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Finite-time leader-follower consensus of a discrete-time system via sliding mode control*

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Abstract: In this study, we solve the finite-time leader-follower consensus problem of discrete-time second-order multi-agent systems (MASs) under the constraints of external disturbances. First, a novel consensus scheme is designed using a novel adaptive sliding mode control theory. Our adaptive controller is designed using the traditional sliding mode reaching law, and its advantages are chatter reduction and invariance to disturbances. In addition, the finite-time stability is demonstrated by presenting a discrete Lyapunov function. Finally, simulation results are presented to prove the validity of our theoretical results.

Key words: Finite-time; Leader-follower consensus; Adaptive sliding mode control; Multi-agent systems
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1 Introduction

In the last few years, distributed cooperative control has been an important research direction within the area of multi-agent systems (MASs) because of its high efficiency and robustness compared to control of the classical single agent (Olfati-Saber and Murray, 2004; Olfati-Saber et al., 2007; Ren, 2008; Ren and Beard, 2008). The distributed cooperative control method is applied to both theoretical and practical areas. The theoretical research includes consensus (Li ZK et al., 2009; Xie DS et al., 2016; Zhang HG et al., 2020b, 2021b), formation control (Oh et al., 2015), coverage control (Atınç et al., 2014), rendezvous (Li P et al., 2018), flocking (Olfati-Saber, 2006), reinforcement learning (Li Q et al., 2021; Xia LN et al., 2022a), and attack (Li Q et al., 2019b; Xia LN et al., 2022b). At the same time, the MAS has been used in many practical areas, such as design of sensor networks (Cruz-Piris et al., 2018) and unmanned aerial quadrotors (Han et al., 2020).

Among the numerous distributed cooperative MAS control methods, the consensus problem is a basic and significant research topic. It is the key to solving other cooperative control problems. Normally, consensus can be classified into three types: leaderless consensus (Cui Q et al., 2020), leader-follower consensus (one leader or a virtual leader) (Li Q et al., 2020; Zhang J et al., 2021), bipartite consensus (Li Q et al., 2019a; Zhang HG et al., 2021a), and containment control (multiple leaders) (Cui GZ et al., 2018; Wang W et al., 2020; Zhang HG et al., 2020a). The consensus problem states that, with the evolution of time, the global or partial MAS states

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tend to be asymptotically stable. Specifically, in the case of leader-follower consensus, it requires that the states of all followers drive into the trajectory of a single leader, which is widely used in the field of formation control as well.

An important performance index of the consensus problem is the convergence rate. In many articles (Chowdhury et al., 2018; Deng et al., 2020; Wang QL and Sun, 2020), the MAS consensus is asymptotic; that is to say, the consensus situation can be reached only when the time tends towards infinity. However, we need to achieve finite-time consensus (FTC) in real applications, which means that the trajectory of the system can converge to the designed equilibrium state within a finite time horizon and remain there since then. Therefore, various kinds of controllers have been constructed to meet the demand of FTC stabilization problems in MAS research. In Zou et al. (2020), second-order FTC was discussed in a switched MAS, and in Shi et al. (2019) it was extended to the FTC of high-order MASs. Ning and Han (2019) and Lu et al. (2021) studied the prescribed and bipartite FTC, respectively. Liu XY et al. (2019) compared the FTC and fixed-time consensus (FxTC), in which the MAS can reach consensus within a fixed time. At the same time, the importance of FTC research was demonstrated because FTC has been used in a wider range of control areas (Min et al., 2018) and extended to practical applications such as the financial sector (Wang YL et al., 2021).

Significantly, most finite-time controllers are discussed based on the continuous models in the MAS field (Tong et al., 2018). In fact, the useful information may not be transmitted continuously in practice due to unreliable communication channels or the limited perceptual capabilities of agents. In addition, in a real-time computer control system, the controller demand must be discrete. Therefore, we focus on the study of discrete-time MASs, in which the agents can obtain only the state information at discrete times from their neighbors. There have been some recent research findings about the MAS discrete-time consensus. Leader-follower and high-order switched discrete MASs have been reported (Liu JW and Huang, 2021; Liu YF and Su, 2021; Zhang YY et al., 2021), and the time-varying situation has been discussed in Zhang JL et al. (2021). Wang B and Tian (2021) and Zhang WL et al. (2021) gave the discrete consensus with multiplicative uncertainties delays and unconfined cyber-attacks in MAS, respectively. Wang QS et al. (2021) presented a new method to achieve the optimal consensus of discrete MAS and Zhou et al. (2021) extended the discrete-time research in the area of financial systems.

Another crucial problem concerns disturbances. Most disturbances originate from uncertainty and the impact of external disturbances, and severely affect the dynamic performance of the MAS. At the same time, MAS control performance and accuracy will be reduced. Research on anti-disturbance control has been carried out using multiple traditional control methods, such as robust control (Wang JY et al., 2021), adaptive control (Liang et al., 2021), back stepping control (Wang XY et al., 2018), fuzzy control (Shao and Ye, 2021), and sliding mode control (SMC) (Yu and Long, 2015; Sinha and Mishra, 2020).

SMC has significant advantages against the uncertainty of the system and the influence of external disturbances, especially for closed-loop systems (Young et al., 1999). The traditional SMC method is essentially a variable structure nonlinear control (Utkin, 1977). It can drive a decentralized controller to reach the sliding surface, which is constructed as a state function of the system in finite time and maintained on the sliding surface thereafter. The programming of the sliding surface is irrelevant to the system parameter of the system and the external disturbances. Many researchers have noticed the excellent performance of SMC, such as robustness and expandability, and have conducted extensive research, especially in the field of MASs (Sun et al., 2018; Wang GD et al., 2018; Qin et al., 2019; Zhang Z et al., 2019; Sinha and Mishra, 2020; Yao et al., 2020). Furthermore, Liu HY et al. (2020) extended the research in fractional-order MASs, Fei et al. (2020) combined the sliding mode method with the neural network, and Chen et al. (2020) introduced SMC in the application of a non-holonomic spherical robot.

However, the SMC algorithm can cause high-frequency chattering of the control system. Because of the discontinuous control structure, the control trajectory may not reach the sliding surface without inertia, and a zigzag trajectory may be superimposed in the practical sliding mode, which causes devastating consequences in the controller. To solve the chattering problem, Gao et al. (1995) first proposed the reaching law theory of discrete-time variable structure control to eliminate the high-frequency chattering of SMC. Recent applications to avoid chattering include the synthetic trigger function (Liu JH et al., 2019) and the adaptive event-triggered controller strategy (Xu and Wu, 2021).

Motivated by the above observations, the main task of this paper is to analyze the FTC of a discrete-time, leader-follower MAS under the controller of discrete-time SMC. The main contributions are summarized below:

1. Aiming at the second-order MAS over a directed graph, based on the traditional SMC method, we propose a new adaptive discrete sliding mode controller that leads the followers to achieve finite-time tracking control of the single leader's position and velocity information in a discrete-time MAS.

2. The existence and reachability of discrete sliding mode are discussed in our research. Using the method of finite-time stability by constructing a discrete Lyapunov function (Hamrah et al., 2019) in a discrete system, it is demonstrated that the presented control algorithm can ensure the stability of the discrete MAS in finite time. Furthermore, the stability is not affected by variation of the discrete sampling time.

3. Compared with the existing discrete SMC method, the adaptive discrete sliding mode controller can adjust the time-varying parameter ε to decrease the inherent chattering of traditional SMC. Meanwhile, the speed at which the sliding mode switching surface is reached is not affected. Using the saturation function $\mathrm{sat}(\cdot)$, which is widely used in the ideal SMC, and substituting the signum function $\mathrm{sgn}(\cdot)$, the advantage of the proposed controller in our research is used to the greatest extent to create robustness and invariance to internal system perturbation and external disturbances.

2 Preliminaries and formulation of problems

2.1 Graph theory

A weighted graph $G=(\mathcal{V},\mathcal{E},\boldsymbol{\mathcal{A}})$ is defined as the MAS communication topology with n agents. For the definition of the graph, we set $\mathcal{V}=\{1,2,\cdots,n\}$ as the node set and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ as the edge set. Then we set the weighted adjacency matrix $\boldsymbol{\mathcal{A}}=[a_{ij}]\in\mathbb{R}^{n\times n}$ as an n-order real matrix. An edge of G from node i to node j is expressed by (i,j), and on behalf of that, node j can consume information from node i. If $(i,j)\in\mathcal{E}$, then $a_{ij}>0$, if not, $a_{ij}=0$, and we set $a_{ii}=0$, which means that the graph has no self-loop. We call $(j,i)\in\mathcal{E}$ because node j is the neighborhood of node i. So, we define $N_i=\{j\mid j\in\mathcal{V},(j,i)\in\mathcal{E}\}$ as the neighbor set of i. The directed spanning tree exists when and only when $\mathcal{V}$ has a root node, and the root node can lead to a directed path to all other nodes. If $(j,i)\in\mathcal{E}\leftrightarrow(i,j)\in\mathcal{E}$, then we define G as an undirected graph. Another crucial definition is the Laplacian matrix $\boldsymbol{\mathcal{L}}=[l_{ij}]\in\mathbb{R}^{n\times n}$, which is based on the adjacency matrix presented above $\boldsymbol{\mathcal{A}}$:

$$l_{ij}=\begin{cases}\sum_{j=1,j\neq i}^{n}a_{ij}, & i=j,\\ -a_{ij}, & i\neq j.\end{cases}$$

From the definition of the Laplacian matrix, we have $\boldsymbol{\mathcal{L}}\cdot\mathbf{1}_n=\mathbf{0}$, where $\mathbf{1}_n=[1,1,\ldots,1]^{\mathrm{T}}$ is a column vector. For a leader-follower MAS, we designate node 0 as the leader and the series $1,2,...,n$ as the followers. Another graph, $\bar{G}=(\bar{\mathcal{V}},\bar{\mathcal{E}},\bar{\boldsymbol{\mathcal{A}}})$, represents the n followers and a single leader in leader-follower MASs. Specifically, $\overline{\mathcal{V}}=\mathcal{V}\cup\{0\}$, and $\overline{\mathcal{E}}$ and $\overline{\boldsymbol{\mathcal{A}}}$ are the corresponding edge set and adjacency matrix, respectively. We set the Laplacian matrix of the leader-follower directed graph as $\bar{\mathcal{L}}$, and denote another column vector $\boldsymbol{\mathcal{B}}=[b_1,b_2,\ldots,b_n]^{\mathrm{T}}$, where $b_i=1$ represents the follower agent with i being a neighborhood of the leader, and $b_i=0$ otherwise. The matrix $\bar{\boldsymbol{\mathcal{B}}}=\mathrm{diag}(b_1,b_2,\cdots,b_n)$. We assume that the followers receive information only from the leader, and that the followers cannot provide information to the leader. We call the leader globally reachable if each follower has a directed path from the leader.

2.2 Problem formation

In this subsection, we propose the issue of leader-follower consensus by considering second-order discrete linear dynamics. The dynamic function includes one single leader and the remaining n agents are followers. The leader is labeled 0, while the followers are labeled $\{1,2,\cdots,n\}$. First, we suppose that the dynamics of follower agents is as follows:

$$\begin{cases} x_i(k+1) = x_i(k) + T_k v_i(k), \\ v_i(k+1) = v_i(k) + T_k[u_i(k) + d_i(k)], \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n$, and $u_i(k), x_i(k), v_i(k) \in \mathbb{R}$ are the control input, position, and velocity of the i^{th} follower agent at sampling time k, respectively. $d_i(k)$ represents the matched slowly varying disturbance acting on the follower agents at time k. T_k indicates the sampling period and k (or $k+1$) is the abbreviation for kT_k (or $(k+1)T_k$). Similarly, the leader's dynamics is presented by another second-order discrete linear system:

$$\begin{cases} x_0(k+1) = x_0(k) + T_k v_0(k), \\ v_0(k+1) = v_0(k). \end{cases} \tag{2}$$

Similar to the above definition, $x_0(k), v_0(k) \in \mathbb{R}$ indicate the position and velocity of the leader, respectively.

We assume $\boldsymbol{\xi}_i(k) = [x_i(k), v_i(k)]^{\text{T}}$, and the leader's column vector is denoted by $\boldsymbol{\xi}_0(k) = [x_0(k), v_0(k)]^{\text{T}}$. Then, from Eqs. (1) and (2), we obtain

$$\begin{cases} \boldsymbol{\xi}_i(k+1) = \boldsymbol{A}\boldsymbol{\xi}_i(k) + \boldsymbol{B}[u_i(k) + d_i(k)], \\ \boldsymbol{\xi}_0(k+1) = \boldsymbol{A}\boldsymbol{\xi}_0(k), \end{cases} \tag{3}$$

where $\boldsymbol{A} = \begin{bmatrix} 1 & T_k \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{B} = [0, T_k]^{\text{T}}$.

To explain the process of consensus controller design and stability analysis, we introduce some necessary assumptions and lemmas.

Assumption 1 The disturbance $d_i(k)$ is bounded and meets the condition $|d_i(k)| \le \overline{d}$, where $\overline{d}$ is a positive number, $i = 1, 2, \ldots, n$.

Assumption 2 The communication topology graph for the above leader-follower MAS has at least one single directed spanning tree, which can be expressed as $\bar{\boldsymbol{\mathcal{B}}} \neq \boldsymbol{0}$.

Lemma 1 (Tsai et al., 2018) To avoid the SMC chattering phenomenon, we can use the saturation function to replace the traditional sign function. The saturation function is defined as

$$\text{sat}(s) = \text{sat}(\boldsymbol{S}_i(k)) = \frac{\boldsymbol{S}_i(k)}{|\boldsymbol{S}_i(k)| + \delta_i},$$

where δ_i is an arbitrary small positive constant.

Lemma 2 We set $\bar{\boldsymbol{\mathcal{L}}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{B}} \in \mathbb{R}^{n\times n}$ as the Laplacian matrix of graph $\bar{G}$. It is positive definite if Assumption 2 holds.

Lemma 3 (Hamrah et al., 2019) Define a discrete-time system output variable $\boldsymbol{y}_k \in \mathbb{R}^n$. Then, consider a positive definite Lyapunov function $V_k = V(\boldsymbol{y}_k), V: \mathbb{R}^n \to \mathbb{R}$. Define the real numbers $\alpha, h \in (0,1)$, and $\gamma_k = \gamma(V_k)$ is also a positive definite function. We have

$$\frac{\gamma_k}{\gamma_0} \ge 1 - h \text{ for } V_k \in (\beta V_0, V_0), 0 < \beta \ll 1, \tag{4}$$

if the following inequality is satisfied:

$$\Delta V_k = V_{k+1} - V_k \le -\gamma_k (V_k)^\alpha. \tag{5}$$

We can say that the discrete system achieves Lyapunov stability. The output variable $\boldsymbol{y}_k$ converges to $\boldsymbol{y} = \boldsymbol{0}$ if $k \ge N$, in which the finite positive integer $N \in \mathbb{N}$.

Proof Based on the definition of Lyapunov stability, the difference of the Lyapunov function ΔV_k is negative definite along the trajectories of the above discrete-time system, while the right-hand side of inequality (4) tends to 0 when, and only when, $V_k = 0$.

Using the definition of γ_k, the equality form of inequality (4) is given as follows:

$$\begin{aligned} V_{k+1} &= V_k - \gamma_k V_k^\alpha \\ &= V_k\left(1 - \frac{\gamma_k}{V_k^{1-\alpha}}\right). \end{aligned} \tag{6}$$

We set the initial value V_0 of the above Lyapunov function as

$$V_0 = g_0(\gamma_0)^{\frac{1}{1-\alpha}}, g_0 > 0. \tag{7}$$

Then, substituting the value of V_0 in inequality (5), we have

$$\begin{aligned} V_1 - g_0(\gamma_0)^{\frac{1}{1-\alpha}} &= -\gamma_0 g_0^\alpha (\gamma_0)^{\frac{\alpha}{1-\alpha}} = -g_0^\alpha(\gamma_0)^{\frac{1}{1-\alpha}} \\ \Rightarrow V_1 &= (g_0 - g_0^\alpha)(\gamma_0)^{\frac{1}{1-\alpha}}. \end{aligned} \tag{8}$$

We define

$$g_1 := g_0 - g_0^\alpha. \tag{9}$$

Thus, the value of V_1 is defined as

$$V_1 = g_1(\gamma_0)^{\frac{1}{1-\alpha}}. \tag{10}$$

The value range of parameter g_0 should be $g_0 > 1$. We have noticed that if $g_0 \le 1$, Eq. (8) means that $g_1 \le 0$. This leads to a paradox unless $g_1 = 0$, because the variable V_1 must be non-negative based on the definition of the Lyapunov function. So, the value of the Lyapunov function has converged to 0 at the first iteration step, which means $k = 1$. To avoid this case, suppose $g_0 > 1$; then, substituting the variable V_1 in inequality (5), we can have a similar iteration expression for V_2 :

$$V_2 = g_2 (\gamma_0)^{\frac{1}{1-\alpha}}, g_2 := g_1 - a_1 g_1^{\alpha}, a_1 := \frac{\gamma_1}{\gamma_0}. \quad (11)$$

If we continue iterating in this method, we can obtain the following expression for V_{k+1} and the recurrence relation involving g_k for a_k:

$$\begin{aligned} &V_{k+1} = g_{k+1} (\gamma_0)^{\frac{1}{1-\alpha}}, k \geq 1, \\ &g_{k+1} := g_k - a_k g_k^{\alpha}, a_k := \tfrac{\gamma_k}{\gamma_n}. \end{aligned} \quad (12)$$

We suppose that V_k is in the scope of inequality (5); then, based on Eq. (11) and inequality (5), we have

$$\begin{aligned} g_{k+1} &\leq g_k - (1-h) g_k^{\alpha} \\ &= h g_k^{\alpha} - \left(1 - g_k^{1-\alpha}\right) g_k^{\alpha}. \end{aligned} \quad (13)$$

Because V_{k+1} is positive definite, g_{k+1} should be greater than or equal to 0 according to Eq. (11). Furthermore, if V_k is within the range given in inequality (5), we have

$$\frac{g_k}{g_0} = \frac{V_k}{V_0} \in (\beta, 1), \quad (14)$$

in which parameter β is arbitrarily small, especially for $\beta g_0 < 1$. Rearrange the right-hand side of inequality (13) as follows:

$$g_{k+1} \leq 0 \Leftrightarrow h \leq 1 - c_k^{1-\alpha} \Leftrightarrow c_k^{1-\alpha} \leq 1 - \varepsilon. \quad (15)$$

When $g_k \to \beta g_0 < 1$ falls in the interval of V_k, a finite positive integer $k = N - 1$ can meet condition (14). For example, $g_{N-1} \leq (1-h)^{\frac{1}{1-\alpha}}$, so $g_N \leq 0$. In addition, $g_N \geq 0$ because $V_N \geq 0$ and $\gamma_0 > 0$. Hence, $g_N = 0$ based on the contradictory conditions. Therefore, we use Eq. (11) again, and it can be concluded that $g_j = 0$ and $V_j = 0$ under $j \geq N$. Consequently, y_j can converge to 0 under $j \geq N$. We now have finite-time stability of the discrete system.

Definition 1 The consensus control for the discrete-time second-order MAS (1) and (2) can be reached in finite time, if for any initial states, there is a point of time $T \in [0, +\infty)$, such that the following conditions are satisfied:

$$\begin{cases} \lim_{k \to T} |x_i(k) - x_0(k)| = 0, \\ \lim_{k \to T} |v_i(k) - v_0(k)| = 0, \ i = 1, 2, \ldots, n, \end{cases} \quad (16)$$

and

$$\begin{cases} x_i(k) = x_0(k), \\ v_i(k) = v_0(k), \ \forall k \geq T, \ i = 1, 2, \ldots, n. \end{cases} \quad (17)$$

3 Main results

In this section, we aim to achieve leader-follower consensus by designing a distributed discrete sliding mode controller. Using the control scheme, the followers can track the position and velocity trajectory of the leader in finite time. We assume that all agents in the designed discrete MAS can update their own position and velocity information at sampling time T_k, and that the position and velocity information can be observed in real time.

First, let us define the position and velocity tracking errors:

$$\begin{cases} \tilde{x}_i(k) = x_i(k) - x_0(k), \\ \tilde{v}_i(k) = v_i(k) - v_0(k), \ i = 1, 2, \ldots, n. \end{cases} \quad (18)$$

We can use $\tilde{\boldsymbol{\xi}}_i(k) = [\tilde{x}_i(k), \tilde{v}_i(k)]^{\mathrm{T}}$ as the vector form of the tracking error:

$$\tilde{\boldsymbol{\xi}}_i(k) = \boldsymbol{\xi}_i(k) - \boldsymbol{\xi}_0(k). \quad (19)$$

Then we define the local neighborhood consensus error for our discrete second-order MAS (1) and (2) as

$$\begin{aligned} \boldsymbol{e}(\boldsymbol{\xi}_i(k)) = &\sum_{j=1}^{n} a_{ij} \left[\boldsymbol{\xi}_i(k) - \boldsymbol{\xi}_j(k)\right] \\ &+ b_i \left[\boldsymbol{\xi}_i(k) - \boldsymbol{\xi}_0(k)\right], \end{aligned} \quad (20)$$

where $i = 1, 2, \ldots, n$.

Based on Eqs. (3) and (20), we obtain

$$\begin{aligned} \boldsymbol{e}(\boldsymbol{\xi}_i(k+1)) = &\boldsymbol{A}\boldsymbol{e}(\boldsymbol{\xi}_i(k)) + (l_{ii} + b_i) \boldsymbol{B} [u_i(k) + d_i(k)] \\ &- \sum_{j=1}^{n} a_{ij} \boldsymbol{B} [u_j(k) + d_j(k)] \\ = &\boldsymbol{A}\boldsymbol{e}(\boldsymbol{\xi}_i(k)) + (l_{ii} + b_i) \boldsymbol{B} u_i(k) \\ &- \sum_{j=1}^{n} a_{ij} \boldsymbol{B} u_j(k) + b_i \boldsymbol{B} d_i(k). \end{aligned} \quad (21)$$

We define l_{ii} as the main diagonal element of the graph Laplacian matrix $\boldsymbol{\mathcal{L}}$. Our aim is to make the consensus error vector $\boldsymbol{e}(\boldsymbol{\xi}_i(k))$ tend toward $\mathbf{0}$ in finite time.

To ensure the convergence of the consensus error in finite time, we construct a new adaptive discrete SMC scheme. First, the sliding mode surface function has been designed as

$$\boldsymbol{S}_i(k)=\boldsymbol{C_e}\boldsymbol{e}_i(k), \tag{22}$$

where $\boldsymbol{e}_i(k)$ is short for the consensus error vector $\boldsymbol{e}(\boldsymbol{\xi}_i(k))$.

Similarly, we have $\boldsymbol{S}_i(k+1)=\boldsymbol{C_e}\boldsymbol{e}_i(k+1)$, where the sliding mode parameter $\boldsymbol{C_e}$ should satisfy the pole-placement approach and the second-order Hurwitz condition such that $\boldsymbol{C_e}=[c_1,\ 1]$, $c_1>0$ and $c_1\in\mathbb{N}$.

Our aim is to design a stable discrete time sliding mode controller based on the classical exponential reaching law. Discrete movement from an arbitrary state, which is driven by our novel control law in Theorem 1, can reach the sliding surface in a finite number of steps, and the system can remain on the sliding surface without zigzagging, that is, the so-called ideal quasi-sliding mode. This can reduce the occurrence of chattering.

The sliding surface function that satisfies the ideal quasi-sliding mode condition is expressed as $\boldsymbol{S}_i(k)=\boldsymbol{C_e}\boldsymbol{e}_i(k)=\boldsymbol{0}$. The sliding mode parameter $\boldsymbol{C_e}$ is a constant, so when the above equation holds, the consensus error $\boldsymbol{e}_i(k)=\boldsymbol{0}$.

Theorem 1 Using the following control law:

$$\begin{aligned}u_i(k)=&\left[\tau\left(l_{ii}+b_i\right)\boldsymbol{C_e}\boldsymbol{B}\right]^{-1}\{[\varepsilon TM_i\operatorname{sat}\left(\boldsymbol{S}_i(k)\right)\\&+\left(qTM_i-\tau\right)\boldsymbol{S}_i(k)]-\tau\boldsymbol{C_e}\boldsymbol{A}\boldsymbol{e}(\boldsymbol{\xi}_i(k))+\varPhi\},\end{aligned} \tag{23}$$

where $\varPhi=\tau\boldsymbol{C_e}\boldsymbol{B}\sum_{j=1}^{n}a_{ij}\left[u_j(k)+d_j(k)\right]$, $q>0,\tau>0$, and $T=T_k<4/(1+2q)$, the leader-follower second-order discrete MAS (1) and (2) under the directed topology can reach the finite-time consensus:

$$\begin{cases}\lim_{k\to T}|x_i(k)-x_0(k)|=0,\\ \lim_{k\to T}|v_i(k)-v_0(k)|=0,\ i=1,2,\ldots,n.\end{cases} \tag{24}$$

Proof First, we design a new discrete adaptive sliding mode controller according to the traditional exponential reaching law (Gao et al., 1995). We define the traditional exponential reaching law as follows:

$$\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)=-qT\boldsymbol{S}_i(k)-\varepsilon T\operatorname{sgn}(\boldsymbol{S}_i(k))+\boldsymbol{D}_i(k), \tag{25}$$

where $\boldsymbol{D}_i(k)=[d_1(k),d_2(k),...,d_i(k)]$, $i=1,2,\ldots,n$, and parameter ε can influence the chattering phenomenon performance and the speed at which the sliding surface is reached. Our ideal parameter ε should change over time. At the beginning of the sliding mode movement, ε should be larger, and with the increase of time, ε needs to decline exponentially to eliminate the chattering.

We use $\varepsilon=|\boldsymbol{S}_i(k)|/2$ as the adaptive parameter instead of a fixed parameter, and replace the signum function $\operatorname{sgn}(\cdot)$ with the saturation function $\operatorname{sat}(\cdot)$. The improved adaptive sliding mode reaching law is defined as

$$\begin{aligned}\Delta\boldsymbol{S}_i(k)&=\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)\\&=-qT\boldsymbol{S}_i(k)-\frac{|\boldsymbol{S}_i(k)|}{2}T\operatorname{sat}\left(\boldsymbol{S}_i(k)\right).\end{aligned} \tag{26}$$

Then we construct a positive definite Lyapunov function of consensus error vector $V_k\left(\boldsymbol{S}_i(k)\right)=\frac{1}{2}\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)>0$. Thus, the iteration of the above Lyapunov function can be calculated as

$$\begin{aligned}\Delta V_k&=V_{k+1}-V_k\\&=\frac{1}{2}\left[\boldsymbol{S}_i(k+1)+\boldsymbol{S}_i(k)\right]\left[\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)\right]^{\mathrm{T}}.\end{aligned} \tag{27}$$

We assume that

$$\boldsymbol{S}_i^{\mathrm{T}}(k+1)=\varPhi'\boldsymbol{S}_i^{\mathrm{T}}(k), \tag{28}$$

in which

$$\varPhi'=\frac{\left[\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)\right]^{1-1/p}-\tau}{\left[\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)\right]^{1-1/p}+\tau},\ \tau>0,1<p<2. \tag{29}$$

Inserting Eq. (29) into Eq. (28), we have

$$\begin{aligned}\Delta\boldsymbol{S}_i(k)&=\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)\\&=-\frac{\tau}{\left[\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)\right]^{1-1/p}}\left[\boldsymbol{S}_i(k+1)+\boldsymbol{S}_i(k)\right].\end{aligned} \tag{30}$$

Then, inserting Eq. (30) into Eq. (27), we can update ΔV_k as

$$\begin{aligned}\Delta V_k=&-\frac{\tau}{2}[\boldsymbol{S}_i(k+1)+\boldsymbol{S}_i(k)][\boldsymbol{S}_i(k+1)+\boldsymbol{S}_i(k)]\\&\cdot\left(\frac{1}{\left[\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)\right]^{1-1/p}}\right)^{\mathrm{T}}\\=&-\rho\left(\boldsymbol{S}_i(k)\right)\left[\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)\right]^{1/p},\end{aligned} \tag{31}$$

where

$$\rho\left(\boldsymbol{S}_i(k)\right)=\rho_k=4\tau\frac{2^{1-1/p}\left(\frac{1}{2}\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)\right)^{2-2/p}}{\left(\left(\boldsymbol{S}_i(k)\boldsymbol{S}_i^{\mathrm{T}}(k)\right)^{1-1/p}+\tau\right)^2}. \tag{32}$$

Therefore, we obtain

$$\Delta V_k\leq-\rho\left(\boldsymbol{S}_i(k)\right)\left(V_k\right)^{1/p} \tag{33}$$

under the following circumstances:

$$0<\rho_k<\frac{4\tau}{2^{1-1/p}}\ \text{ for } 0<2V_k<\infty, \tag{34}$$

to ensure the monotonic decrease of $\Delta V_k<0$.

Based on Lemma 3, and from inequality (5), we can have $V_k\left(\boldsymbol{S}_i(k)\right)$, and the consensus error vector $\boldsymbol{e}_i(k)$ converges to $\mathbf{0}$ in a finite number of steps of the discrete-time domain. So, the local neighborhood consensus error tends to $\mathbf{0}$ in finite time. This means that the discrete MAS achieves consensus in finite time.

Based on the sliding mode function (22) and the improved adaptive sliding mode reaching law (26), we can use the iteration of the sliding mode function $\Delta\boldsymbol{S}=\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)$ to construct the new adaptive sliding mode controller.

Using the definition of the local neighborhood consensus error vector $\boldsymbol{e}(\boldsymbol{\xi}_i(k))$ and the iteration form of the error vector $\boldsymbol{e}(\boldsymbol{\xi}_i(k+1))$, the adaptive sliding mode reaching law (26) can be updated using the following iterative form of the sliding mode surface function $\boldsymbol{S}_i(k+1)=\boldsymbol{C_e}\boldsymbol{e}_i(k+1)$:

$$\begin{aligned}\boldsymbol{S}_i(k+1)=&\boldsymbol{C_e}\boldsymbol{e}\left(\boldsymbol{\xi}_i(k+1)\right)\\=&\boldsymbol{C_e}\boldsymbol{A}\boldsymbol{e}\boldsymbol{\xi}_i(k)+\left(l_{ii}+b_i\right)\boldsymbol{C_e}\boldsymbol{B}\left[u_i(k)\right.\\&\left.+d_i(k)\right]-\sum_{j=1}^{n}a_{ij}\boldsymbol{C_e}\boldsymbol{B}\left[u_j(k)+d_j(k)\right].\end{aligned} \tag{35}$$

Then, inserting Eq. (35) in Eq. (26) and reorganizing the function, we have

$$\begin{aligned}&\tau\boldsymbol{C_e}\{\boldsymbol{A}\boldsymbol{e}\left(\boldsymbol{\xi}_i(k)\right)+\left(l_{ii}+b_i\right)\boldsymbol{B}\left[u_i(k)+d_i(k)\right]\\&-\sum_{i=1}^{n}a_{ij}\boldsymbol{B}\left[u_j(k)+d_j(k)\right]\}\\=&-(qTM+\tau)\boldsymbol{S}_i(k)-\varepsilon TM_i\operatorname{sat}\left(\boldsymbol{S}_i(k)\right).\end{aligned} \tag{36}$$

Finally, we arrive at the adaptive sliding mode controller:

$$\begin{aligned}u_i(k)=&\left[\tau\left(l_{ii}+b_i\right)\boldsymbol{C_e}\boldsymbol{B}\right]^{-1}\{\left[\varepsilon TM_i\operatorname{sat}\left(\boldsymbol{S}_i(k)\right)\right.\\&\left.+\left(qTM_i-\tau\right)\boldsymbol{S}_i(k)\right]-\tau\boldsymbol{C_e}\boldsymbol{A}\boldsymbol{e}(\boldsymbol{\xi}_i(k))+\varPhi\}.\end{aligned} \tag{37}$$

Remark 1 To guarantee the stability of the designed control scheme, the existence and reachability of our novel adaptive discrete sliding mode controller are discussed. We set another Lyapunov function $\boldsymbol{V}_{k1}=\boldsymbol{S}_i^2(k)/2$. Once the following condition

$$\Delta\boldsymbol{V}_{k1}=\boldsymbol{S}_i^2(k+1)-\boldsymbol{S}_i^2(k)<0,\ \boldsymbol{S}_i(k)\neq\mathbf{0} \tag{38}$$

is met based on the Lyapunov stability theorem, $\boldsymbol{S}_i(k)=\mathbf{0}$ is a globally asymptotically stable switched sliding surface, which means that any initial position of the state (velocity) will tend toward the switched sliding surface. Taking the reachability condition as

$$\boldsymbol{S}_i^2(k+1)<\boldsymbol{S}_i^2(k) \tag{39}$$

when the sampling time T_k is small enough, the existence and reachability of the discrete sliding mode can be described as

$$\begin{cases}\left[\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)\right]\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)<0,\\\left[\boldsymbol{S}_i(k+1)+\boldsymbol{S}_i(k)\right]\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)>0.\end{cases} \tag{40}$$

Substituting function (25) with the adaptive parameter $\varepsilon=|\boldsymbol{S}_i(k)|/2$ and the adaptive sliding mode reaching law

$$\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)=-qT\boldsymbol{S}_i(k)-\frac{|\boldsymbol{S}_i(k)|}{2}T\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right), \tag{41}$$

we have

$$\begin{aligned}&\left[\boldsymbol{S}_i(k+1)-\boldsymbol{S}_i(k)\right]\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)\\=&\left(-qT\boldsymbol{S}_i(k)-\frac{|\boldsymbol{S}_i(k)|}{2}T\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)\right)\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)\\=&-(q+0.5)T\left|\boldsymbol{S}_i(k)\right|<0,\end{aligned} \tag{42}$$

$$\begin{aligned}&\left[\boldsymbol{S}_i(k+1)+\boldsymbol{S}_i(k)\right]\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)\\=&\left((2-qT)\boldsymbol{S}_i(k)-\frac{|\boldsymbol{S}_i(k)|}{2}T\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)\right)\\&\cdot\operatorname{sgn}\left(\boldsymbol{S}_i(k)\right)\\=&(2-0.5T-qT)\left|\boldsymbol{S}_i(k)\right|>0.\end{aligned} \tag{43}$$

This completes the proof of existence and reachability of our novel adaptive discrete sliding mode.

Remark 2 As a typical controller design method of variable structure control sliding mode, the reaching law method has the advantage of causing the motion to get close to the sliding surface monotonically. Using the reaching law method, we can solve the SMC high-frequency chattering problem very well.

Remark 3 Based on the proof of Theorem 1, we can obtain the time-variant adaptive parameter ε in the process of system movement. To guarantee the convergence performance of sliding mode surface function $\boldsymbol{S}_i(k)$, we first analyze the traditional exponential reaching law:

$$\boldsymbol{S}_i(k+1) - \boldsymbol{S}_i(k) = -qT\boldsymbol{S}_i(k) - \varepsilon T \,\mathrm{sgn}\, \boldsymbol{S}_i(k), \quad (44)$$

$$\begin{aligned}\boldsymbol{S}_i(k+1) &= (1-qT)\boldsymbol{S}_i(k) - \varepsilon T \frac{\boldsymbol{S}_i(k)}{|\boldsymbol{S}_i(k)|} \\ &= \left(1 - qT - \frac{\varepsilon T}{|\boldsymbol{S}_i(k)|}\right)\boldsymbol{S}_i(k) = p\boldsymbol{S}_i(k).\end{aligned} \quad (45)$$

Thus,

$$|p| = \frac{|\boldsymbol{S}_i(k+1)|}{|\boldsymbol{S}_i(k)|}, p = 1 - qT - \frac{\varepsilon T}{|\boldsymbol{S}_i(k)|}. \quad (46)$$

Obviously, $p < 1$, and when $|\boldsymbol{S}_i(k)| > \frac{\varepsilon T}{2-qT}$, we have

$$p > 1 - Tq - \frac{\varepsilon T(2-qT)}{\varepsilon T} = 1 - Tq - (2-qT) = -1. \quad (47)$$

We can have $|p| < 1$, which means $|\boldsymbol{S}_i(k+1)| < |\boldsymbol{S}_i(k)|$, and $|\boldsymbol{S}_i(k)|$ is decreasing; thus, the convergence of sliding function $\boldsymbol{S}_i(k)$ can be guaranteed.

Based on the decreasing condition $|\boldsymbol{S}_i(k)| > \varepsilon T/(2-qT)$, $Tq + \varepsilon T/|\boldsymbol{S}_i(k)| < 2$ is required. Then the adaptive parameter is reorganized as

$$\varepsilon < \frac{1}{T}(2-qT)\,|\boldsymbol{S}_i(k)|\,. \quad (48)$$

Thus, the sampling time T_k meets the condition

$$T_k < \frac{4}{1+2q}, \quad (49)$$

and $\boldsymbol{S}_i(k)$ can monotonically approach the sliding mode surface.

4 Simulations

In this section, the efficacy of the proposed sliding mode consensus is verified in a discrete-time MAS through simulations.

We consider the directed communication topology graph with five followers and one leader. The graph is shown in Fig. 1. We assume that the leader node is labeled 0, and that the follower nodes are labeled 1–5.

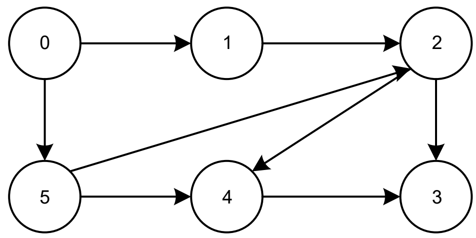

Fig. 1 Communication topology graph of the second-order discrete-time multi-agent system

Based on Fig. 1, we can define the adjacency matrix $\boldsymbol{\mathcal{A}}$, pinning gain matrix $\boldsymbol{\mathcal{B}}$, and Laplacian matrix $\boldsymbol{\mathcal{L}}$ as follows:

$$\boldsymbol{\mathcal{A}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{\mathcal{B}} = \mathrm{diag}\,(1,0,0,0,1)\,,$$

$$\boldsymbol{\mathcal{L}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Let the discrete sampling interval be $T = 0.01$. The external disturbance was applied to each follower as $d_i(k) = 0.002\cos(0.86k)$. The system matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ were set as

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}.$$

The useful parameters were defined as follows: sliding gain $\boldsymbol{C_e} = [2.5, 1]$, reaching law gain $q = 1.1$, and the Lyapunov gain $\tau = 10$.

Then we set the initial position and velocity state of each follower agent as $[1,1]$, $[2,2]$, $[3,3]$, $[4,4]$, and $[5,5]$.

From Figs. 2 and 3, we can see that all the follower position and velocity trajectories can reach finite-time consensus under the proposed control scheme. Furthermore, the consensus time was less than 10 s.

Fig. 4 shows the trajectories of the control input $u_i(k)$ of each follower agent. Compared with previous research results, the use of the saturation function to replace the sign function and the time varying parameter ε can greatly reduce the chattering of the traditional control law.

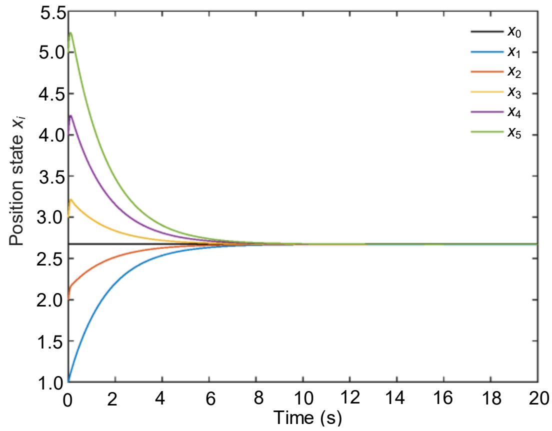

Fig. 2 Position state of finite-time leader-follower consensus with disturbances

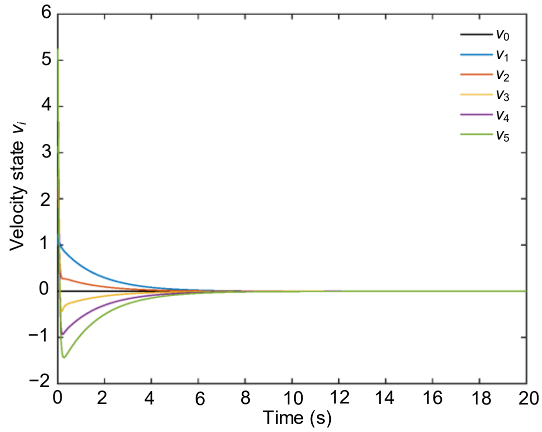

Fig. 3 Velocity state of finite-time leader-follower consensus with disturbances

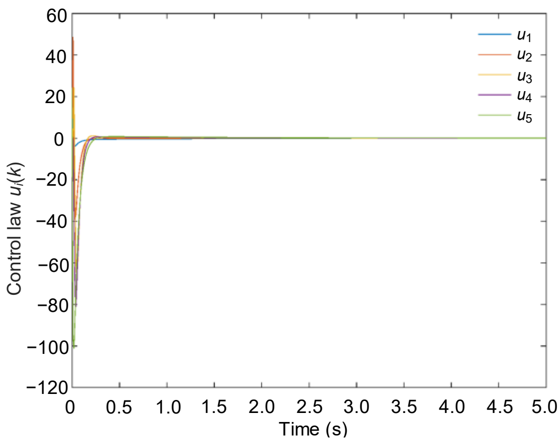

Fig. 4 Control input of finite-time leader-follower consensus with disturbances

Fig. 5 presents the sliding surface function of each follower agent. The sliding surface reached and remained at a quasi-sliding mode band within (−0.1, 0.1) in finite time.

We performed a comparative experiment to show the effectiveness of our controller. First, we replaced the sign function as our saturation function, as defined in Lemma 1. The contrast effect of the trajectories in sliding surface $\boldsymbol{S}_i(k)$ with the sign function is shown in Fig. 6.

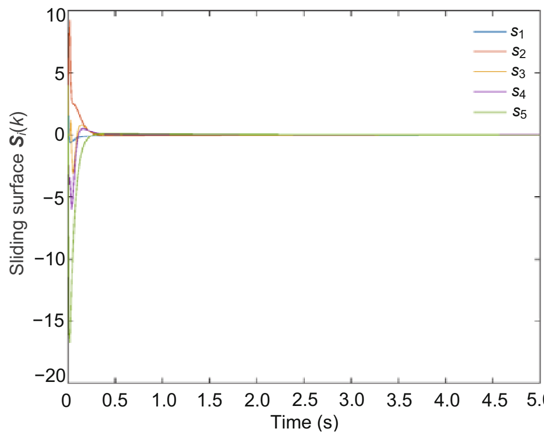

Fig. 5 Sliding surface function of finite-time leader-follower consensus with disturbances

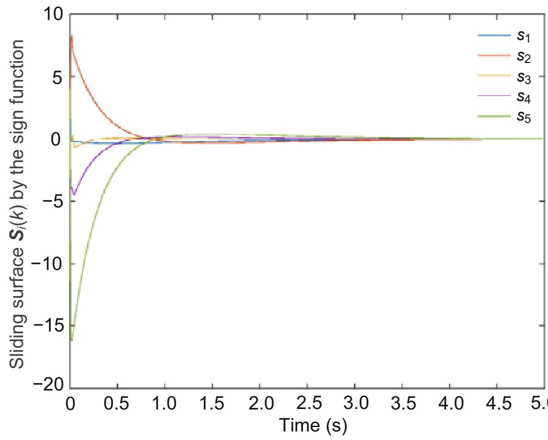

Fig. 6 Sliding surface function of finite-time leader-follower consensus with disturbances by the sign function

From Fig. 6, the sliding surface $\boldsymbol{S}_i(k)$ reached the steady-state region with a delay of more than 2 s compared to using the saturation function.

Second, we considered parameter ε. This is the main parameter for the SMC system to overcome external disturbances. Using our proposed controller, the time varying parameter was updated to the adaptive form: $\varepsilon = |\boldsymbol{S}_i(k)|/2$. Compared with the fixed value, the adaptive parameter ε was larger at the beginning to accelerate toward stability, and then decreased to reduce the chattering. The contrast effect of the trajectories in sliding surface $\boldsymbol{S}_i(k)$ with a fixed ε is shown in Fig. 7.

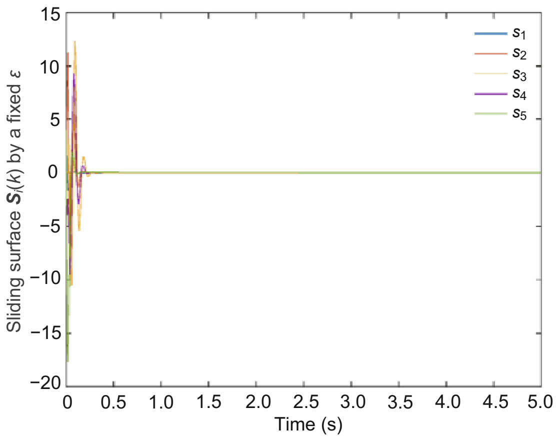

Fig. 7 Sliding surface function of finite-time leader-follower consensus with disturbances by a fixed ε

The chattering phenomenon was severe and may have a bad effect on the performance of the system. Our aim is to avoid the damage and extend the life of the system.

Based on the above comparison of Figs. 6 and 7 with Fig. 5, we conclude that the use of the saturation function and adaptive sliding mode parameter in the proposed controller is helpful and effective.

5 Conclusions

In this study, we proposed a new control algorithm to solve the finite-time leader-follower consensus problem of second-order discrete-time multi-agent systems with external disturbances. We built a distributed adaptive sliding mode controller to realize finite-time consensus. Specifically, a time-varying sliding mode parameter ε was adopted and a saturation function was designed for use in the novel sliding mode function to eliminate the influence of chattering. The results of the numerical simulations showed the efficiency and effectiveness of our control protocol.

Contributors

Ruizhuo SONG and Shi XING designed the research. Shi XING processed the data. Ruizhuo SONG, Shi XING, and Zhen XU drafted the paper. Ruizhuo SONG and Shi XING revised and finalized the paper.

Compliance with ethics guidelines

Ruizhuo SONG, Shi XING, and Zhen XU declare that they have no conflict of interest.

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