

## Perspective:

# Theoretical foundation of a decision network for urban development\*

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Received July 21, 2015; Revision accepted Nov. 10, 2016; Crosschecked Aug. 1, 2017

**Abstract:** Planning problems are challenging and complex in that they usually involve multiple stakeholders with multi-attribute preferences. Thus few, if any, planning tools are useful in helping planners address such problems. Decision analysis is less useful than expected in dealing with planning problems because it focuses overwhelmingly on making a single decision for a particular decision-maker. In this paper, we describe the theoretical foundation of a planning tool called the ‘decision network’, which aims to help planners make multiple and linked decisions when facing multiple stakeholders with multi-attribute preferences. The research provides a starting point for a fully fledged technology that is useful for dealing with complex planning problems. We first provide a general formulation of the planning problem that the decision network intends to address. We then introduce an efficient solution algorithm for this problem, with a numerical example to demonstrate how the algorithm works. The proposed solution algorithm is efficient, allowing computerization of the planning tool. We also demonstrate that the diagrammatic representation of the decision network is more efficient than that of a decision tree. Therefore, when dealing with challenging and complex planning problems, using the decision network to make multiple and linked decisions may yield more benefits than making such decisions independently.

**Key words:** Decision making; Linked decisions; Decision network; Planning

<http://dx.doi.org/10.1631/FITEE.1510000>

**CLC number:** F291.1

## 1 Introduction

Planning is an important topic in the field of artificial intelligence (Ghallab *et al.*, 2004; Pollock, 2006). However, in real-world situations, most planning problems are complex and challenging, not only because the problems are ambiguous and difficult to define, but also because they involve multiple stakeholders with multi-attribute preferences. These preferences are difficult to elicit. Planners have to make more than one decision at a time, contradicting the view that making linked decisions is rare (Keeney, 2004). In short, planning problems are ill-defined

(Hopkins, 1984); therefore, their solution algorithms must be different from those of well-defined problems. Traditional techniques developed in decision analysis (such as the decision tree) focus on a single decision-maker with unidimensional attributes (such as utility), and evaluate a given set of alternatives to select the best. In real situations, the complex nature of planning problems renders such techniques less useful than expected in helping decision-makers figure out what to do. As an alternative to traditional techniques, we propose a theoretical foundation for a technique called the ‘decision network’ for making multiple and linked decisions that involve multiple stakeholders with multi-attribute preferences. The decision network is derived mainly from the ideas of the decision tree (Kirkwood, 1996), the strategic choice approach (Friend and Hickling, 2005), and the garbage can model (Cohen *et al.*, 1972). A detailed

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\* Project supported by the Ministry of Science and Technology of Taiwan (No. NSC101-2410-H305-066-MY2)

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description of the conceptual framework of the decision network was provided by Han and Lai (2011), together with a description of its application to the management of urban growth boundaries (Han and Lai, 2012). Here, we provide a formal and general formulation of the framework, and describe a solution algorithm for the formulation.

## 2 Formulation

Consider  $m$  decision-makers eligible to attend any of  $n$  decision situations. Decision-makers are individuals or coherent groups with authority and capability to make decisions. Decision situations are the choice opportunities in which problems and solutions are discussed and evaluated by decision-maker(s). Assume that there are  $p$  problems and  $q$  solutions under consideration. A utility is the affected individual's (or stakeholder's) level of satisfaction imposed by decision-makers, problems, and solutions in a decision situation. Whether a decision is made in a decision situation depends on whether the amount of supplied utilities exceeds that of the utilities demanded by the decision situation. Thus, utility can be either positive or negative (disutility). Note that decision situations can be either deterministic (denoted as decision nodes) or stochastic (denoted as chance nodes); i.e., some decision situations are pre-determined, whereas others are probabilistic in occurrence. Given these definitions, we can define the variables and parameters of the decision network problem as shown in Table 1.

With these variables and parameters, there are three structures in a decision network, namely, decision structure, access structure, and solution structure. All the structures are represented by 0-1 matrices. The only difference is that the rows in these matrices are decision-makers, problems, and solutions, respectively, with the columns representing decision situations. '1' in these matrices implies that the element in the row is related to the decision situation in its corresponding column, and '0' means that no such relationship exists.

For example, in the decision structure, if the value of the cell in row three (e.g., a planner) and column five (e.g., a public hearing) is 1, it means that the planner is eligible to participate in public hearing

**Table 1 Definitions of variables and parameters of the formulation**

Decision situation	Notation	Probability	Utility
Decision node 1	$d_1$	1.0	Not applicable
Decision node 2	$d_2$	1.0	Not applicable
⋮	⋮	⋮	⋮
Decision node $i$	$d_i$	1.0	Not applicable
Chance node $i+1$	$d_{i+1}$	$p_{i+1}$	Not applicable
Chance node $i+2$	$d_{i+2}$	$p_{i+2}$	Not applicable
⋮	⋮	⋮	⋮
Chance node $n$	$d_n$	$p_n$	Not applicable

Decision-maker	Notation	Probability	Utility
1	$m_1$	Not applicable	$u_1$
2	$m_2$	Not applicable	$u_2$
⋮	⋮	⋮	⋮
$m$	$m_m$	Not applicable	$u_m$

Problem	Notation	Probability	Utility
1	$r_1$	Not applicable	$v_1$
2	$r_2$	Not applicable	$v_2$
⋮	⋮	⋮	⋮
$p$	$r_p$	Not applicable	$v_p$

Solution	Notation	Probability	Utility
1	$s_1$	Not applicable	$w_1$
2	$s_2$	Not applicable	$w_2$
⋮	⋮	⋮	⋮
$q$	$s_q$	Not applicable	$w_q$

for the decision-making. The generic forms of the three matrices are shown in Tables 2–4.

Note that  $a_{ij} \in \{0, 1\}$  ( $i=1, 2, \dots, m$  and  $j=1, 2, \dots, n$ ),  $b_{ij} \in \{0, 1\}$  ( $i=1, 2, \dots, p$  and  $j=1, 2, \dots, n$ ), and  $c_{ij} \in \{0, 1\}$  ( $i=1, 2, \dots, q$  and  $j=1, 2, \dots, n$ ).

The variables and parameters in Table 1, together with the structural constraints specified in Tables 2–4, form the basic information for the decision network problem, which can be represented in a directed graph, as demonstrated by Han and Lai (2011) in a numerical example.

The task is then to make a 'plan' by assigning the given  $m$  decision-makers,  $p$  problems, and  $q$  solutions to  $n$  decision situations to yield the highest overall expected utility under the structural constraints. Mathematically, this task can be easily formulated as a 0-1 integer program:

$$\max \sum_{j=1}^n p_j \left( \sum_{i=1}^m x_{ij} u_i + \sum_{k=1}^p y_{kj} v_k + \sum_{l=1}^q z_{lj} w_l \right) \quad (1)$$

$$\text{s.t. } \sum_{j=1}^n y_{kj} = 1, \quad k = 1, 2, \dots, p,$$

$$\sum_{j=1}^n z_{lj} = 1, \quad l = 1, 2, \dots, q,$$

where  $p_j=1$  if  $d_j$  is a decision node, and  $0 < p_j < 1$  otherwise, and  $x_{ij}, y_{kj}, z_{lj} = 0, 1$  for  $j=1, 2, \dots, n$ .

Note that in Eq. (1), the two constraints require problems and solutions be assigned to one, and only one, decision situation. Also, note that for each given decision network problem under consideration, Eq. (1) is problem-dependent in that the objective function changes in relation to the three structural constraints specified in Tables 2–4.

**Table 2 The 0-1 matrix for decision structure**

	$d_1$	$d_2$	...	$d_n$
$m_1$	$a_{11}$	$a_{12}$	...	$a_{1n}$
$m_2$	$a_{21}$	$a_{22}$	...	$a_{2n}$
⋮	⋮	⋮	⋮	⋮
$m_m$	$a_{m1}$	$a_{m2}$	...	$a_{mn}$

**Table 3 The 0-1 matrix for access structure**

	$d_1$	$d_2$	...	$d_n$
$r_1$	$b_{11}$	$b_{12}$	...	$b_{1n}$
$r_2$	$b_{21}$	$b_{22}$	...	$b_{2n}$
⋮	⋮	⋮	⋮	⋮
$r_p$	$b_{p1}$	$b_{p2}$	...	$b_{pn}$

**Table 4 The 0-1 matrix for solution structure**

	$d_1$	$d_2$	...	$d_n$
$s_1$	$c_{11}$	$c_{12}$	...	$c_{1n}$
$s_2$	$c_{21}$	$c_{22}$	...	$c_{2n}$
⋮	⋮	⋮	⋮	⋮
$s_q$	$c_{q1}$	$c_{q2}$	...	$c_{qn}$

### 3 Solution algorithm

For a small- or medium-sized problem, solving Eq. (1) is straightforward using a commercial package, such as LINDO. When the problem size becomes large enough to involve thousands of decision-makers, problems, solutions, and decision situations, it would be cumbersome to construct the model and solve it through LINDO. An algorithm proposed by Kirkwood (1993) for solving large models involving sequential decisions under uncertainty could be applied. However, it would require the modeler reconstruct the

decision network problem into a decision tree. With thousands of variables and parameters as shown in Table 1, this reconstruction would render the solution algorithm inextricable. Alternatively, we develop a solution algorithm that is specific for solving large-scale decision network problems.

Consider the matrices in Tables 2–4 and denote them as  $\mathbf{D}$ ,  $\mathbf{A}$ , and  $\mathbf{S}$  respectively, shown as follows:

$$\mathbf{D} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pn} \end{pmatrix},$$

$$\mathbf{S} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{q1} & c_{q2} & \cdots & c_{qn} \end{pmatrix}. \tag{2}$$

Furthermore, suppose that  $\mathbf{e}$  and  $\mathbf{e}_k$  are the unit matrix and the  $k$ th unit matrix, respectively, where the  $k$ th unit vector in the  $k$ th unit matrix is equal to 1 or 0. Note that  $\mathbf{e}'$  and  $\mathbf{e}'_k$  are the transposes of  $\mathbf{e}$  and  $\mathbf{e}_k$ , respectively. More details on these symbolic representations of the unit matrix and the  $k$ th unit matrix are as follows:

$$\mathbf{e} = \begin{pmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \mathbf{e}' = [1 \cdots 1 \cdots 1], \mathbf{e}_k = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \mathbf{e}'_k = [0 \cdots 1 \cdots 0]. \tag{3}$$

The following steps summarize and describe the solution algorithm:

Step 1: Retrieve the row vectors for the access structure, i.e.,  $\mathbf{p}_i = \mathbf{e}'_i \cdot \mathbf{A}$ , where  $\mathbf{p}_i$  denotes the vector of the  $i$ th row, where  $i=1, 2, \dots, m$ .

Step 2: Identify the number of elements for each row where the values of the elements are equal to 1 to obtain  $\Omega_a$  decomposed matrices, i.e.,  $\mathbf{u}_i = \{p_{ij}=1, j=1, 2, \dots, n, i=1, 2, \dots, p\}$ , which is the set of indices of columns for row  $i$  where the element  $p_{ij}$  is equal to 1. Note that  $\langle \mathbf{u}_i \rangle$  is the number of elements in set  $\mathbf{u}_i$ , which by definition is greater than or equal to 1 for all  $i$ 's. Let  $\Omega_a$  be the number of all combinations of

non-zero elements across the rows for the access structure, with one, and only one, non-zero element in each row, and we have

$$\Omega_a = \langle \mathbf{u}_1 \rangle \cdot \langle \mathbf{u}_2 \rangle \cdot \dots \cdot \langle \mathbf{u}_m \rangle. \quad (4)$$

Each combination stands for a matrix where each row has one, and only one, element, whose value is equal to 1.

Step 3: Decompose the solution structure following steps 1 and 2 to obtain  $\Omega_s$  decomposed matrices.

Step 4: For each combination of the decomposed matrices of the access and solution structures, compute the overall expected utility and select the combination that yields the highest overall expected utility as the solution; i.e., there are a total of  $\Omega_a \cdot \Omega_s$  combinations of decomposed matrices across the access and solution structures. For each combination, we can compute the overall expected utility by summing the expected utility for each decision node. Mathematically, for each decision (or chance) node, the expected utility is equal to

$$p_l \left( \sum_i u_{il} \sum_j v_{jl} \sum_k w_{kl} \right), \quad (5)$$

where  $u_{il}$ ,  $v_{jl}$ , and  $w_{kl}$  are the (dis)utilities associated with decision node  $l$  for decision-maker(s) in the decision structure, problem(s) in the access structure, and solution(s) in the solution structure, respectively, and  $p_l$  is the probability that a decision (or chance) is obtained by node  $l$ .

### 4 Numerical examples

Following Han and Lai (2011), we use the same numerical example here to show how the algorithm works. Assume that  $\mathbf{D}$ ,  $\mathbf{A}$ , and  $\mathbf{S}$  are given as follows:

$$\mathbf{D} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad (6)$$

$$\mathbf{S} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

Assume that the probabilities associated with the five decision (chance) nodes are 1.0, 1.0, 1.0, 0.7, and 0.5, respectively, that the utilities associated with the two decision-makers are 0.7 and 0.3, respectively, that the disutilities associated with the three problems are  $-0.6$ ,  $-0.5$ , and  $-0.7$ , respectively, and that the utilities associated with the four solutions are 0.6, 0.3, 0.7, and 0.5, respectively. We show first how the access structure is decomposed according to steps 1–3 and finally demonstrate how the solution is obtained.

Step 1: Retrieve the row vectors for the access structure:

$$\begin{cases} \mathbf{p}_1 = (0 & 1 & 0 & 0 & 1), \\ \mathbf{p}_2 = (0 & 0 & 0 & 1 & 0), \\ \mathbf{p}_3 = (0 & 0 & 1 & 0 & 0). \end{cases} \quad (7)$$

Step 2: Identify the number of elements for each row where the values of the elements are equal to 1 to obtain  $\Omega_a$  decomposed matrices.  $u_1 = \{2, 5\}$ ,  $u_2 = \{4\}$ ,  $u_3 = \{3\}$ , and  $\Omega_a = 2 \times 1 \times 1 = 2$ , and we have

$$\mathbf{A}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{A}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \quad (8)$$

Step 3: Decompose the solution structure following steps 1 and 2 to obtain  $\Omega_s$  decomposed matrices.  $\Omega_s = 2 \times 1 \times 1 \times 2 = 4$ , and we have

$$\mathbf{S}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{S}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

$$\mathbf{S}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{S}_4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Step 4: For each combination of the decomposed matrices of the access and solution structures, compute the overall expected utility and select the combination that yields the highest overall expected utility as the solution. Take the combination of  $\mathbf{A}_1$  and  $\mathbf{S}_3$  as an example. Let  $U(n_i)$  denote the total expected utility for decision node  $i$  ( $i=1, 2, 3, 4, 5$ ). We have

$$\begin{aligned}
 U(n_1) &= 1.0 \times (0 \ 1) \times \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} + 1.0 \times (0 \ 0 \ 0) \times \begin{pmatrix} -0.6 \\ -0.5 \\ -0.7 \end{pmatrix} \\
 &\quad + 1.0 \times (0 \ 1 \ 0 \ 0) \times \begin{pmatrix} 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \end{pmatrix} \quad (10) \\
 &= 0.3 + 0 + 0.3 = 0.6,
 \end{aligned}$$

$$\begin{aligned}
 U(n_2) &= 1.0 \times (1 \ 0) \times \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} + 1.0 \times (1 \ 0 \ 0) \times \begin{pmatrix} -0.6 \\ -0.5 \\ -0.7 \end{pmatrix} \\
 &\quad + 1.0 \times (0 \ 0 \ 0 \ 0) \times \begin{pmatrix} 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \end{pmatrix} \quad (11) \\
 &= 0.7 - 0.6 + 0 = 0.1,
 \end{aligned}$$

$$\begin{aligned}
 U(n_3) &= 1.0 \times (1 \ 0) \times \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} + 1.0 \times (0 \ 0 \ 1) \times \begin{pmatrix} -0.6 \\ -0.5 \\ -0.7 \end{pmatrix} \\
 &\quad + 1.0 \times (1 \ 0 \ 0 \ 1) \times \begin{pmatrix} 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \end{pmatrix} \quad (12) \\
 &= 0.7 - 0.7 + (0.6 + 0.5) = 1.1,
 \end{aligned}$$

$$\begin{aligned}
 U(n_4) &= 0.7 \times (0 \ 1) \times \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} + 0.7 \times (0 \ 1 \ 0) \times \begin{pmatrix} -0.6 \\ -0.5 \\ -0.7 \end{pmatrix} \\
 &\quad + 0.7 \times (0 \ 0 \ 1 \ 0) \times \begin{pmatrix} 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \end{pmatrix} \quad (13) \\
 &= 0.21 - 0.35 + 0.49 = 0.35,
 \end{aligned}$$

$$\begin{aligned}
 U(n_5) &= 0.5 \times (1 \ 0) \times \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} + 0.5 \times (0 \ 0 \ 0) \times \begin{pmatrix} -0.6 \\ -0.5 \\ -0.7 \end{pmatrix} \\
 &\quad + 0.5 \times (0 \ 0 \ 0 \ 0) \times \begin{pmatrix} 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \end{pmatrix} \quad (14) \\
 &= 0.35 + 0 + 0 = 0.35.
 \end{aligned}$$

Therefore, for the combination of  $A_1$  and  $S_3$ , the overall expected utility across the five decision nodes is  $0.6+0.1+1.1+0.35+0.35=2.5$ . The overall expected utilities for other combinations of the access and solution structures can be computed in a similar way. The best 'plan', or combination, of the access and solution structures is the one that yields the highest overall expected utility, which is  $A_2S_1$  or  $A_2S_3$  in this example, yielding an overall expected utility of 2.8.

## 5 Discussions

One might ask what the benefits are of using a decision network compared to a decision tree in solving complex problems. The following hypothetical planning problem shows how a decision network and a decision tree frame the problem differently while coming up with the same answer (Hopkins, 2001). Consider a residential construction project consisting of two decisions: infrastructure and housing constructions. These two decisions fall within the authority of two decision-makers, namely the infrastructure provider and the housing builder. Assume that the infrastructure provider could construct either a high-density system of 500 units on 100 acres or a low-density system of 500 units on 250 acres; the housing builder could construct a high-density community of 500 dwelling units on 100 acres or a low-density community of 200 dwelling units on 100 acres.

Based on decision tree analysis, Hopkins (2001) demonstrated algebraically that making plans by simultaneously considering the two decisions, namely infrastructure and housing decisions, yields more benefits in monetary terms than making these decisions independently. Without delving into detailed numerical calculations, we show that considering multiple stakeholders in a decision network framework reinforces Hopkins' argument that making multiple and linked decisions matters. First, the infrastructure decision node can be represented by the diagram shown in Fig. 1.

In Fig. 1, the circle denotes the decision situation, in which there are two options: high-density or low-density development. The decision-maker (infrastructure provider), problem (infrastructure demand), and solution (infrastructure construction) are

associated with the three inward arrows, whereas the arrow emanating from the decision situation represents the outcome, i.e., high-density development in this case, according to Hopkins' original calculations (Hopkins, 2001). Similarly, the housing decision represented in the decision network framework is shown in Fig. 2. Note that the housing builder should choose low-density development, if the situation is considered independently, given a set of unit costs and revenues for different types of development (Hopkins, 2001).

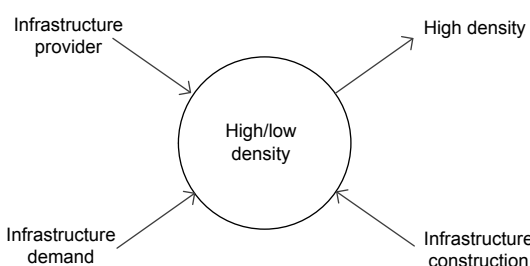


Fig. 1 Decision network diagram for infrastructure decision

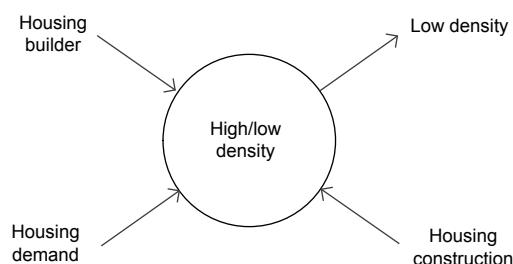


Fig. 2 Decision network diagram for housing decision

Fig. 3 shows that when the two decisions are considered simultaneously, the decision outcome for the infrastructure changes from high- to low-density development, whereas the decision outcome for the housing builder remains the same.

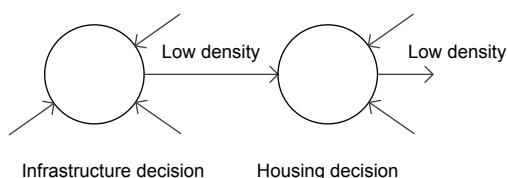


Fig. 3 Decision network diagram for combined infrastructure and housing decisions

Note that in Fig. 3, the outcome of the infrastructure decision becomes one of the four inputs of

the housing decision. Compared with decision tree representation, the decision network diagram is more succinct, more flexible, and richer while conveying more information. For example, in the two diagrams for the infrastructure and housing decisions, we can add more information about the options inside the decision situations and the infrastructure and housing demands; i.e., the infrastructure and housing demands and supplies can be added without altering the diagrams significantly. This is not the case in decision tree analysis. These advantages become more significant when the size of the problem is large and hundreds or thousands of decision nodes are involved. Of course, a deductive comparison of effectiveness between the decision network and decision tree frameworks demands future work.

We have not delved into the multi-attribute preferences that characterize most planning problems; however, a straightforward extension of the existing formulation of the decision network would, in theory, suffice to cover this issue. One way of doing this is to replace the unidimensional utilities, i.e.,  $u_i$ ,  $v_j$ , and  $w_k$ , with multi-dimensional utilities (Keeney and Raiffa, 1993). Considerable literature exists on multi-attribute decision-making, which we cannot delve into here because of limited space. However, the literature provides a basis for future exploration of incorporating multi-attribute preferences into the decision network technology.

Most planning problems deal with spatial issues. In its current form, a decision network can easily be modified to incorporate the spatial dimension into its formulation. Specifically, we can add location as the fourth element into the decision situation, in addition to problems, solutions, and decision-makers, together with a spatial structure linking those locations to decision situations. This has been done in an attempt to simulate how an urban system works, based on the garbage can model proposed by Cohen *et al.* (1972) and Lai (2006). The solution algorithm introduced here remains the same, regardless of the addition of the spatial dimension to the decision network formulation.

It is arguably true that, because the real world is non-linear, the linear model of the decision network described in the 0-1 integer program in Eq. (1) is far from being realistic. We agree that the model does not map reality faithfully, since reality is far more

complex than what can be described mathematically. However, the 0-1 integer program simplifies the real problem and serves as a good and approximate basis from which promising solutions can be derived through means other than mathematics, such as graphic and verbal communications. In other words, to take advantage of the problem-solving logic presented in this paper, the decision network could be developed into a fully fledged technology to incorporate means of problem solving, whether mathematical or non-mathematical, in dealing with complex planning problems.

## 6 Conclusions

Planning problems are characterized by difficulty and complexity. Traditional decision analysis techniques commonly used by planners are focused overwhelmingly on making independent decisions for a single decision-maker. Effective planning tools must address multiple stakeholders and multi-attribute preferences at the same time. Here, we have provided a theoretical basis for the decision network. It attempts to make multiple and linked decisions with multiple stakeholders and multi-attribute preferences. In its current form, the decision network is by no means a mature planning tool because much ambiguity remains to be worked out. With sufficient and persistent effort, the theoretical foundation introduced in this paper could serve as a starting point for development into a fully fledged technology that helps planners deal confidently with challenging planning problems.

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