from conditions (S21)–(S23), one can obtain that

$$\mathbb{E}\{\|z(t)\|^2\} \leq \upsilon_2\mathrm{e}^{-\upsilon_1(t-t_0h)}\mathbb{E}\{\|z(t_0h)\|^2 + \|\eta(t_0h)\|^2\},$$
$$\mathbb{E}\{\|\eta(t)\|^2\} \leq \upsilon_2\mathrm{e}^{-\upsilon_1(t-t_0h)}\mathbb{E}\{\|z(t_0h)\|^2 + \|\eta(t_0h)\|^2\},$$

where $\upsilon_2 = \max\{\alpha/\lambda_{\min}(P), \alpha/\lambda_{\min}(G_1 - \omega W)\}$. Then the system (Eqs. (10) and (11)) subject to condition (12) is exponentially mean-square stable. For any nonzero $h(t)$, we have

$$\begin{aligned} &\mathbb{E}\{\mathcal{L}\mathcal{V}(t) - d^{\mathrm{T}}(t)Wd(t)\} \\ &\leq \sum_{i=1}^{3}\sum_{j=1}^{3}\varsigma_i(t)\varsigma_j(t-\tau(t)) \begin{bmatrix} \vartheta(t) \\ h(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \tilde{\mathcal{E}}_{ij}(t) & (\epsilon_1^{\mathrm{T}}X_1 + \epsilon_2^{\mathrm{T}}X_2)D \\ \star & 0 \end{bmatrix} \begin{bmatrix} \vartheta(t) \\ h(t) \end{bmatrix} \\ &+\mathbb{E}\{\eta^{\mathrm{T}}(t)\eta(t)\} - \mathbb{E}\{\eta^{\mathrm{T}}(t)\eta(t)\} + \gamma^2h^{\mathrm{T}}(t)h(t) - \gamma^2h^{\mathrm{T}}(t)h(t). \end{aligned}$$

If condition (17) is ensured, it can be derived that

$$\mathbb{E}\{\mathcal{L}\mathcal{V}(t) - d^{\mathrm{T}}(t)Wd(t) + \eta^{\mathrm{T}}(t)\eta(t)\} - h^{\mathrm{T}}(t)h(t) \leq 0 \tag{S24}$$

holds for $t \in \mathcal{F}_{k,i}$ ($i = 0, 1, \ldots, \mathcal{J}_k, \forall k \in \mathbb{N}$), and

$$\begin{aligned}
&\int_{t_0 h}^{\infty} \mathbb{E}\{\mathcal{L}\mathcal{V}(s) - d^{\mathrm{T}}(s) W d(s)\} \mathrm{d}s \\
\geq &\sum_{k=0}^{\infty} \mathbb{E}\{\mathcal{V}(f_{t_{k+1}}^{-}) - \mathcal{V}(f_{t_k}) - d^{\mathrm{T}}(f_{t_k}) W d(f_{t_k})\} \\
\geq &\sum_{k=0}^{\infty} \mathbb{E}\{\mathcal{V}(f_{t_{k+1}}) + d^{\mathrm{T}}(f_{t_k}) W d(f_{t_k})\} - \sum_{k=0}^{\infty} \mathbb{E}\{\mathcal{V}(f_{t_k}) + d^{\mathrm{T}}(f_{t_k}) W d(f_{t_k})\} \\
= &\mathbb{E}\{\mathcal{V}(t_\infty) - \mathcal{V}(t_0 h)\}.
\end{aligned} \tag{S25}$$

Considering $\mathbb{E}\{\mathcal{V}(t_\infty)\} > 0$ and $\mathbb{E}\{\mathcal{V}(t_0 h)\} = 0$ under $z(t_0 h) = 0$ and $d(t_0 h) = 0$, one can conclude from conditions (S24) and (S25) that $\mathbb{E}\{\|\eta(t)\|_2\} \leq \gamma \|h(t)\|_2$. Combining with the proof in (1)–(3), we complete the proof.

## 2 Proof of Theorem 2

**Proof** Let $X = X_1^{-\mathrm{T}} = \varrho X_2^{-\mathrm{T}}$, $Y_i = K_i X$ $(i = 1, 2, 3)$, $\mathscr{X}_1 = \mathrm{diag}\{X, X\}$, $\mathscr{X}_2 = \mathrm{diag}\{X, X, X\}$, $\mathscr{X}_3 = \mathrm{diag}\{X, X, \ldots, X\}_{10n \times 10n}$, $\mathscr{X}_4 = \mathrm{diag}\{\mathscr{X}_1, \mathscr{X}_1\}$, $\tilde{P} = \mathscr{X}_1{}^{\mathrm{T}} P \mathscr{X}_1$, $\tilde{Q} = X^{\mathrm{T}} Q X$, $\tilde{R} = X^{\mathrm{T}} R X$, $\tilde{S} = \mathscr{X}_2{}^{\mathrm{T}} S \mathscr{X}_2$, $\tilde{T} = X^{\mathrm{T}} T X$, $\tilde{W} = X^{\mathrm{T}} W X$, $\tilde{G}_i = X^{\mathrm{T}} G_i X$, $\tilde{M}_i = \mathscr{X}_2^{\mathrm{T}} M_i \mathscr{X}_2$, $\tilde{N}_i = \mathscr{X}_2^{\mathrm{T}} N_i \mathscr{X}_2$, $\tilde{U}_i = \mathscr{X}_3^{\mathrm{T}} U_i \mathscr{X}_1$ $(i = 1, 2, 3)$, $\tilde{\Pi}_1 = \mathscr{X}_1^{\mathrm{T}} \Pi_1 \mathscr{X}_1$, $\tilde{\Pi}_2 = \mathscr{X}_1^{\mathrm{T}} \Pi_2 \mathscr{X}_1$, $\tilde{\Pi}_3 = \mathscr{X}_2^{\mathrm{T}} \Pi_3 \mathscr{X}_2$, and $\tilde{H}^{(i,j)} = \mathscr{X}_2^{\mathrm{T}} H^{(i,j)} \mathscr{X}_2$ $(i, j = 1, 2, \ldots, 6)$. Then pre- and post-multiplying both sides of $\Xi_{lij}$ $(l = 1, 3)$ with $\mathrm{diag}\{\mathscr{X}_3, \mathscr{X}_1, 1\}^{\mathrm{T}}$ and its transpose, and $\Xi_{lij}$ $(l = 2, 4)$ with $\mathrm{diag}\{\mathscr{X}_3, \mathscr{X}_4, 1\}^{\mathrm{T}}$ and its transpose, respectively, and using the Schur complement, one can obtain conditions (20)–(23). The proof is completed.

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