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E-mail: jzus@zju.edu.cn

Supplementary materials for

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1 Proof of Property 1

Differentiating V_1^z with respect to time yields

$$\dot{V}_1^z = \frac{\partial V_1^z}{\partial z_1}\dot{z}_1 + \frac{\partial V_1^z}{\partial y_\mathrm{d}}\dot{y}_\mathrm{d} + \frac{\partial V_1^z}{\partial k_\mathrm{a1}}\dot{k}_\mathrm{a1} + \frac{\partial V_1^z}{\partial k_\mathrm{b1}}\dot{k}_\mathrm{b1}. \tag{S1}$$

The second term on the right side of Eq. (S1) satisfies

$$\frac{\partial V_1^z}{\partial y_\mathrm{d}} = z_1\left[\frac{(k_\mathrm{a1}+k_\mathrm{b1})^2}{(k_\mathrm{a1}+z_1+y_\mathrm{d})(k_\mathrm{b1}-z_1-y_\mathrm{d})} + \Psi_1\right], \tag{S2}$$

where $\Psi_1 = \dfrac{k_\mathrm{a1}+k_\mathrm{b1}}{z_1}\ln\dfrac{(k_\mathrm{a1}+y_\mathrm{d})(k_\mathrm{b1}-z_1-y_\mathrm{d})}{(k_\mathrm{a1}+z_1+y_\mathrm{d})(k_\mathrm{b1}-y_\mathrm{d})}$.

The third term on the right side of Eq. (S1) satisfies

$$\frac{\partial V_1^z}{\partial k_\mathrm{a1}} = z_1\left(\frac{k_\mathrm{a1}+k_\mathrm{b1}}{k_\mathrm{a1}+z_1+y_\mathrm{d}} + I_1\right), \tag{S3}$$

where $I_1 = \dfrac{2k_\mathrm{a1}+k_\mathrm{b1}+y_\mathrm{d}}{z_1}\ln\dfrac{k_\mathrm{a1}+y_\mathrm{d}}{k_\mathrm{a1}+z_1+y_\mathrm{d}} + \dfrac{k_\mathrm{b1}-y_\mathrm{d}}{z_1}\ln\dfrac{k_\mathrm{b1}-y_\mathrm{d}}{k_\mathrm{b1}-z_1-y_\mathrm{d}}$.

The fourth term on the right side of Eq. (S1) satisfies

$$\frac{\partial V_1^z}{\partial k_\mathrm{b1}} = z_1\left(-\frac{k_\mathrm{a1}+k_\mathrm{b1}}{k_\mathrm{b1}-z_1-y_\mathrm{d}} + Q_1\right), \tag{S4}$$

where $Q_1 = \dfrac{k_\mathrm{a1}+2k_\mathrm{b1}-y_\mathrm{d}}{z_1}\ln\dfrac{k_\mathrm{b1}-y_\mathrm{d}}{k_\mathrm{b1}-z_1-y_\mathrm{d}} + \dfrac{k_\mathrm{a1}+y_\mathrm{d}}{z_1}\ln\dfrac{k_\mathrm{a1}+y_\mathrm{d}}{k_\mathrm{a1}+z_1+y_\mathrm{d}}$.

Using L'Hôpital's rule, we can solve for the limit values of Ψ_1, I_1, and Q_1. Applying L'Hôpital's rule to Ψ_1 results in

$$\lim_{z_1\to 0}\Psi_1 = \lim_{z_1\to 0}\frac{k_\mathrm{a1}+k_\mathrm{b1}}{z_1}\ln\frac{(k_\mathrm{a1}+y_\mathrm{d})(k_\mathrm{b1}-z_1-y_\mathrm{d})}{(k_\mathrm{a1}+z_1+y_\mathrm{d})(k_\mathrm{b1}-y_\mathrm{d})} = -\frac{(k_\mathrm{a1}+k_\mathrm{b1})^2}{(k_\mathrm{a1}+y_\mathrm{d})(k_\mathrm{b1}-y_\mathrm{d})}. \tag{S5}$$

Applying L'Hôpital's rule to I_1 results in

$$\lim_{z_1\to 0}I_1 = \lim_{z_1\to 0}\frac{2k_\mathrm{a1}+k_\mathrm{b1}+y_\mathrm{d}}{z_1}\ln\frac{k_\mathrm{a1}+y_\mathrm{d}}{k_\mathrm{a1}+z_1+y_\mathrm{d}} + \frac{k_\mathrm{b1}-y_\mathrm{d}}{z_1}\ln\frac{k_\mathrm{b1}-y_\mathrm{d}}{k_\mathrm{b1}-z_1-y_\mathrm{d}} = -\frac{2k_\mathrm{a1}+k_\mathrm{b1}+y_\mathrm{d}}{k_\mathrm{a1}+y_\mathrm{d}} + 1. \tag{S6}$$

Applying L'Hôpital's rule to Q_1 results in

$$\lim_{z_1\to 0}Q_1 = \lim_{z_1\to 0}\frac{k_\mathrm{a1}+2k_\mathrm{b1}-y_\mathrm{d}}{z_1}\ln\frac{k_\mathrm{b1}-y_\mathrm{d}}{k_\mathrm{b1}-z_1-y_\mathrm{d}} + \frac{k_\mathrm{a1}+y_\mathrm{d}}{z_1}\ln\frac{k_\mathrm{a1}+y_\mathrm{d}}{k_\mathrm{a1}+z_1+y_\mathrm{d}} = \frac{k_\mathrm{a1}+2k_\mathrm{b1}-y_\mathrm{d}}{k_\mathrm{b1}-y_\mathrm{d}} - 1. \tag{S7}$$

From Assumption 2, y_d is within the predefined constraint. Thus, Ψ_1, I_1, and Q_1 are well defined in a neighborhood of $z_1 = 0$.

2 Proof of Theorem 1

Consider the following Lyapunov function candidate for the z_1-subsystem:

$$V_1 = \overline{q}_1 V_1^z + \frac{1}{2}\tilde{\boldsymbol{W}}_{\mathrm{c1}}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c1}} + \frac{1}{2}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a1}}, \tag{S8}$$

where $\tilde{\boldsymbol{W}}_{\mathrm{c1}}$ and $\tilde{\boldsymbol{W}}_{\mathrm{a1}}$ represent the critic and actor NN estimation errors respectively and ensure that $\tilde{\boldsymbol{W}}_{\mathrm{c1}} = \hat{\boldsymbol{W}}_{\mathrm{c1}} - \boldsymbol{W}_1^*$ and $\tilde{\boldsymbol{W}}_{\mathrm{a1}} = \hat{\boldsymbol{W}}_{\mathrm{a1}} - \boldsymbol{W}_1^*$ hold.

Based on Eqs. (S1) and (S8), the time derivative of V_1 is obtained as

$$\dot{V}_1 = \overline{q}_1\left(z_1\varPhi_1\dot{z}_1 + \frac{\partial V_1^z}{\partial y_{\mathrm{d}}}\dot{y}_{\mathrm{d}} + \frac{\partial V_1^z}{\partial k_{\mathrm{a1}}}\dot{k}_{\mathrm{a1}} + \frac{\partial V_1^z}{\partial k_{\mathrm{b1}}}\dot{k}_{\mathrm{b1}}\right) + \tilde{\boldsymbol{W}}_{\mathrm{c1}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}_{\mathrm{c1}} + \tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}_{\mathrm{a1}}. \tag{S9}$$

So, we have

$$\dot{V}_1 = \overline{q}_1\left(g_1\varPhi_1 z_1 z_2 - \varPhi_1 z_1\hat{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1 - \eta_1\varPhi_1^2 z_1^2 - k_1\varPhi_1 z_1^2\right) + \tilde{\boldsymbol{W}}_{\mathrm{c1}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}_{\mathrm{c1}} + \tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{W}}}_{\mathrm{a1}}. \tag{S10}$$

From Eqs. (8) and (10) and $\boldsymbol{\omega}_1$, the HJB equation of the z_1-subsystem can be presented as follows:

$$H_1\left(z_1, \alpha_1^*, \frac{\partial J_1^*(z_1)}{\partial z_1}\right) = \overline{q}_1 V_1^z - \frac{P_1^2}{g_1^2} + \frac{2P_1}{g_1^2}(f_1 - \dot{y}_{\mathrm{d}}) + \frac{2}{g_1^2}(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1(f_1 - P_1 - \dot{y}_{\mathrm{d}}) - \frac{1}{g_1^2}(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^* = 0. \tag{S11}$$

Consider the following equation:

$$\overline{q}_1 V_1^z - \frac{P_1^2}{g_1^2} + \frac{2P_1}{g_1^2}(f_1 - \dot{y}_{\mathrm{d}}) = -\frac{2}{g_1^2}\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{W}_1^* - \frac{2}{g_1^2}\hat{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^* + \frac{1}{g_1^2}(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^*. \tag{S12}$$

From Eqs. (S11) and (S12), Eq. (S10) can become

$$\begin{aligned}\dot{V}_1 =& \overline{q}_1 g_1\varPhi_1 z_1 z_2 - \overline{q}_1\varPhi_1 z_1\hat{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1 - \overline{q}_1\eta_1\varPhi_1^2 z_1^2 - \overline{q}_1 k_1\varPhi_1 z_1^2 - \frac{\xi_{\mathrm{c1}}\tilde{\boldsymbol{W}}_{\mathrm{c1}}^{\mathrm{T}}\boldsymbol{\omega}_1}{\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1 + 1}\left[\frac{2}{g_1^2}\boldsymbol{\omega}_1^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c1}} + \frac{1}{g_1^2}\hat{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\right.\\ &\left.\cdot\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}} - \frac{2}{g_1^2}\hat{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^* + \frac{1}{g_1^2}(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^*\right] + \overline{q}_1\varPhi_1 z_1\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1 - \xi_{\mathrm{a1}}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}}\\ &+ \frac{\xi_{\mathrm{c1}}}{g_1^2(\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1 + 1)}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}}\boldsymbol{\omega}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{c1}},\end{aligned} \tag{S13}$$

where

$$\frac{1}{g_1^2}\hat{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}} - \frac{2}{g_1^2}\hat{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^* + \frac{1}{g_1^2}(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^* = \frac{1}{g_1^2}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}} - \frac{1}{g_1^2}(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a1}}.$$

Rewrite Eq. (S13) as

$$\begin{aligned}\dot{V}_1 =& \overline{q}_1 g_1\varPhi_1 z_1 z_2 - \overline{q}_1\varPhi_1 z_1(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1 - \overline{q}_1\eta_1\varPhi_1^2 z_1^2 - \overline{q}_1 k_1\varPhi_1 z_1^2 - \frac{\xi_{\mathrm{c1}}\tilde{\boldsymbol{W}}_{\mathrm{c1}}^{\mathrm{T}}\boldsymbol{\omega}_1}{\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1 + 1}\left[\frac{2}{g_1^2}\boldsymbol{\omega}_1^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c1}} + \frac{1}{g_1^2}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\right.\\ &\left.\cdot\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}} - \frac{1}{g_1^2}(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a1}}\right] - \xi_{\mathrm{a1}}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}} + \frac{\xi_{\mathrm{c1}}}{g_1^2(\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1 + 1)}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}}\boldsymbol{\omega}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{c1}},\end{aligned} \tag{S14}$$

where

$$\begin{aligned}&-\frac{\xi_{\mathrm{c1}}}{g_1^2(\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1 + 1)}\tilde{\boldsymbol{W}}_{\mathrm{c1}}^{\mathrm{T}}\boldsymbol{\omega}_1\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}} + \frac{\xi_{\mathrm{c1}}}{g_1^2(\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1 + 1)}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}}\boldsymbol{\omega}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{c1}}\\ =&\frac{\xi_{\mathrm{c1}}}{g_1^2(\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1 + 1)}\tilde{\boldsymbol{W}}_{\mathrm{a1}}^{\mathrm{T}}\boldsymbol{\varphi}_1(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\omega}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a1}}.\end{aligned}$$

Eq. (S14) can be rewritten as

$$\begin{aligned}\dot{V}_1 =&\overline{q}_1 g_1 \varPhi_1 z_1 z_2 - \overline{q}_1 \varPhi_1 z_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 - \overline{q}_1 \eta_1 \varPhi_1^2 z_1^2 - \overline{q}_1 k_1 \varPhi_1 z_1^2 - \frac{\xi_{\mathrm{c}1} \tilde{\boldsymbol{W}}_{\mathrm{c}1}^{\mathrm{T}} \boldsymbol{\omega}_1}{\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1} \left[\frac{2}{g_1^2} \boldsymbol{\omega}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{c}1} \right. \\ &\left. - \frac{1}{g_1^2} (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1} \right] - \xi_{\mathrm{a}1} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1} + \frac{\xi_{\mathrm{c}1}}{g_1^2 (\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\omega}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1}.\end{aligned} \tag{S15}$$

Based on the fact that $\tilde{\boldsymbol{W}}_{\mathrm{a}1} = \hat{\boldsymbol{W}}_{\mathrm{a}1} - \boldsymbol{W}_1^*$, the following equation holds:

$$\xi_{\mathrm{a}1} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1} = \frac{\xi_{\mathrm{a}1}}{2} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1} + \frac{\xi_{\mathrm{a}1}}{2} \hat{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1} - \frac{\xi_{\mathrm{a}1}}{2} (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \boldsymbol{W}_1^*.$$

According to Young's inequality, there is

$$-\overline{q}_1 \varPhi_1 z_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 \leq \frac{\overline{q}_1}{2} \left((\boldsymbol{W}_1^*)^{\mathrm{T}} \right)^2 + \frac{\overline{q}_1}{2} (\varPhi_1 z_1)^2.$$

Substituting the above inequality into Eq. (S15), we have

$$\begin{aligned}\dot{V}_1 \leq& - \left(\overline{q}_1 \eta_1 - \frac{\overline{q}_1}{2} \right) \varPhi_1^2 z_1^2 + \overline{q}_1 g_1 \varPhi_1 z_1 z_2 - \overline{q}_1 k_1 \varPhi_1 z_1^2 - \frac{\xi_{\mathrm{a}1}}{2} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1} - \frac{2\xi_{\mathrm{c}1}}{g_1^2(x_1)(\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} \tilde{\boldsymbol{W}}_{\mathrm{c}1}^{\mathrm{T}} \boldsymbol{\omega}_1 \\ &\cdot \boldsymbol{\omega}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{c}1} + \frac{\xi_{\mathrm{c}1}}{g_1^2 (\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\omega}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1} + \frac{\xi_{\mathrm{c}1}}{g_1^2 (\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} \tilde{\boldsymbol{W}}_{\mathrm{c}1}^{\mathrm{T}} \boldsymbol{\omega}_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1} \\ &- \frac{\xi_{\mathrm{a}1}}{2} \hat{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1} + \frac{\xi_{\mathrm{a}1} + \overline{q}_1}{2} \left((\boldsymbol{W}_1^*)^{\mathrm{T}} \right)^2.\end{aligned} \tag{S16}$$

Based on Cauchy's inequality and Young's inequality, two inequalities can be derived as follows:

$$\frac{1}{g_1^2} \frac{\xi_{\mathrm{c}1} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\omega}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1}}{\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1} \leq \frac{1}{g_1^2} \left[\frac{1}{4} \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\omega}_1 \boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{W}_1^* \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1} + \xi_{\mathrm{c}1}^2 \hat{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1} \right], \tag{S17}$$

$$\frac{1}{g_1^2} \frac{\xi_{\mathrm{c}1} \tilde{\boldsymbol{W}}_{\mathrm{c}1}^{\mathrm{T}} \boldsymbol{\omega}_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1}}{\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1} \leq \frac{1}{g_1^2} \left[\frac{1}{4(\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} \tilde{\boldsymbol{W}}_{\mathrm{c}1}^{\mathrm{T}} \boldsymbol{\omega}_1 (\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \boldsymbol{W}_1^* \boldsymbol{\omega}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{c}1} + \xi_{\mathrm{c}1}^2 \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1} \right]. \tag{S18}$$

Adding the inequalities obtained above to inequality (S16) yields

$$\begin{aligned}\dot{V}_1 \leq& - \left(\overline{q}_1 \eta_1 - \frac{\overline{q}_1}{2} \right) \varPhi_1^2 z_1^2 + \overline{q}_1 g_1 \varPhi_1 z_1 z_2 - \overline{q}_1 k_1 \varPhi_1 z_1^2 - \left[\frac{\xi_{\mathrm{a}1}}{2} - \frac{\xi_{\mathrm{c}1}^2}{g_1^2} - \frac{(\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\omega}_1 \boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{W}_1^*}{4 g_1^2} \right] \tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{a}1} \\ &- \frac{1}{g_1^2 (\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} \left[2\xi_{\mathrm{c}1} - \frac{(\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1}{4} \boldsymbol{\varphi}_1^{\mathrm{T}} \boldsymbol{W}_1^* \right] \tilde{\boldsymbol{W}}_{\mathrm{c}1}^{\mathrm{T}} \boldsymbol{\omega}_1 \boldsymbol{\omega}_1^{\mathrm{T}} \tilde{\boldsymbol{W}}_{\mathrm{c}1} \\ &- \left(\frac{\xi_{\mathrm{a}1}}{2} - \frac{\xi_{\mathrm{c}1}^2}{g_1^2} \right) \hat{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \hat{\boldsymbol{W}}_{\mathrm{a}1} + \frac{\xi_{\mathrm{a}1} + \overline{q}_1}{2} \left((\boldsymbol{W}_1^*)^{\mathrm{T}} \right)^2.\end{aligned} \tag{S19}$$

Design parameters $\xi_{\mathrm{c}1}$, $\xi_{\mathrm{a}1}$, $\overline{q}_1$, η_1, and $\boldsymbol{\omega}_1$, which satisfy the following inequalities:

$$\begin{cases} \overline{q}_1 \eta_1 - \dfrac{\overline{q}_1}{2} > 0, \\ \dfrac{\xi_{\mathrm{a}1}}{2} - \dfrac{\xi_{\mathrm{c}1}^2}{g_1^2} - \dfrac{(\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\omega}_1 \boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{W}_1^*}{4 g_1^2} > 0, \\ \dfrac{2\xi_{\mathrm{c}1}}{g_1^2 (\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} - \dfrac{(\boldsymbol{W}_1^*)^{\mathrm{T}} \boldsymbol{\varphi}_1 \boldsymbol{\varphi}_1^{\mathrm{T}} \boldsymbol{W}_1^*}{4 g_1^2 (\boldsymbol{\omega}_1^{\mathrm{T}} \boldsymbol{\omega}_1 + 1)} > 0, \\ \dfrac{\xi_{\mathrm{a}1}}{2} - \dfrac{\xi_{\mathrm{c}1}^2}{g_1^2} > 0. \end{cases}$$

Define

$$m_{\mathrm{a}1}=\inf_{t\geq 0}\left\{\frac{\pi_1\xi_{\mathrm{a}1}}{2}-\frac{\pi_1\xi_{\mathrm{c}1}^2}{g_1^2}-\frac{\pi_1(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\omega}_1\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{W}_1^*}{4g_1^2}\right\}$$

and

$$m_{\mathrm{c}1}=\inf_{t\geq 0}\left\{\frac{2\xi_{\mathrm{c}1}\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1}{g_1^2(\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1+1)}-\frac{\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1(\boldsymbol{W}_1^*)^{\mathrm{T}}\boldsymbol{\varphi}_1\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{W}_1^*}{4g_1^2(\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\omega}_1+1)}\right\},$$

where π_1 is the dimension of $\hat{\boldsymbol{W}}_{\mathrm{c}1}$ and $\hat{\boldsymbol{W}}_{\mathrm{a}1}$. So, inequality (S19) can become

$$\dot{V}_1\leq -\overline{q}_1k_1\varPhi_1z_1^2+\overline{q}_1g_1\varPhi_1z_1z_2-m_{\mathrm{a}1}\tilde{\boldsymbol{W}}_{\mathrm{a}1}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}1}-m_{\mathrm{c}1}\tilde{\boldsymbol{W}}_{\mathrm{c}1}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}1}+d_1, \tag{S20}$$

where $d_1=\dfrac{\xi_{\mathrm{a}1}+\overline{q}_1}{2}\left((\boldsymbol{W}_1^*)^{\mathrm{T}}\right)^2$.

Similarly, consider the following Lyapunov function candidate for the z_i-subsystem:

$$V_i=\overline{q}_iV_i^z+\frac{1}{2}\tilde{\boldsymbol{W}}_{\mathrm{c}i}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}i}+\frac{1}{2}\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}i}, \tag{S21}$$

where $\tilde{\boldsymbol{W}}_{\mathrm{c}i}$ and $\tilde{\boldsymbol{W}}_{\mathrm{a}i}$ represent the critic and actor NN estimation errors respectively, and ensure that $\tilde{\boldsymbol{W}}_{\mathrm{c}i}=\hat{\boldsymbol{W}}_{\mathrm{c}i}-\boldsymbol{W}_i^*$ and $\tilde{\boldsymbol{W}}_{\mathrm{a}i}=\hat{\boldsymbol{W}}_{\mathrm{a}i}-\boldsymbol{W}_i^*$ hold.

Based on Eq. (S21), the time derivative of V_i is obtained as

$$\dot{V}_i=\overline{q}_i\left(z_i\varPhi_i\dot{z}_i+\dot{\hat{\alpha}}_{i-1}\frac{\partial V_i^z}{\partial\hat{\alpha}_{i-1}}+\dot{k}_{\mathrm{a}i}\frac{\partial V_i^z}{\partial k_{\mathrm{a}i}}+\dot{k}_{\mathrm{b}i}\frac{\partial V_i^z}{\partial k_{\mathrm{b}i}}\right)+\tilde{\boldsymbol{W}}_{\mathrm{c}i}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{c}i}+\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{a}i}. \tag{S22}$$

Thus, Eq. (S22) becomes

$$\dot{V}_i=\overline{q}_i\left(-g_{i-1}\varPhi_{i-1}z_{i-1}z_i+g_i\varPhi_iz_iz_{i+1}-\varPhi_iz_i\hat{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i-\eta_i\varPhi_i^2z_i^2-k_i\varPhi_iz_i^2\right)+\tilde{\boldsymbol{W}}_{\mathrm{c}i}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{c}i}+\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{a}i}. \tag{S23}$$

From Eqs. (23) and (25) and $\boldsymbol{\omega}_i$, the HJB equation of the z_i-subsystem can be presented as

$$\begin{aligned}&H_i\left(z_i,\alpha_i^*,\frac{\partial J_i^*(z_i)}{\partial z_i}\right)\\ =&\overline{q}_iV_i^z-\frac{P_i^2}{g_i^2}+\frac{2P_i}{g_i^2}(f_i-\dot{\hat{\alpha}}_{i-1})+\frac{2}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\left(f_i-P_i-\dot{\hat{\alpha}}_{i-1}\right)-\frac{1}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*\\ =&0.\end{aligned} \tag{S24}$$

Consider the following equation:

$$\overline{q}_iV_i^z-\frac{P_i^2}{g_i^2}+\frac{2P_i}{g_i^2}\left(f_i-\dot{\hat{\alpha}}_{i-1}\right)=-\frac{2}{g_i^2}\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{W}_i^*-\frac{2}{g_i^2}\hat{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*+\frac{1}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*. \tag{S25}$$

From Eqs. (S24) and (S25), Eq. (S23) can become

$$\begin{aligned}\dot{V}_i=&-\overline{q}_ig_{i-1}\varPhi_{i-1}z_{i-1}z_i+\overline{q}_ig_i\varPhi_iz_iz_{i+1}-\overline{q}_i\varPhi_iz_i\hat{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i-\overline{q}_i\eta_i\varPhi_i^2z_i^2-\overline{q}_ik_i\varPhi_iz_i^2\\ &-\frac{\xi_{\mathrm{c}i}\tilde{\boldsymbol{W}}_{\mathrm{c}i}^{\mathrm{T}}\boldsymbol{\omega}_i}{\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1}\left[\frac{2}{g_i^2}\boldsymbol{\omega}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}i}+\frac{1}{g_i^2}\hat{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}i}-\frac{2}{g_i^2}\hat{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*+\frac{1}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*\right]\\ &+\overline{q}_i\varPhi_iz_i\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i-\xi_{\mathrm{a}i}\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}i}+\frac{\xi_{\mathrm{c}i}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}i}\boldsymbol{\omega}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}i},\end{aligned} \tag{S26}$$

where

$$\frac{1}{g_i^2}\hat{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}i}-\frac{2}{g_i^2}\hat{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*+\frac{1}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*=\frac{1}{g_i^2}\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}i}-\frac{1}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}i}.$$

Rewrite Eq. (S26) as

$$\begin{aligned}\dot{V}_i =& -\overline{q}_i g_{i-1}\Phi_{i-1}z_{i-1}z_i + \overline{q}_i g_i \Phi_i z_i z_{i+1} - \overline{q}_i \Phi_i z_i (\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i - \overline{q}_i \eta_i \Phi_i^2 z_i^2 - \overline{q}_i k_i \Phi_i z_i^2 \\ & - \frac{\xi_{ci}\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i}{\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1}\left[\frac{2}{g_i^2}\boldsymbol{\omega}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ci} + \frac{1}{g_i^2}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} - \frac{1}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai}\right] - \xi_{ai}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} \\ & + \frac{\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai}\boldsymbol{\omega}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ci},\end{aligned} \tag{S27}$$

where

$$\begin{aligned}& -\frac{\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} + \frac{\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\cdot\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai}\boldsymbol{\omega}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ci} \\ =& \frac{\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai}.\end{aligned}$$

Eq. (S27) can be rewritten as

$$\begin{aligned}\dot{V}_i =& -\overline{q}_i g_{i-1}\Phi_{i-1}z_{i-1}z_i + \overline{q}_i g_i \Phi_i z_i z_{i+1} - \overline{q}_i \Phi_i z_i (\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i - \overline{q}_i \eta_i \Phi_i^2 z_i^2 - \overline{q}_i k_i \Phi_i z_i^2 \\ & - \frac{\xi_{ci}\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i}{\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1}\left[\frac{2}{g_i^2}\boldsymbol{\omega}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ci} - \frac{1}{g_i^2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai}\right] - \xi_{ai}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} \\ & + \frac{\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai}.\end{aligned} \tag{S28}$$

Based on the fact that $\tilde{\boldsymbol{W}}_{ai} = \hat{\boldsymbol{W}}_{ai} - \boldsymbol{W}_i^*$, we have

$$\xi_{ai}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} = \frac{\xi_{ai}}{2}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} + \frac{\xi_{ai}}{2}\hat{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} - \frac{\xi_{ai}}{2}(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*.$$

According to Young's inequality, we have $-\overline{q}_i\Phi_i z_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i \le \frac{\overline{q}_i}{2}\left((\boldsymbol{W}_i^*)^{\mathrm{T}}\right)^2 + \frac{\overline{q}_i}{2}(\Phi_i z_i)^2$. Substituting the above inequality into Eq. (S28) yields

$$\begin{aligned}\dot{V}_i \le& -\left(\overline{q}_i\eta_i - \frac{\overline{q}_i}{2}\right)\Phi_i^2 z_i^2 - \overline{q}_i g_{i-1}\Phi_{i-1}z_{i-1}z_i + \overline{q}_i g_i\Phi_i z_i z_{i+1} - \overline{q}_i k_i \Phi_i z_i^2 - \frac{\xi_{ai}}{2}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} \\ & - \frac{2\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\omega}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ci} + \frac{\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} + \frac{\xi_{ci}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)} \\ & \cdot\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} - \frac{\xi_{ai}}{2}\hat{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} + \frac{\xi_{ai}+\overline{q}_i}{2}\left((\boldsymbol{W}_i^*)^{\mathrm{T}}\right)^2.\end{aligned} \tag{S29}$$

Similar to inequalities (S17) and (S18), two inequalities can be derived as follows:

$$\frac{1}{g_i^2}\frac{\xi_{ci}}{\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} \le \frac{1}{g_i^2}\left[\frac{1}{4}\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{W}_i^*\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} + \xi_{ci}^2\hat{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai}\right], \tag{S30}$$

$$\frac{1}{g_i^2}\frac{\xi_{ci}}{\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1}\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} \le \frac{1}{g_i^2}\left[\frac{1}{4(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*\boldsymbol{\omega}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ci} + \xi_{ci}^2\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai}\right]. \tag{S31}$$

Adding the inequalities obtained above to inequality (S29) yields

$$\begin{aligned}\dot{V}_i \le& -\left(\overline{q}_i\eta_i - \frac{\overline{q}_i}{2}\right)\Phi_i^2 z_i^2 - \overline{q}_i g_{i-1}\Phi_{i-1}z_{i-1}z_i + \overline{q}_i g_i\Phi_i z_i z_{i+1} - \overline{q}_i k_i\Phi_i z_i^2 - \left(\frac{\xi_{ai}}{2} - \frac{\xi_{ci}^2}{g_i^2}\right. \\ & \left. - \frac{(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{W}_i^*}{4g_i^2}\right)\tilde{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ai} - \frac{1}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\left(2\xi_{ci} - \frac{(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*}{4}\right)\tilde{\boldsymbol{W}}_{ci}^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\omega}_i^{\mathrm{T}}\tilde{\boldsymbol{W}}_{ci} \\ & - \left(\frac{\xi_{ai}}{2} - \frac{\xi_{ci}^2}{g_i^2}\right)\hat{\boldsymbol{W}}_{ai}^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\hat{\boldsymbol{W}}_{ai} + \frac{\xi_{ai}+\overline{q}_i}{2}\left((\boldsymbol{W}_i^*)^{\mathrm{T}}\right)^2.\end{aligned} \tag{S32}$$

Design parameters $\xi_{\mathrm{c}i}$, $\xi_{\mathrm{a}i}$, $\overline{q}_i$, η_i, and $\boldsymbol{\omega}_i$, which satisfy the following inequalities:

$$\begin{cases} \overline{q}_i\eta_i - \dfrac{\overline{q}_i}{2} > 0, \\ \dfrac{\xi_{\mathrm{a}i}}{2} - \dfrac{\xi_{\mathrm{c}i}^2}{g_i^2} - \dfrac{(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{W}_i^*}{4g_i^2} > 0, \\ \dfrac{2\xi_{\mathrm{c}i}}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)} - \dfrac{(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*}{4g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)} > 0, \\ \dfrac{\xi_{\mathrm{a}i}}{2} - \dfrac{\xi_{\mathrm{c}i}^2}{g_i^2} > 0. \end{cases}$$

Define

$$m_{\mathrm{a}i} = \inf_{t\geq 0}\left\{\frac{\pi_i\xi_{\mathrm{a}i}}{2} - \frac{\pi_i\xi_{\mathrm{c}i}^2}{g_i^2} - \frac{\pi_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\omega}_i\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{W}_i^*}{4g_i^2}\right\}$$

and

$$m_{\mathrm{c}i} = \inf_{t\geq 0}\left\{\frac{2\xi_{\mathrm{c}i}\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i}{g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)} - \frac{\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i(\boldsymbol{W}_i^*)^{\mathrm{T}}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^{\mathrm{T}}\boldsymbol{W}_i^*}{4g_i^2(\boldsymbol{\omega}_i^{\mathrm{T}}\boldsymbol{\omega}_i+1)}\right\},$$

where π_i is the dimension of $\hat{\boldsymbol{W}}_{\mathrm{c}i}$ and $\hat{\boldsymbol{W}}_{\mathrm{a}i}$. So, inequality (S32) can become

$$\dot{V}_i \leq -\overline{q}_i k_i \varPhi_i z_i^2 - \overline{q}_i g_{i-1}\varPhi_{i-1}z_{i-1}z_i + \overline{q}_i g_i \varPhi_i z_i z_{i+1} - m_{\mathrm{a}i}\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}i} - m_{\mathrm{c}i}\tilde{\boldsymbol{W}}_{\mathrm{c}i}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}i} + d_i, \tag{S33}$$

where $d_i = \dfrac{\xi_{\mathrm{a}i}+\overline{q}_i}{2}\left((\boldsymbol{W}_i^*)^{\mathrm{T}}\right)^2$.

Similarly, consider the following Lyapunov function candidate for the z_n-subsystem:

$$V_n = \overline{q}_n V_n^z + \frac{1}{2}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n} + \frac{1}{2}\tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}n}, \tag{S34}$$

where $\tilde{\boldsymbol{W}}_{\mathrm{c}n}$ and $\tilde{\boldsymbol{W}}_{\mathrm{a}n}$ represent the critic and actor NN estimation errors respectively, and ensure that $\tilde{\boldsymbol{W}}_{\mathrm{c}n} = \hat{\boldsymbol{W}}_{\mathrm{c}n} - \boldsymbol{W}_n^*$ and $\tilde{\boldsymbol{W}}_{\mathrm{a}n} = \hat{\boldsymbol{W}}_{\mathrm{a}n} - \boldsymbol{W}_n^*$ hold.

Based on Eq. (S34), the time derivative of V_n is obtained as

$$\dot{V}_n = \overline{q}_n\left(z_n\varPhi_n\dot{z}_n + \dot{\hat{\alpha}}_{n-1}\frac{\partial V_n^z}{\partial\hat{\alpha}_{n-1}} + \dot{k}_{\mathrm{a}n}\frac{\partial V_n^z}{\partial k_{\mathrm{a}n}} + \dot{k}_{\mathrm{b}n}\frac{\partial V_n^z}{\partial k_{\mathrm{b}n}}\right) + \tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{c}n} + \tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{a}n}. \tag{S35}$$

Then, we have

$$\dot{V}_n = \overline{q}_n\left(-g_{n-1}\varPhi_{n-1}z_{n-1}z_n - \varPhi_n z_n\hat{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n - \eta_n\varPhi_n^2 z_n^2 - k_n\varPhi_n z_n^2\right) + \tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{c}n} + \tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\dot{\hat{\boldsymbol{W}}}_{\mathrm{a}n}. \tag{S36}$$

From Eqs. (38) and (40) and $\boldsymbol{\omega}_n$, the HJB equation of the z_n-subsystem can be presented as follows:

$$\begin{aligned} &H_n\left(z_n, u^*, \frac{\partial J_n^*(z_n)}{\partial z_n}\right) \\ =&\overline{q}_n V_n^z - \frac{P_n^2}{g_n^2} + \frac{2P_n}{g_n^2}\left(f_n - \dot{\hat{\alpha}}_{n-1}\right) + \frac{2}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\left(f_n - P_n - \dot{\hat{\alpha}}_{n-1}\right) - \frac{1}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^* \\ =&0. \end{aligned} \tag{S37}$$

Consider the following equation:

$$\overline{q}_n V_n^z - \frac{P_n^2}{g_n^2} + \frac{2P_n}{g_n^2}\left(f_n - \dot{\hat{\alpha}}_{n-1}\right) = -\frac{2}{g_n^2}\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{W}_n^* - \frac{2}{g_n^2}\hat{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^* + \frac{1}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^*. \tag{S38}$$

From Eqs. (S37) and (S38), Eq. (S36) can become

$$\begin{aligned}\dot{V}_n =& -\overline{q}_n g_{n-1}\Phi_{n-1}z_{n-1}z_n - \overline{q}_n\Phi_n z_n\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n - \overline{q}_n\eta_n\Phi_n^2 z_n^2 - \overline{q}_n k_n\Phi_n z_n^2 - \frac{\xi_{\mathrm{c}n}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n}{\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1}\\ &\cdot\left[\frac{2}{g_n^2}\boldsymbol{\omega}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n} + \frac{1}{g_n^2}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} - \frac{2}{g_n^2}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^* + \frac{1}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^*\right]\\ &+\overline{q}_n\Phi_n z_n\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n - \xi_{\mathrm{an}}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} + \frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}\boldsymbol{\omega}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{c}n},\end{aligned}\tag{S39}$$

where

$$\frac{1}{g_n^2}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} - \frac{2}{g_n^2}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^* + \frac{1}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^* = \frac{1}{g_n^2}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} - \frac{1}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{an}}.$$

Rewrite Eq. (S39) as

$$\begin{aligned}\dot{V}_n =& -\overline{q}_n g_{n-1}\Phi_{n-1}z_{n-1}z_n - \overline{q}_n\Phi_n z_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n - \overline{q}_n\eta_n\Phi_n^2 z_n^2 - \overline{q}_n k_n\Phi_n z_n^2 - \frac{\xi_{\mathrm{c}n}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n}{\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1}\\ &\cdot\left[\frac{2}{g_n^2}\boldsymbol{\omega}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n} + \frac{1}{g_n^2}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} - \frac{1}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{an}}\right] - \xi_{\mathrm{an}}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}\\ &+\frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}\boldsymbol{\omega}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{c}n},\end{aligned}\tag{S40}$$

where

$$\begin{aligned}&-\frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} + \frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}\boldsymbol{\omega}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{c}n}\\ =&\frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}.\end{aligned}$$

Then, Eq. (S40) can be rewritten as

$$\begin{aligned}\dot{V}_n =& -\overline{q}_n g_{n-1}\Phi_{n-1}z_{n-1}z_n - \overline{q}_n\Phi_n z_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n - \overline{q}_n\eta_n\Phi_n^2 z_n^2 - \overline{q}_n k_n\Phi_n z_n^2 - \frac{\xi_{\mathrm{c}n}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n}{\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1}\\ &\cdot\left[\frac{2}{g_n^2}\boldsymbol{\omega}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n} - \frac{1}{g_n^2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{an}}\right] - \xi_{\mathrm{an}}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}\\ &+\frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}.\end{aligned}\tag{S41}$$

Based on the fact that $\tilde{\boldsymbol{W}}_{\mathrm{an}} = \hat{\boldsymbol{W}}_{\mathrm{an}} - \boldsymbol{W}_n^*$, the following equation holds:

$$\xi_{\mathrm{an}}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} = \frac{\xi_{\mathrm{an}}}{2}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{an}} + \frac{\xi_{\mathrm{an}}}{2}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} - \frac{\xi_{\mathrm{an}}}{2}(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^*.$$

According to Young's inequality, we have

$$-\overline{q}_n\Phi_n z_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n \le \frac{\overline{q}_n}{2}\left((\boldsymbol{W}_n^*)^{\mathrm{T}}\right)^2 + \frac{\overline{q}_n}{2}(\Phi_n z_n)^2.$$

Substituting the above inequality into Eq. (S41), we have

$$\begin{aligned}\dot{V}_n \le& -\left(\overline{q}_n\eta_n - \frac{\overline{q}_n}{2}\right)\Phi_n^2 z_n^2 - \overline{q}_n g_{n-1}\Phi_{n-1}z_{n-1}z_n - \overline{q}_n k_n\Phi_n z_n^2 - \frac{\xi_{\mathrm{an}}}{2}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}(z_n)\tilde{\boldsymbol{W}}_{\mathrm{an}}\\ &-\frac{2\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\omega}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n} + \frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}}\\ &+\frac{\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{an}} - \frac{\xi_{\mathrm{an}}}{2}\hat{\boldsymbol{W}}_{\mathrm{an}}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{an}} + \frac{\xi_{\mathrm{an}}+\overline{q}_n}{2}\left((\boldsymbol{W}_n^*)^{\mathrm{T}}\right)^2.\end{aligned}\tag{S42}$$

Similar to inequalities (S30) and (S31), two inequalities can be derived as follows:

$$\frac{1}{g_n^2}\frac{\xi_{cn}}{\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1}\tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}n} \le \frac{1}{g_n^2}\left[\frac{1}{4}\tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{W}_n^*\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}n}+\xi_{\mathrm{c}n}^2\hat{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}n}\right], \tag{S43}$$

$$\begin{aligned}&\frac{1}{g_n^2}\frac{\xi_{cn}}{\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}n}\\ \le&\frac{1}{g_n^2}\left[\frac{1}{4(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^*\boldsymbol{\omega}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n}+\xi_{\mathrm{c}n}^2\tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}n}\right].\end{aligned} \tag{S44}$$

Adding the inequalities obtained above to inequality (S42) yields

$$\begin{aligned}\dot{V}_n \le &-\left(\overline{q}_n\eta_n-\frac{\overline{q}_n}{2}\right)\Phi_n^2 z_n^2-\overline{q}_n g_{n-1}\Phi_{n-1}z_{n-1}z_n-\overline{q}_n k_n\Phi_n z_n^2-\left[\frac{\xi_{\mathrm{a}n}}{2}-\frac{\xi_{\mathrm{c}n}^2}{g_n^2}-\frac{(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{W}_n^*}{4g_n^2}\right]\\ &\cdot\tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}n}-\frac{1}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\left[2\xi_{\mathrm{c}n}-\frac{(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n}{4}\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^*\right]\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\omega}_n^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n}\\ &-\left(\frac{\xi_{\mathrm{a}n}}{2}-\frac{\xi_{\mathrm{c}n}^2}{g_n^2}\right)\hat{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\hat{\boldsymbol{W}}_{\mathrm{a}n}+\frac{\xi_{\mathrm{a}n}+\overline{q}_n}{2}\left((\boldsymbol{W}_n^*)^{\mathrm{T}}\right)^2.\end{aligned} \tag{S45}$$

Design parameters ξ_{cn}, $\xi_{\mathrm{a}n}$, $\overline{q}_n$, η_n, and $\boldsymbol{\omega}_n$, which satisfy the following inequalities:

$$\begin{cases}\overline{q}_n\eta_n-\dfrac{\overline{q}_n}{2}>0,\\ \dfrac{\xi_{\mathrm{a}n}}{2}-\dfrac{\xi_{\mathrm{c}n}^2}{g_n^2}-\dfrac{(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{W}_n^*}{4g_n^2}>0,\\ \dfrac{2\xi_{\mathrm{c}n}}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}-\dfrac{(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^*}{4g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}>0,\\ \dfrac{\xi_{\mathrm{a}n}}{2}-\dfrac{\xi_{\mathrm{c}n}^2}{g_n^2}>0.\end{cases}$$

Define

$$m_{\mathrm{a}n}=\inf_{t\ge 0}\left\{\frac{\pi_n\xi_{\mathrm{a}n}}{2}-\frac{\pi_n\xi_{\mathrm{c}n}^2}{g_n^2}-\frac{\pi_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\omega}_n\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{W}_n^*}{4g_n^2}\right\}$$

and

$$m_{\mathrm{c}n}=\inf_{t\ge 0}\left\{\frac{2\xi_{\mathrm{c}n}\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n}{g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}-\frac{\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n(\boldsymbol{W}_n^*)^{\mathrm{T}}\boldsymbol{\varphi}_n\boldsymbol{\varphi}_n^{\mathrm{T}}\boldsymbol{W}_n^*}{4g_n^2(\boldsymbol{\omega}_n^{\mathrm{T}}\boldsymbol{\omega}_n+1)}\right\},$$

where π_n is the dimension of $\hat{\boldsymbol{W}}_{\mathrm{c}n}$ and $\hat{\boldsymbol{W}}_{\mathrm{a}n}$. So, inequality (S45) can become

$$\dot{V}_n \le -\overline{q}_n k_n\Phi_n z_n^2-\overline{q}_n g_{n-1}\Phi_{n-1}z_{n-1}z_n-m_{\mathrm{a}n}\tilde{\boldsymbol{W}}_{\mathrm{a}n}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}n}-m_{\mathrm{c}n}\tilde{\boldsymbol{W}}_{\mathrm{c}n}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}n}+d_n, \tag{S46}$$

where $d_n=\dfrac{\xi_{\mathrm{a}n}+\overline{q}_n}{2}\left((\boldsymbol{W}_n^*)^{\mathrm{T}}\right)^2$.

Finally, construct the whole IBLF as

$$V_l=\sum_{i=1}^{n}V_i. \tag{S47}$$

From inequalities (S20), (S33), and (S46), the time derivative of V_l can be written as

$$\dot{V}_l \le -\sum_{i=1}^{n}\overline{q}_i k_i\Phi_i z_i^2-\sum_{i=1}^{n}m_{\mathrm{a}i}\tilde{\boldsymbol{W}}_{\mathrm{a}i}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{a}i}-\sum_{i=1}^{n}m_{\mathrm{c}i}\tilde{\boldsymbol{W}}_{\mathrm{c}i}^{\mathrm{T}}\tilde{\boldsymbol{W}}_{\mathrm{c}i}+d, \tag{S48}$$

where $d=\sum_{i=1}^{n}d_i=\frac{1}{2}\sum_{i=1}^{n}(\xi_{\mathrm{a}i}+\overline{q}_i)\left((\boldsymbol{W}_i^*)^{\mathrm{T}}\right)^2$. Since all terms of d are bounded, there exists a positive constant η such that $|d|<\eta$. Thus, inequality (S48) becomes

$$\dot{V}_l < -cV_l+\eta, \tag{S49}$$

where $c = \min\limits_{1 \leq i \leq n} \{k_i, 2m_{\mathrm{a}i}, 2m_{\mathrm{c}i}\}$. Therefore, the following inequality can be obtained:

$$V_l < V_l(0)\mathrm{e}^{-c(t-t_0)} + \frac{\eta}{c}. \tag{S50}$$

We can conclude that z_i, $\varPhi_i$, $\tilde{\boldsymbol{W}}_{\mathrm{c}i}$, and $\tilde{\boldsymbol{W}}_{\mathrm{a}i}$ are bounded by Proposition 2 and Assumption 2. Then, we have that x_i, $\hat{\boldsymbol{W}}_{\mathrm{c}i}$, and $\hat{\boldsymbol{W}}_{\mathrm{a}i}$ are bounded signals. From Eqs. (14), (29), and (44), it can be found that the optimal virtual controller α_i^* and the optimal actual controller u^* are both bounded and convergent, because each is a function formed by the simple combination of above-bounded signals. Thus, all signals of the system are bounded.