Electronic Supplementary Materials

For https://doi.org/10.1631/jzus.A2200014

Numerical modelling and experimental investigation of a two-phase sink vortex and its fluid-solid vibration characteristics

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S1 Two-phase filed spatial discretization method

A spatial discretization solution is needed to replace the continuous space with a finite number of discrete points. It divides the calculated distance into many non-overlapping subspaces and confirms the position of the node in each subspace and the control volume it represents. In this study, the high order ENO method was adopted to calculate the change of the phase interface. If ϕ and u are in the center of each Cartesian coordinate system lattice, then

$$u\cdot\nabla\phi=u_{ij}(\phi_{i+j/2}-\phi_{i-j/2})/\Delta x+v_{ij}(\phi_{ij+1/2}-\phi_{ij-1/2})/\Delta y, \tag{S1}$$

$$u\cdot\nabla u=u_{ij}-(u_{i+j/2}-u_{i-j/2})/\Delta x+v_{ij}(u_{ij+1/2}-u_{ij-1/2})/\Delta y. \tag{S2}$$

In the above equation:

$$u_{ij}=(\tilde{u}_{i+j/2}+\tilde{u}_{i-j/2})/2, \tag{S3}$$

$$v_{ij}=(\tilde{v}_{i+j/2}+\tilde{v}_{i-j/2})/2, \tag{S4}$$

$$\tilde{u}_{i+j/2}=(s_{i+j/2+1/2}+s_{i+j/2-1/2})/\Delta y, \tag{S5}$$

$$\tilde{v}_{ij+1/2}=(s_{i+j/2+1/2}+s_{i-j/2-1/2})/\Delta x. \tag{S6}$$

where i, j are the subscripts of each discrete point. The relationship between u and v can be expressed as

$$(\tilde{u}_{i+j/2}-\tilde{u}_{i-j/2})/\Delta x+(\tilde{v}_{ij+1/2}-\tilde{v}_{ij-j/2})/\Delta y=0. \tag{S7}$$

The boundary values such as $\phi_{i\pm j/2}$, $u_{i\pm j/2}$, $\phi_{ij\pm 1/2}$, and $u_{ij\pm 1/2}$ can be calculated using the high order ENO method. Taking $f_{i+j/2}$ (the f value at the (i+1/2, j) point) as an example:

(1) First order

$$f_{i+1/2,j}^{(1)} = f_{k_1,j}. \tag{S8}$$

(2) Second order

$$a = \frac{f_{k_1,j} - f_{k_1-1,j}}{\Delta x}, \tag{S9}$$

$$b = \frac{f_{k_1+1,j} - f_{k_1,j}}{\Delta x}, \tag{S10}$$

$$c = \begin{cases} a, & |a| \le |b| \\ b, & |a| > |b| \end{cases}, \tag{S11}$$

$$f_{i+j/2}^{(2)} = f_{i+1/2,j}^{(1)} + \frac{\Delta x}{2} c(1-2(k_1 - i)). \tag{S12}$$

(3) Third order

$$a = \frac{f_{k_2-1,j} - 2f_{k_2,j} + f_{k_2+1,j}}{\Delta x^2}, \tag{S13}$$

$$b = \frac{f_{k_2,j} - 2f_{k_2+1,j} + f_{k_2+2,j}}{\Delta x^2}, \tag{S14}$$

$$c = \begin{cases} a, & |a| \le |b| \\ b, & |a| > |b| \end{cases}, \tag{S15}$$

$$f_{i+j/2}^{(3)} = f_{i+1/2,j}^{(2)} + \frac{\Delta x^2}{3} c(3(k_2 - i)^2 - 1). \tag{S16}$$

Time discretization takes the high-order TVD format. It is assumed that the time of the phase interface at a certain point is t^n, so that $\phi^{n}=\phi(t^n)$ represents the current value, and $\phi^{n+1}=\phi(t^{n+1})$ represents the ϕ value after Δt. Thus, the discrete form is obtained as follows:

$$\bar{u}_{n+1} = u_n + \Delta t u_{tn}, \tag{S17}$$

$$u_{n+1} = u_n + \frac{\Delta t}{2}(\bar{u}_{tn+1} + u_{tn}). \tag{S18}$$

In the calculation, the Courant-Friedrichs-Lewy (CFL) condition is a necessary criterion for stability and convergence while solving certain partial differential equations (usually hyperbolic PDEs) numerically. For the discrete value to meet the CFL condition, Δt is required to be set as

$$\Delta t^{n+1} = \frac{1}{2}\min(\Delta t_v, \Delta t_s, \Delta t_c). \tag{S19}$$

where Δt_s, Δt_v, Δt_c are the corresponding infinitesimal time intervals of gravity, viscosity, and surface tension, respectively.

S2 Mesh independence study

Mesh quantity dramatically affects the accuracy of vortex flow field transient simulation results. To guarantee both the discrete and round-off errors to be within the acceptable range, a mesh independence study for a mechanical dynamic model was performed to satisfy the precision and repeatability of the simulation results. The velocity and pressure data in flow field are shown in Fig. S1 under the condition that the outlet d=14 mm at the drainage time t=1.2 s.

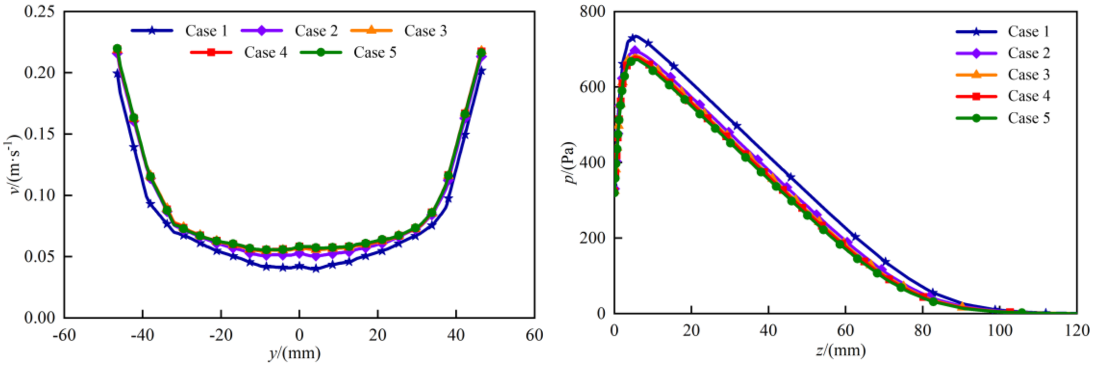

Fig. S1 Mesh independence study for the mechanical dynamic model (a) Velocity (b) Total pressure

Five computational models with different mesh numbers (cases 1 to 5) were tested. The mesh size, number, and quantity are listed in Table S1. The numerical results of vortex velocity and total pressure showed similar variation trends. As the mesh quantity reached a specific value, the velocity *v* and pressure *p* curves (cases 3 and 4) showed uniform patterns inconsistent with the variable values (case 5). However, lower mesh resolutions (cases 1 and 2) did not provide sufficiently accurate results. Therefore, the mesh resolutions with a numerical error of less than 5% had already satisfied the requirement of mesh independence and could guarantee simulation accuracy and repeatability. We adopted the mesh size consistent with case 3 to save computation time.

Table S1 Computation matrix of mesh size and quantity

Case number	Number of elements	Maximum mesh size	Minimum mesh size
1	95839	7.43 mm	0.540 mm
2	241063	4.73 mm	0.203 mm
3	475098	3.63 mm	0.135 mm
4	783429	2.98 mm	0.051 mm
5	1048019	2.71 mm	0.027 mm