Electronic Supplementary Materials
For https://doi.org/10.1631/jzus.A2200142

Effect of hydraulic fracture deformation hysteresis on CO_2 huff-n-puff performance in shale gas reservoirs

Xia YAN[1], Pi-yang LIU[2], Zhao-qin HUANG[1], Hai SUN[1], Kai ZHANG[1,2], Jun-feng WANG[3], Jun YAO[1]✉

[1]*School of Petroleum Engineering, China University of Petroleum (East China), Qingdao 266580, China*

[2]*Civil Engineering School, Qingdao University of Technology, Qingdao 266520, China*

[3]*Chuandong Drilling Company, CNPC Chuanqing Drilling Engineering Company Limited, Chongqing 401120, China*

✉ Jun YAO, RCOGFR_UPC@126.com

S1 Multi-component flow model and geomechanics equations discretization

S1.1 Multi-component flow model

K_n is the Knudsen number, and α_K is the rarefaction coefficient, they are defined as (Song et al., 2016; Fan et al., 2019; Lijun et al., 2019):

$$K_\mathrm{n} = \frac{\lambda}{r_\mathrm{h}}, \quad \alpha_\mathrm{K} = \frac{1.358}{1 + 0.170 K_\mathrm{n}^{-0.4348}} \tag{S1}$$

where λ is the mean free paths of gas molecules, and r_h is the pore radius (Song et al., 2016).

$$\lambda = \frac{\mu}{p}\sqrt{\frac{\pi z R T}{2 M_\mathrm{g}}}, \quad r_\mathrm{h} = 2\sqrt{2\tau_0}\sqrt{\frac{k_\infty}{\phi_0}} \tag{S2}$$

where z is the gas compressibility factor; T is the reservoir temperature; R is the universal gas constant; M_g is the gas molar weight; τ_0 is the initial tortuosity, and ϕ_0 is the initial porosity.

S1.2 Geomechanics equations discretization

In this study, we added the Heaviside fracture tip asymptotic and junction enrichment functions to the standard finite element space for simulating the displacement discontinuity at hydraulic fractures. Then, the expression of the enriched displacement field is given as

$$\begin{aligned}\boldsymbol{u}(\boldsymbol{x}) = & \sum_{i\in I} \boldsymbol{N}_i(\boldsymbol{x})\boldsymbol{u}_i + \sum_{j=1}^{N_\mathrm{dis}} \sum_{i\in I^\mathrm{dis}} \boldsymbol{N}_i(\boldsymbol{x})(H(\boldsymbol{x}) - H(\boldsymbol{x}_i))\boldsymbol{a}_i + \sum_{k=1}^{N_\mathrm{tip}} \sum_{i\in I^\mathrm{tip}} \boldsymbol{N}_i(\boldsymbol{x}) \sum_{\alpha=1}^{4} (\boldsymbol{F}_\mathrm{tip}^\alpha(\boldsymbol{x}) - \boldsymbol{F}_\mathrm{tip}^\alpha(\boldsymbol{x}_i))\boldsymbol{b}_i^\alpha \\ & + \sum_{l=1}^{N_\mathrm{jun}} \sum_{i\in I^\mathrm{jun}} \boldsymbol{N}_i(\boldsymbol{x})(J(\boldsymbol{x}) - J(\boldsymbol{x}_i))\boldsymbol{c}_i\end{aligned} \tag{S3}$$

where I denotes the node set of a grid; subscript i denotes the i-node; $\boldsymbol{N}$ is the standard shape function; I^{dis}, I^{tip}, and I^{jun} are the node subsets to enrich for the fracture discontinuity, tip and junction, respectively; N_{dis}, N_{tip}, and N_{jun} are the numbers of fractures, fracture tips and fracture junctions, respectively; $\boldsymbol{u}$, $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are the standard and additional degrees of freedom (DOFs), respectively; H, $\boldsymbol{F}$, and J represent the Heaviside function, fracture tip asymptotic function and junction function, respectively (Khoei, 2014). For convenience, Eq. S3 can be written as

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{N}_u^u(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{N}_u^a(\boldsymbol{x})\boldsymbol{a} + \boldsymbol{N}_u^b(\boldsymbol{x})\boldsymbol{b} + \boldsymbol{N}_u^c(\boldsymbol{x})\boldsymbol{c} \tag{S4}$$

where, $\boldsymbol{N}_u^u(\boldsymbol{x})$, $\boldsymbol{N}_u^a$, $\boldsymbol{N}_u^b$, and $\boldsymbol{N}_u^c$ are the matrix of the standard and enriched shape functions, respectively. Therefore, the corresponding strain vector is given by

$$\boldsymbol{\varepsilon}(\boldsymbol{x}) = \boldsymbol{B}_u^u(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{B}_u^a(\boldsymbol{x})\boldsymbol{a} + \boldsymbol{B}_u^b(\boldsymbol{x})\boldsymbol{b} + \boldsymbol{B}_u^c(\boldsymbol{x})\boldsymbol{c} \tag{S5}$$

where $\boldsymbol{B}_u^u(\boldsymbol{x}) = \boldsymbol{L}\boldsymbol{N}_u^u(\boldsymbol{x})$, $\boldsymbol{B}_u^a(\boldsymbol{x}) = \boldsymbol{L}\boldsymbol{N}_u^a(\boldsymbol{x})$, $\boldsymbol{B}_u^b(\boldsymbol{x}) = \boldsymbol{L}\boldsymbol{N}_u^b(\boldsymbol{x})$, $\boldsymbol{B}_u^c(\boldsymbol{x}) = \boldsymbol{L}\boldsymbol{N}_u^c(\boldsymbol{x})$, and $\boldsymbol{L}$ indicates the matrix differential operator

$$\boldsymbol{L} = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \end{bmatrix} \tag{S6}$$

Now, we substitute Eqs. S4 and S5 into Eq. 15, and consider that Eq. 15 must apply for any kinematically admissible test function, then Eq. 15 is written as

$$\boldsymbol{f}_\alpha^{\mathrm{int}} = \boldsymbol{f}_\alpha^{\mathrm{ext}}, \quad \alpha = u, a, b, c \tag{S7}$$

with

$$\boldsymbol{f}_\alpha^{\mathrm{int}} = \int_\Omega \left(\boldsymbol{B}_u^\alpha\right)^{\mathrm{T}} \boldsymbol{C}_a \boldsymbol{\varepsilon} \mathrm{d}\Omega \tag{S8}$$

$$\begin{aligned} \boldsymbol{f}_\alpha^{\mathrm{ext}} = {} & \int_\Omega \left(\boldsymbol{B}_u^\alpha\right)^{\mathrm{T}} \boldsymbol{m} p_a \mathrm{d}\Omega + \int_{\Gamma_{\mathrm{HF}}} [\![\boldsymbol{N}_u^\alpha]\!]^{\mathrm{T}} \left(p_{\mathrm{HF}} + \sigma_{\mathrm{n}}\right) \cdot \boldsymbol{n}_{\mathrm{HF}} \mathrm{d}\Gamma + \int_\Omega \left(\boldsymbol{N}_u^\alpha\right)^{\mathrm{T}} \boldsymbol{f} \mathrm{d}\Omega + \int_{\Gamma \mathrm{t}} \left(\boldsymbol{N}_u^\alpha\right)^{\mathrm{T}} \bar{\boldsymbol{t}} \mathrm{d}\Gamma \\ & \underbrace{- \int_{\Gamma_{\mathrm{HF}}} \frac{\tau}{2M} \left(- \frac{\partial \sigma_{\mathrm{n}}}{\partial \varepsilon_{\mathrm{n}}} \frac{\boldsymbol{n}_{\mathrm{HF}}}{d_{\mathrm{HF0}}} \right) \cdot \left([\![\boldsymbol{N}_u^\alpha]\!] - \Pi [\![\boldsymbol{N}_u^\alpha]\!] \right)^{\mathrm{T}} \left(\sigma_{\mathrm{n}} - \Pi \sigma_{\mathrm{n}} \right) \mathrm{d}\Gamma}_{\text{Stabilizing term}} \end{aligned} \tag{S9}$$

where $\boldsymbol{f}^{\mathrm{int}}$ and $\boldsymbol{f}^{\mathrm{ext}}$ represent the internal and external force vectors; $\boldsymbol{m}$=[1, 1, 1, 0, 0, 0]$^{\mathrm{T}}$.