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A NUMERICAL METHOD FOR ORTHOGONAL GRID GENERATION BY LAPLACE SYSTEM

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Abstract: A new method for the generation of orthogonal body-fitted grids is presented in this paper. The positions of boundary points in the physical domain are adjusted to obtain orthogonal curvilinear grids without changing the value of ξ and η during the numerical process. The densities of ξ and η are given in advance. Good results were achieved in the application of this method on the cooling system of an internal combustion engine and a trail duct.

Key words: orthogonal grids, curvilinear coordinates, numerical method
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INTRODUCTION

Many flows and heat exchanges in engineering occur in complex regions. In the study of computational fluid dynamics, the generation of body-fitted coordinates for these complicated areas is generally a critical requirement for accurate numerical flow simulations. A poorly constructed grid system may cause erroneous results and bring about slow numerical convergence, even divergence.

The existing methods for the generation of a curvilinear body-fitted coordinate system are based on two different principles: conformal mapping and the solution of elliptic partial differential equations.

The general method proposed by D. E. Papantonis and N. A. Athanassiadis(1985) is based on the solution by finite difference means of a set of Laplace equations and application of a relaxation method. In this paper, a simple and effective numerical method for the generation of orthogonal grids is presented and used to illustrate the capabilities and characteristics of the meshes which are generated. The orthogonal grids are generated in the cooling system of an internal combustion engine and a trail duct.

GOVERNING EQUATIONS

The elliptic grid generation equations pioneered by J. F. Thompson (1974) are employed. The system utilized for generating body-fitted grids is based on the formulation of elliptic partial differential equations. In vector form, the equation may be written:

$$\nabla^2 \xi^i = P^i(\xi, \eta) \quad (i = 1,2) \qquad (1)$$

The curvilinear grid is non-orthogonal with the method described above. It is known that the governing equations can be briefly expressed by the adoption of an orthogonal curvilinear grid system and rapid numerical convergence and satisfactory results can be obtained.

Suppose the set of lines $\varphi(x, y) = c_1$ and $\Psi(x, y) = c_2$ in the Cartesian coordinates system satisfies the Cauchy-Riemann condition:

$$\varphi_x = \Psi_y ; \Psi_x = -\varphi_y$$

If $\xi(x, y)$ and $\eta(x, y)$ represent the functions φ and Ψ respectively, the desired curvilinear system (ξ, η) satisfies a set of Laplace equations in the physical domain (x, y):

$$\nabla^2 \xi^i = 0 \quad (i = 1,2) \qquad (2)$$

It is known that the solution of the equations above corresponds to a set of $\xi(x, y)$ = constant and $\eta(x, y)$ = constant lines which are mutually orthogonal.

Considering the Cartesian coordinates x and y as functions of ξ and η respectively, i.e. $x = (\xi, \eta)$ and $y = y(\xi, \eta)$, the Laplace equation (2) in the physical domain, taking into account the orthogonality of the ξ = constant and η = constant lines, is equivalent to the equations below in body-fitted coordinates:

$$\nabla^2 x^i = 0 \qquad (3)$$

where $x^i = x, y \quad (i = 1,2)$

By iterative procedure, we introduce into the set of Laplace equations (3) a scaling factor h which is equal to the ratio of the scale factors L_ξ and L_η associated with the orthogonal coordinates ξ and η respectively, described by K. A. Antonopoulos(1979). That is

$$h = \frac{L_\xi}{L_\eta}$$

where L_ξ is the total length of all the ξ = constant lines and L_η is the total length of all the η = constant lines. The following set of equations is obtained:

$$x_{\xi\xi} + x_{\eta\eta}/h^2 = 0 \qquad (4a)$$

$$y_{\xi\xi} + y_{\eta\eta}/h^2 = 0 \qquad (4b)$$

The following finite difference equation is obtained by integrating the equations (4) over control volumes centered on the grid point $P(\xi, \eta)$:

$$A_P Z_P = A_E Z_E + A_W Z_W + A_S Z_S + A_N Z_N \qquad (5)$$

where Z represents x and y respectively, the grid points surrounding the grid point P are characterized by the subscripts E, W, N, and S, with

$$A_E = \frac{1}{(\xi_E - \xi_W)(\xi_E - \xi_P)}$$

$$A_W = \frac{1}{(\xi_E - \xi_W)(\xi_P - \xi_W)}$$

$$A_N = \frac{1}{(\eta_N - \eta_S)(\eta_N - \eta_P)h^2}$$

$$A_S = \frac{1}{(\eta_N - \eta_S)(\eta_P - \eta_S)h^2}$$

$$A_P = A_E + A_W + A_S + A_N$$

The finite difference equation (5) is solved numerically by applying an iterative relaxation method and appropriate boundary conditions.

BOUNDARY CONDITIONS AND NUMERICAL PROCEDURE

The boundaries of the physical domain are also grid lines, so the physical domain boundaries must be mutually perpendicular between them.

For simplicity, as seen in Fig.1, the line of the (ξ, η) domain $\eta = 0$ is associated with the boundary of physical domain AB, The line $\eta = 1$ is associated with DC, the line $\xi = 0$ is associated with AD, the line $\xi = 1$ is associated with BC.

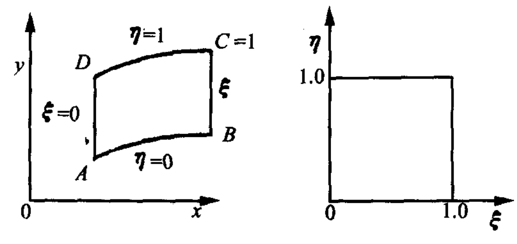

Fig.1 schematic of physical (a) and computational (b) domain

In this paper, finite difference equation (5) is solved with the successive underrelax iterative method. The block-correction method is used to accelerate convergence. In the references Y. Liren (1989), the set of Laplace equations is solved by iterative procedure. The value of ξ and η is gradually changed to obtain the orthogonal grid by using the expressions $\delta\xi = C\delta\varphi_1$ and $\delta\eta = C\delta\varphi_2$ during the computational process, where C is the coefficient associated with the computational convergence, $\delta\varphi_1$ and $\delta\varphi_2$ are the angles of the correction for orthogonality.

IMPROVED NUMERICAL PROCEDURE

Due to the difficulty in the choice of coefficient C, this paper brings forward a numerical method. The distribution of ξ and η is given in advance for the requirements of computational grids and is not changed during the whole iteration. The grid points on the boundaries are adjusted in order to make the internal grid lines perpendicular to the boundaries.

If the grid points $B(I,2)$ are not located at the normals to the boundary from the corresponding points $A(I,1)$, as is seen in Fig.2, for the next iteration, the grid points on the boundaries must be adjusted for the requirement of orthogonality. The new grid points $A'(I,1)$ on the boundary are taken as the intersection point of the boundary with the normals to the boundary from the grid points $B(I,2)$. $A'(I,1)$ is the new boundary condition for the next iteration.

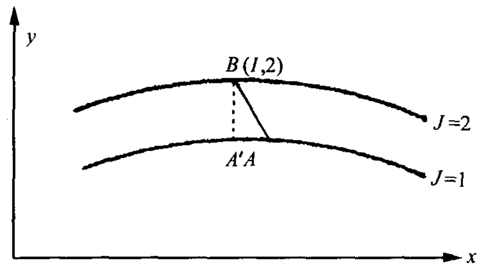

Fig.2 Adjustment of boundary grid points

The grid points on the other boundaries are also obtained with the same process described above. The finite difference equation (5) is solved for all the internal nodes of the grid. The new physical coordinates x and y of the grid points are then obtained. The block-correction method is used to rapidly transfer the new boundary conditions to the domain inside, and the value of ξ and η is not changed. As a final result, good convergence and a brief program can be obtained. The method for the solving process is described below:

1. The densities of ξ and η are ascertained in advance according to the needs of computational grids.

2. Set initial the boundary conditions and the values of internal domain for the solution.

3. Adjust the grid points on the boundaries to achieve orthogonality by the method described above.

4. Solve the finite difference equation (5).

5. Calculate the maximum value of grid points on the boundaries, i. e, max $x = \left|\frac{x^{n+1} - x^n}{x^n}\right|$ and max$y = \left|\frac{y^{n+1} - y^n}{y^n}\right|$. The whole computation ends and the outputs are given if max x and max y are less than the value ε which is the parameter controlling convergence, or return to step 3 until the convergence is achieved.

EXAMPLES

To illustrate the capabilities of the proposed method for generating body-fitted orthogonal curvilinear coordinates, the trail duct and cooling system of an internal combustion ȇngine are applied.

An example of the cooling system is given in Fig.3 and Fig.4. Fig.3 displays a non-orthogonal grid generated by the TTM method by using Poisson equations. Fig.4 illustrates a grid system generated by the improved method described in this paper. The grid system in Fig.4 has good orthogonality, compared with Fig.3.

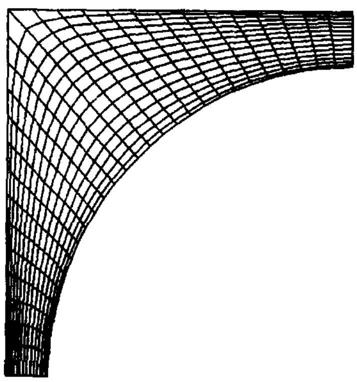

Fig.3 Grid distribution for cooling system by TTM method

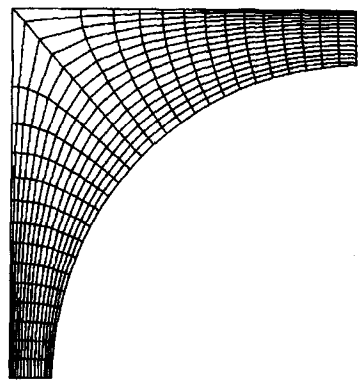

Fig.4 Orthogonal grid by the method in this paper

An example of the trail duct grid system is given in Fig.5 and Fig.6 to illustrate the capability for adjusting densities of ξ and η for controlling densities of grid lines. The densities of ξ and η are different, so different dense grids are obtained for different purposes for numerical computations.

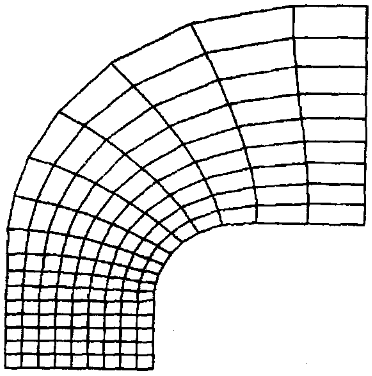

Fig.5 Grid distribution for trail duct

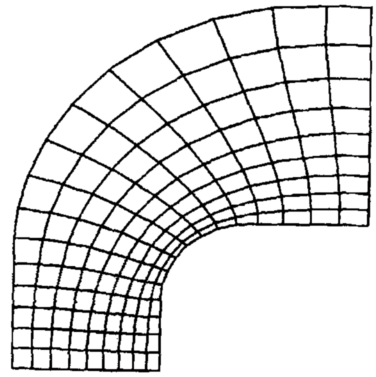

Fig.6 Grid distribution for different densities of ξ and η

CONCLUSIONS

A grid generation algorithm based on the Laplace equations for achieving grid orthogonality was developed in this paper. A modification of the numerical method of Papantonis was introduced that improves convergence rates and makes brief programs possible. The densities of ξ and η are set in advance for the computational requirements and are not changed during the whole process. Different grids are obtained by using different densities of ξ and η.

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