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Dynamic response of bilayered saturated porous media based on fractional thermoelastic theory

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Data S1 Derivation of the analytical solution for Eqs. (25) and (26)

For clarity, Eqs. (25) and (26) can be specifically rewritten for medium ① as:

$$\left[\frac{\partial^2}{\partial x^2}-\left(\frac{b_1 \mathrm{i}\omega}{(\lambda_1+2\mu_1)\xi_2}+\frac{\rho_{\mathrm{f}1}\omega^2}{\rho_1}-\omega^2\right)\right]e_1=\left[\frac{\beta_1 T_0}{\lambda_1+2\mu_1}\frac{\partial^2}{\partial x^2}-\frac{b_1\alpha_{\mathrm{f}1}\mathrm{i}\omega T_0}{(\lambda_1+2\mu_1)\xi_2}\right]\theta_1, \tag{A-1}$$

$$\frac{\beta_1 \mathrm{i}\omega}{m_1}\left(1+\frac{\tau_1^{\alpha_1}(\mathrm{i}\omega)^{\alpha_1}}{\alpha_1!}\right)e_1=\left[\frac{\partial^2}{\partial x^2}-\mathrm{i}\omega\left(1+\frac{\tau_1^{\alpha_1}(\mathrm{i}\omega)^{\alpha_1}}{\alpha_1!}\right)\right]\theta_1, \tag{A-2}$$

Eqs. (A-1) and (A-2) can be further simplified as:

$$\left(\frac{\partial^2}{\partial x^2}-a_1\right)e_1=\left(a_4\frac{\partial^2}{\partial x^2}-a_5\right)\theta_1, \tag{A-3}$$

$$a_3 e_1=\left(\frac{\partial^2}{\partial x^2}-a_2\right)\theta_1, \tag{A-4}$$

where $a_1=\dfrac{b_1\mathrm{i}\omega}{(\lambda_1+2\mu_1)\xi_2}+\dfrac{\rho_{\mathrm{f}j}\omega^2}{\rho_1}-\omega^2$, $a_2=\mathrm{i}\omega\left(1+\dfrac{\tau_1^{\alpha_1}(\mathrm{i}\omega)^{\alpha_1}}{\alpha_1!}\right)$, $a_3=\dfrac{\beta_1\mathrm{i}\omega}{m_1}\left(1+\dfrac{\tau_1^{\alpha_1}(\mathrm{i}\omega)^{\alpha_1}}{\alpha_1!}\right)$, $a_4=\dfrac{\beta_1 T_0}{\lambda_1+2\mu_1}$,

and $a_5=\dfrac{b_1\mathrm{i}\omega\alpha_{\mathrm{f}1}T_0}{(\lambda_1+2\mu_1)\xi_2}$.

Combining Eq. (A-3) and Eq. (A-4), one obtains the following fourth order partial differential equation:

$$\left(\frac{\partial^4}{\partial x^4}-A_1\frac{\partial^2}{\partial x^2}+A_2\right)\theta_1=0, \tag{A-5}$$

where $A_1=a_1+a_2+a_3a_4$ and $A_2=a_1a_2+a_3a_5$.

In a similar manner, the following expression can be derived:

$$\left(\frac{\partial^4}{\partial x^4}-A_1\frac{\partial^2}{\partial x^2}+A_2\right)e_1=0, \tag{A-6}$$

Eq. (A-5) can be further decomposed to:

$$\left(\frac{\partial^2}{\partial x^2}-M_1^2\right)\left(\frac{\partial^2}{\partial x^2}-M_2^2\right)\theta_1=0, \tag{A-7}$$

where $M_1^2=\dfrac{A_1+\sqrt{A_1^2-4A_2}}{2}$ and $M_2^2=\dfrac{A_1-\sqrt{A_1^2-4A_2}}{2}$ are the characteristic roots of Eq. (A-5).

Consequently, the solution of Eq. (A-5) can be given by:

$$\theta_1=\sum_{i=1}^{2}B_i e^{-M_i x}+\sum_{i=1}^{2}C_i e^{M_i x}, \tag{A-8}$$

where B_i and C_i are undetermined coefficients.

Data S2　Derivation of the analytical solution for medium 2

And similarly, the governing equations of medium ② are obtained as:

$$\left[\frac{\lambda_2+2\mu_2}{\rho_2\xi_1^2}\frac{\partial^2}{\partial x^2}-\left(\frac{b_2\mathrm{i}\omega}{\rho_2\xi_1^2\xi_2}+\frac{\rho_{\mathrm{f}2}\omega^2}{\rho_2}-\omega^2\right)\right]e_2=\left(\frac{\beta_2T_0}{\rho_2\xi_1^2}\frac{\partial^2}{\partial x^2}-\frac{b_2\mathrm{i}\omega\alpha_{\mathrm{f}2}T_0}{\rho_2\xi_1^2\xi_2}\right)\theta_2, \tag{B-1}$$

$$\frac{\beta_2\mathrm{i}\omega}{k_2\xi_2}\left(1+\frac{\tau_2^{\alpha_2}(\mathrm{i}\omega)^{\alpha_2}}{\alpha_2!}\right)e_2=\left[\frac{\partial^2}{\partial x^2}-\frac{m_2\mathrm{i}\omega}{k_2\xi_2}\left(1+\frac{\tau_2^{\alpha_2}(\mathrm{i}\omega)^{\alpha_2}}{\alpha_2!}\right)\right]\theta_2, \tag{B-2}$$

Eqs. (B-1) and (B-2) can be respectively simplified as:

$$\left(\eta_1\frac{\partial^2}{\partial x^2}-\eta_2\right)e_2=\left(\eta_3\frac{\partial^2}{\partial x^2}-\eta_4\right)\theta_2, \tag{B-3}$$

$$\eta_5e_2=\left(\frac{\partial^2}{\partial x^2}-\eta_6\right)\theta_2, \tag{B-4}$$

where $\eta_1=\frac{\lambda_2+2\mu_2}{\rho_2\xi_1^2}$, $\eta_2=\frac{b_2\mathrm{i}\omega}{\rho_2\xi_1^2\xi_2}+\frac{\rho_{\mathrm{f}2}\omega^2}{\rho_2}-\omega^2$, $\eta_3=\frac{\beta_2T_0}{\rho_2\xi_1^2}$, $\eta_4=\frac{b_2\mathrm{i}\omega\alpha_{\mathrm{f}2}T_0}{\rho_2\xi_1^2\xi_2}$, $\eta_5=\frac{\beta_2\mathrm{i}\omega\left(1+\frac{\tau_2^{\alpha_2}(\mathrm{i}\omega)^{\alpha_2}}{\alpha_2!}\right)}{k_2\xi_2}$, and $\eta_6=\frac{m_2\mathrm{i}\omega}{k_2\xi_2}\left(1+\frac{\tau_2^{\alpha_2}(\mathrm{i}\omega)^{\alpha_2}}{\alpha_2!}\right)$.

Similar to the derivation process of medium ①, a similar equation for medium ② is obtained from Eqs. (B-3) and (B-4) as:

$$\left(\eta_1\frac{\partial^4}{\partial x^4}-A_3\frac{\partial^2}{\partial x^2}+A_4\right)\theta_2=0, \tag{B-5}$$

where $A_3=\eta_1\eta_6+\eta_2+\eta_3\eta_5$ and $A_4=\eta_2\eta_6+\eta_4\eta_5$.

In a similar manner, the following expression can be derived:

$$\left(\eta_1\frac{\partial^4}{\partial x^4}-A_3\frac{\partial^2}{\partial x^2}+A_4\right)e_2=0, \tag{B-6}$$

Eq. (B-5) can be factorized as:

$$\left(\frac{\partial^2}{\partial x^2}-N_1^2\right)\left(\frac{\partial^2}{\partial x^2}-N_2^2\right)\theta_2=0, \tag{B-7}$$

where $N_1^2=\frac{A_3+\sqrt{A_3^2-4\eta_1A_4}}{2\eta_1}$ and $N_2^2=\frac{A_3-\sqrt{A_3^2-4\eta_1A_4}}{2\eta_1}$ are the characteristic roots of Eq. (B-5).

Consequently, according to the boundary conditions $x\to\infty$, $\theta_2=0$, $u_2=0$, and $p_{\mathrm{f}2}=0$, the solution of Eq. (B-5) can be given by:

$$\theta_2=\sum_{i=1}^{2}E_i e^{-N_i x} , \tag{B-8}$$

Similarly, the following expressions can be derived:

$$e_1=\sum_{i=1}^{2}G_i e^{-M_i x}+\sum_{i=1}^{2}H_i e^{M_i x} , \tag{B-9}$$

$$e_2=\sum_{i=1}^{2}I_i e^{-N_i x} , \tag{B-10}$$

where E_i, G_i, H_i, and I_i are undetermined coefficients.

Substituting Eqs. (A-8) and (B-9) into Eq. (A-4), the relationship between B_i and G_i, as well as that between C_i and H_i can be derived as:

$$G_i=\delta_i B_i ,\quad H_i=\delta_i C_i , \tag{B-11}$$

where $\delta_i=\dfrac{M_i^2-a_2}{a_3}$.

In addition, substituting Eqs. (B-8) and (B-10) into Eq. (B-3), the relationship between E_i and I_i can be derived:

$$I_i=\gamma_i E_i , \tag{B-12}$$

where $\gamma_i=\dfrac{N_i^2-\eta_6}{\eta_5}$.

Therefore, e_1 and e_2 can be respectively expressed as:

$$e_1=\sum_{i=1}^{2}\delta_i B_i e^{-M_i x}+\sum_{i=1}^{2}\delta_i C_i e^{M_i x} , \tag{B-13}$$

$$e_2=\sum_{i=1}^{2}\gamma_i E_i e^{-N_i x} , \tag{B-14}$$

According to $e_j=\partial u_j/\partial x$ and the boundary condition given by Eqs. (3)~(5), u_1 and u_2 can be respectively expressed as:

$$u_1=-\sum_{i=1}^{2}\frac{\delta_i}{M_i}B_i e^{-M_i x}+\sum_{i=1}^{2}\frac{\delta_i}{M_i}C_i e^{M_i x}+J_1 , \tag{B-15}$$

$$u_2=-\sum_{i=1}^{2}\frac{\gamma_i}{N_i}E_i e^{-N_i x} , \tag{B-16}$$

Substituting Eqs. (A-8) and (B-15) into Eq. (20), the pore water pressure in the first layer can be written as:

$$p_{\mathrm{f1}}=\sum_{i=1}^{2}\left[\frac{\lambda_1+2\mu_1}{\mu_1}\delta_i-\frac{\beta_1 T_0}{\mu_1}+\frac{\omega^2(\lambda_1+2\mu_1)}{\mu_1}\frac{\delta_i}{M_i^2}\right]B_i e^{-M_i x}+\sum_{i=1}^{2}\left[\frac{\lambda_1+2\mu_1}{\mu_1}\delta_i-\frac{\beta_1 T_0}{\mu_1}+\frac{\omega^2(\lambda_1+2\mu_1)}{\mu_1}\frac{\delta_i}{M_i^2}\right]C_i e^{M_i x}+\frac{\omega^2(\lambda_1+2\mu_1)}{\mu_1}J_1 x+J_2, \tag{B-17}$$

where J_1 and J_2 are undetermined coefficients.

Substituting Eqs. (B-8) and (B-16) into Eq. (20), the pore water pressure in the second layer can be written as:

$$p_{\mathrm{f2}}=\sum_{i=1}^{2}\left[\frac{\lambda_2+2\mu_2}{\mu_1}\gamma_i-\frac{\beta_2 T_0}{\mu_1}+\frac{\omega^2\rho_2\xi_1^2}{\mu_1}\frac{\gamma_i}{N_i^2}\right]E_i e^{-N_i x}, \tag{B-18}$$

Substituting Eqs. (A-8), (B-15), and (B-16) into Eq. (19), the stresses in the first and second layers can be obtained as:

$$\sigma_1=-\sum_{i=1}^{2}\left[\frac{\omega^2(\lambda_1+2\mu_1)}{\mu_1}\frac{\delta_i}{M_i^2}\right]B_i e^{-M_i x}-\sum_{i=1}^{2}\left[\frac{\omega^2(\lambda_1+2\mu_1)}{\mu_1}\frac{\delta_i}{M_i^2}\right]C_i e^{M_i x}-\frac{\omega^2(\lambda_1+2\mu_1)}{\mu_1}J_1 x-J_2, \tag{B-19}$$

$$\sigma_2=-\sum_{i=1}^{2}\frac{\omega^2\rho_2\xi_1^2}{\mu_1}\frac{\gamma_i}{N_i^2}E_i e^{-N_i x}. \tag{B-20}$$

Performing normalization on Eqs. (1), (2), and (6)~(11), the corresponding expressions can be written in frequency domain as follows:

$$\sigma_1\big|_{x=0}=0, \tag{B-21}$$

$$\theta_1\big|_{x=0}=\theta_0, \tag{B-22}$$

$$p_{\mathrm{f1}}\big|_{x=0}=0, \tag{B-23}$$

$$X_{\mathrm{T}}\frac{\partial\theta_1}{\partial x}\bigg|_{x=h}=-\left[1+\frac{\tau_1^{\alpha_1}}{\alpha_1!}(\mathrm{i}\omega)^{\alpha_1}\right](\theta_1\big|_{x=h}-\theta_2\big|_{x=h}). \tag{B-24}$$

where $X_{\mathrm{T}}=k_1\xi_1^2\xi_2^2/hm$ denotes the dimensionless thermal contact resistance.

$$\frac{\partial\theta_1}{\partial x}\bigg|_{x=h}=\frac{k_2}{k_1}\frac{\left[1+\frac{\tau_1^{\alpha_1}}{\alpha_1!}(\mathrm{i}\omega)^{\alpha_1}\right]}{\left[1+\frac{\tau_2^{\alpha_2}}{\alpha_2!}(\mathrm{i}\omega)^{\alpha_2}\right]}\frac{\partial\theta_2}{\partial x}\bigg|_{x=h}, \tag{B-25}$$

$$u_1\big|_{x=h}=u_2\big|_{x=h}, \tag{B-26}$$

$$\frac{\sigma_2\big|_{x=h}}{\sigma_1\big|_{x=h}}=\frac{2}{1+Z_1/Z_2}, \tag{B-27}$$

$$p_{\mathrm{f1}}\big|_{x=h}=p_{\mathrm{f2}}\big|_{x=h}. \tag{B-28}$$

Substituting Eqs. (A-8), (B-8) and (B-15)~(B-20) into Eqs. (B-21)~(B-28), the undetermined coefficients B_i, C_i, E_i, and J_i can be derived, which form the analytical solutions to the THM coupling response of the bilayered saturated porous media.