Electronic supplementary materials

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Finite-time path following control of a sailboat with actuator failure and unknown sideslip angle

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Lemma S1: (Yu et al., 2018) For system

$$\dot{x} = f\left(x\right). \tag{S1}$$

Suppose there exists a continuously differentiable function $V(x)$, which satisfies

$$\dot{V}\left(x\right) \le -a_3 V\left(x\right) - b_1 V^{\rho}\left(x\right) + c_1, \tag{S2}$$

where $a_3 > 0$, $b_1 > 0$, $0 < \rho < 1$, $0 < c_1 < \infty$.

Then the trajectory of system $\dot{x} = f\left(x\right)$ can achieve finite time stability, and the residual set of the solution of system $\dot{x} = f\left(x\right)$ is given by

$$\left\{ \lim_{t \to T_{\mathrm{f}}} | V\left(x\right) \le \min\left\{ \frac{c_1}{\left(1-\theta_0\right) a_3}, \left[\frac{c_1}{\left(1-\theta_0\right) b_1} \right]^{\frac{1}{\rho}} \right\} \right\}, \tag{S3}$$

where $0 < \theta_0 < 1$. In addition, the finite setting time T_{f} satisfies

$$\begin{aligned} T_{\mathrm{f}} = \max\Bigg\{ & t_0 + \frac{1}{a_3 \theta_0 \left(1-l\right)} \ln \frac{\theta_0 a_3 V^{1-\rho}\left(t_0\right) + b_1}{b_1}, \\ & t_0 + \frac{1}{a_3 \left(1-l\right)} \ln \frac{a_3 V^{1-\rho}\left(t_0\right) + \theta_0 b_1}{\theta_0 b_1} \Bigg\}. \end{aligned} \tag{S4}$$

Lemma S2: (Shtessel et al., 2007) The following system

$$
\begin{aligned}
&\dot{\sigma}_0 = -\lambda_0 L^{\frac{1}{n+1}} \left|\sigma_0\right|^{\frac{n}{n+1}} \operatorname{sign}\left(\sigma_0\right) + \sigma_1, \\
&\dot{\sigma}_1 = -\lambda_1 L^{\frac{1}{n}} \left|\sigma_1 - \dot{\sigma}_0\right|^{\frac{n-1}{n}} \operatorname{sign}\left(\sigma_1 - \dot{\sigma}_0\right) + \sigma_2, \\
&\quad \vdots \\
&\dot{\sigma}_{n-1} = -\lambda_{n-1} L^{\frac{1}{2}} \left|\sigma_{n-1} - \dot{\sigma}_{n-2}\right|^{1/2} \operatorname{sign}\left(\sigma_{n-1} - \dot{\sigma}_{n-2}\right) + \sigma_n, \\
&\dot{\sigma}_n \in -\lambda_n L \operatorname{sign}\left(\sigma_{n-1} - \dot{\sigma}_{n-2}\right) + \left[-L, L\right]
\end{aligned}
\tag{S5}
$$

is finite-time stable, where $L > 0$, $\lambda_i > 0$, $i = 0,1,2,...,n$.

Lemma S3: (Basin et al., 2017) The following system

$$
\begin{aligned}
&\dot{\eta}_1 = -\lambda_1 \left|\eta_1 - g\right|^{\alpha_1} \operatorname{sign}\left(\eta_1 - g\right) + \eta_2, \\
&\dot{\eta}_i = -\lambda_i \left|\eta_1 - g\right|^{\alpha_i} \operatorname{sign}\left(\eta_1 - g\right) + \eta_{i+1}, \\
&\dot{\eta}_n = -\lambda_n \left|\eta_1 - g\right|^{\alpha_n} \operatorname{sign}\left(\eta_1 - g\right),
\end{aligned}
\tag{S6}
$$

where $\lambda_i \left(i = 0, 1, ..., n\right)$ are positive constants and $\alpha_i \in \left(0, 1\right)$, satisfying $\alpha_i = i\alpha - \left(i - 1\right)$ and $\alpha_1 \in \left(1 - \sigma, 1\right)$ for a sufficiently small $\sigma > 0$. Gains λ_i are assigned such that the matrix $\boldsymbol{A}$ is Hurwitz, which is the dynamics matrix of the observer estimation error system obtained from Eq. (65), upon setting $\alpha_i = 1$. The convergence time can be estimated as $T \le V^{\rho}\left[\boldsymbol{\xi}\left(\boldsymbol{e}\left(t_0\right)\right)\right] / (\gamma\rho)$, where $V\left(\boldsymbol{\xi}\right) = \boldsymbol{\xi}^{\mathrm{T}}\left(e\right)\boldsymbol{P}\boldsymbol{\xi}\left(e\right)$, $\boldsymbol{\xi} = \left[e_1^{\alpha_1}, e_2^{\alpha_2}, ..., e_n^{\alpha_n}\right]^{\mathrm{T}} \in \boldsymbol{R}^n$, $\boldsymbol{e}\left(t\right) = \left[e_1, e_2, ..., e_n\right]^{\mathrm{T}} \in \boldsymbol{R}^{\mathrm{T}}$, $e_i = \eta_i - g^{i-1}$, $\rho = 1 - \alpha$, $\gamma = \lambda_{\min}\left(\boldsymbol{Q}\right) / \lambda_{\max}\left(\boldsymbol{P}\right)$, $\boldsymbol{Q} \in R^{n\times m}$ is a symmetric positive-definite matrix, satisfying $\boldsymbol{PA} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} = -\boldsymbol{Q}$, $\lambda_{\min}\left(\boldsymbol{Q}\right)$ and $\lambda_{\max}\left(\boldsymbol{P}\right) > 0$.

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