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USE OF SUPERPOSITION PRINCIPLE TO DERIVE A GENERAL MATHEMATICAL MODEL TO SIMULATE ONE-TO-ONE, ONE-TO-MULTI AND MULTI-TO-MULTI SAW FILTER DESIGNS

Issa Haitham M.A.* ZHU Da-zhong(朱大中), QIU Pei-liang(仇佩亮)
(*Department of Information and Electronic Engineering, Zhejiang University, Hangzhou 310027, China*)
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Abstract: This paper explains and summarizes a new attempt to derive a general mathematical model [GMM] to simulate surface acoustic wave (SAW) filters, using the superposition principle and delta function model. GMM can be used to simulate One-to-One, One-to-Multi and Multi-to-Multi SAW filter devices. The simulation program was written using MATLAB (the language of technical computing). Four-design structures (One-to-One, One-to-Two, One-to-Three and Ten-to-Ten) ware selected to test the correctness of GMM. The frequency response of the simulation and test results are similar in center frequency and 3-dB bandwidth, but the insertion loss is different, because of some second order effects (Issa Haitham, 1999).

Key words: surface acoustic wave filter and interdigital transducer (IDT)
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INTRODUCTION

Among models used to mathematically analyze the SAW filter; the most important are (1) delta-function model; (2) the cross-field model; (3) the impulse-response model. Each model can describe mathematically the SAW filter, consider some second-order effects, and can reveal some important features for SAW filter design. In this paper, we will only consider the delta function model and use it to derive the general mathematical model to simulate the frequency response of linear SAW filters, that are matched and partially programmed (Kosinski et al., 1997).

DELTA FUNCTION MODEL

The delta function model provides basic information on the transfer function response of a SAW filter (Issa Haitham, 1999). It can only yield a relative insertion loss as a function of frequency. When the voltage is applied at the electrodes of an IDT structure (Fig.1a), it produces an electric field in the gaps between electrodes. At any instant, adjacent electrodes have opposite voltage polarity and opposite charges which accumulate at the edges of the IDT fingers. The resultant charge distribution can be modeled as delta function sources of electric field intensity E_y at the finger edges (Fig.1b). Each electrode finger will have two delta sources of electric field intensity associated with it, whose amplitudes are proportional to the applied voltage. The directions and relative field polarities will alternate

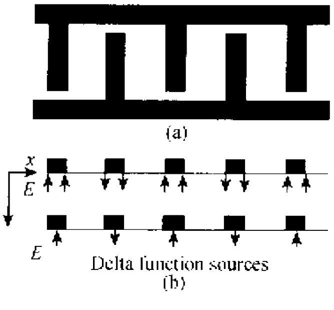

Fig.1 Bi-directional IDT of uniform apodization and its delta-function modeling
(a) bi-directional IDT with uniform apodization.
(b) delta-function modeling of E-field distribution under excited IDT with sources at each finger edge.

* Ph.D. student of Zhejiang University, from Ministry of Water and Irrigation of Jordan, E-mail: etedalissa@yahoo.com.

in pairs from finger to finger. The summation of these delta sources can be used to simulate the resultant electric field intensity and yield the frequency response $H(f)$. The intensity of the electric field distribution will be proportional to the instantaneous charge accumulation on adjacent electrode fingers (Compbell, 1998).

THE SUPERPOSITION PRINCIPLE

A linear system, in continuous or discrete time, is a system that possesses the property of superposition. If an input consists of the weighted sum of several signals (e. g. surface acoustic waves (Morgan, 1991), then the output is the superposition of the responses of the system to each of those signals (Oppenheim et al., 1997). More precisely, if $x_k[n]$, $k = 1, 2, 3, \ldots$, is a set of inputs to a linear system with corresponding outputs $y_k[n]$, $k = 1, 2, 3, \ldots$, then the response to a linear combination of these inputs is given by

$$x[n] = \sum_k a_k x_k[n] - a_1 x_1[n] + a_2 x_2[n] + \cdots \quad (1)$$

is

$$y[n] = \sum_k a_k y_k[n] - a_1 y_1[n] + a_2 y_2[n] + \cdots \quad (2)$$

DERIVATION OF THE GENERAL MODEL

Fig.2 represents the SAW filter with unapodized input and output IDTs, every group of IDTs forms what we called a rung (for more information about the concept of a rung, see the book of Colin K. Campbell, 1998, Chapter 12), so, we have several input and output rungs. Now, let the first IDT of the first input rung be located at the zero x-axis and consider it as a reference for the other input and output rung IDTs, as shown in Fig.2 below.

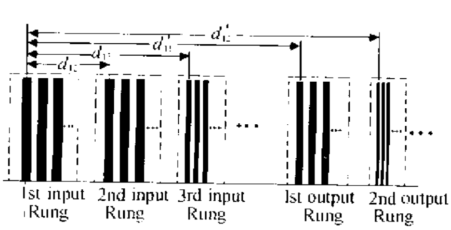

Fig.2 SAW filter of several input and output rungs

d_{1k}: The distance between the first IDT input of the first rung and the first IDT of input rung k;

d'_{1k}: The distance between the first IDT input of the first rung and the first IDT output of rung k.

To calculate the frequency response of the input and the output rungs, we will select a reference point to accumulate all the summations on it. For simplicity, we have selected the zero x-axis as a reference point. The general model for calculating the frequency response of any input or output rung will be (Issa Haitham, 1999):

$$H_{iq/ol}(f) \sum_{n=1}^{N} (-1)^n \exp(-i\beta x_n) \quad (3)$$

Whereas,

$H_{iq/ol}(f)$: The frequency response of q input (iq) rung or l output (ol) rung $= [F.\ R.\ Rung]_{iq/ol}$;

$(-1)^n$: Refers to the alternating electrode polarity.

β: The phase constant;

x_n: The distance between the centers of the input/output IDTs and the zero x-axis;

N: The number of input or output electrodes.

Now, according to Eq.(3), the whole frequency response of the first input rung and the first output rung $[F.R.\ Rung]_{11}$ will take the form:

$$[F.R.\ Rung]_{11} = [F.R.\ Rung]_{i1} \times [F.R.\ Rung]_{o1} = H_{i1}(f) \times H_{o1}(f) =$$

$$\sum_{n=1}^{N} (-1)^n \exp(-i\beta x_n) \sum_{m=1}^{M} (-1)^m \exp(-i\beta x_m) \quad (4)$$

(Issa Haitham et al., 2000).

By the same way, we can calculate the frequency response of every input rung with each output rung and get these responses:

$[F.R.\ Rung]_{12} = H_{i1}(f) \times H_{o2}(f)$

$[F.R.\ Rung]_{13} = H_{i1}(f) \times H_{o3}(f)$

also

$[F.R.\ Rung]_{21} = H_{i2}(f) \times H_{o1}(f)$

$[F.R.\ Rung]_{22} = H_{i2}(f) \times H_{o2}(f)$

$[F.R.\ Rung]_{1l} = H_{i1}(f) \times H_{ol}(f)$

$[F.R.\ Rung]_{2l} = H_{i2}(f) \times H_{ol}(f)$

and $[F.R.\ Rung]q_1 = H_{iq}(f) \times H_{o1}(f)$

$[F.R.\ Rung]q_2 = H_{iq}(f) \times H_{o2}(f)$

$[F.R.\ Rung]q_l = H_{iq}(f) \times H_{ol}(f)$

Now, as the SAW filter is considered to be a linear one, so, it should obey the super-position principle, therefore, the summation of all the previous frequency responses will be equal to the whole frequency response of the SAW filter of Fig.2.

The SAW filter frequency response $= H(f)$ $= [F.R.\ Rung]_{11} + [F.R.\ Rung]_{12} + \cdots + [F.R.\ Rung]_{11} + [F.R.\ Rung]_{21} + [F.R.\ Rung]_{22} + \cdots + [F.R.\ Rung]_{21} + \cdots + [F.R.\ Rung]q_1 + [F.R.\ Rung]q_2 + \cdots + [F.R.\ Rung]q_1$ (5)

Now, The general mathematical model (GMM) of Eq. (5) can be written as the multiplication of the summation of the frequency responses of the input rungs with the summation of the frequency responses of the output rungs:

$$H(f) = \{([F.R.\ Rung]_{i1} + [F.R.\ Rung]_{i2} + \cdots + [F.R.\ Rung]_{iq}) \times ([F.R.\ Rung]_{o1} + [F.R.\ Rung]_{o2} + \cdots + [F.R.\ Rung]_{ol})\} \quad (6)$$

APPLYING GMM TO SIMULATE THE FREQUENCY RESPONSE OF FOUR SAW FILTER DESIGNS

1. One-to-One SAW filter design

In this filter design (Issa Haitham, 1999), we have one input rung of five electrodes and center frequency of 121.4375 MHz and one output rung of nine electrodes and center frequency of 161.9167 MHz (Fig.3).

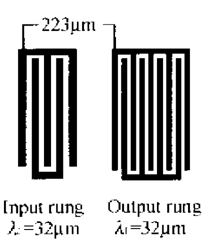

Fig.3 The SAW filter design in One-to- One case

For One-to-One case, the GMM of Equation (6) will be minimized to:

$$H(f) = ([F.R.\ Rung]_{i1}) \times ([F.R.\ Rung]_{o1}) \quad (8)$$

By using the programming language of MATLAB we can simulate Eq.(8). Fig.4 shows the simulation frequency response $H(f)$ of One-to-One SAW filter.

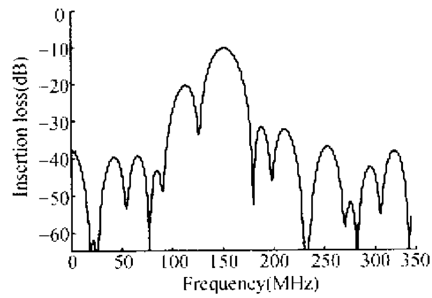

Fig.4 The frequency response of One-to-One SAW filter design, the whole center frequency f_0 = 150.05 MHz, the insertion loss (IL) = − 10.29 dB and the 3 − dB band width = 23 MHz (Issa Haitham, 1999).

2. One-to-Multi SAW filter design

One-to-Two SAW filter design

For this design, we will use an input rung of nine electrodes of center frequency 161.92 MHz and two output rungs (the first has five electrodes of center frequency 161.92 MHz and the second has also five electrodes of center frequency of 194.3 MHz) (Issa Haitham, 1999). For more details, see Fig.5.

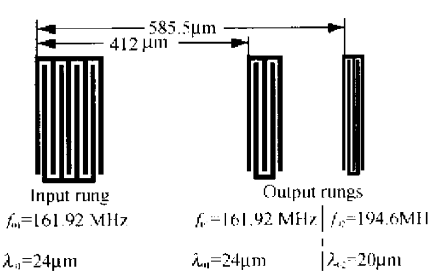

Fig.5 The SAW filter design of One-to-Two case

The frequency response of this SAW filter can be extracted from the GMM of Eq.(6).

$$H(f) = ([F.R.Rung]_{i1}) \times ([F.R.Rung]_{o1} + [F.R.\ Rung]_{o2}) \quad (9)$$

Fig.6 shows the simulation frequency response of One-to-Two SAW filter design.

$$H(f) = \sum_{n=1}^{9}(-1)^n \exp(-i\pi nf/16192)[\sum_{m_1=1}^{5}(-1)^{m_1} \exp(i\pi m_1 f/16192) \times \exp(i\beta 412) + \sum_{m_2=2}^{5}(-1)^{m_2} \exp(im_2 f/1943)\exp(i\beta 5855)] \quad (10)$$

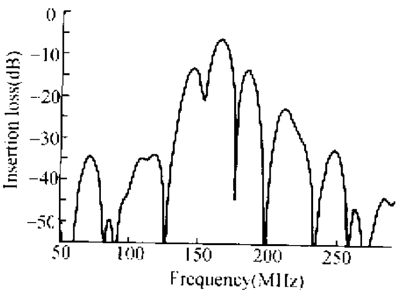

Fig.6 The frequency response of One-to-Two SAW filter design. The whole center frequency f_0 = 164 MHz, the insertion loss (IL) = −6.23 dB and the 3-dB bandwidth = 10.35 MHz
(Issa Haitham, 1999).

One-to-Three SAW filter design

In this case, we will use one input rung and three output rungs. All the rungs have the same center frequency (161.92 MHz); the input rung has three electrodes and each output rung has five electrodes (Fig.7).

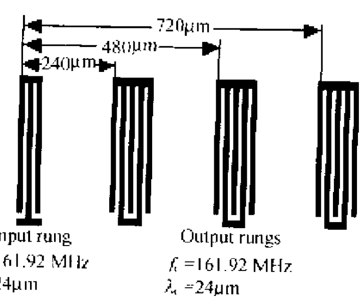

Fig.7 The SAW filter design of One-to- Three case

From the GMM derived in Eq.(6), we can calculate the proper frequency response model for such design:

$$H(f) = ([F.R.\ Rung]_{i1}) \times ([F.R.\ Rung]_{o1} + [F.R.\ Rung]_{o2} + [F.R.\ Rung]_{o3}) \quad (11)$$

$$H(f) = \sum_{n=1}^{9}(-1)^n \exp(-i\pi nf/161.92) \times [\sum_{m_1=1}^{5}(-1)^{m_1}\exp(i\pi m_1 f/161.92) \times \exp(i.\beta 4240) + \sum_{m_2=2}^{5}(-1)^{m_2} \times \exp(im_2 f/161.92)\exp(i.\beta 480) + \sum_{m_2=1}^{5}(-1)^{m_3}\exp(im_3 f/161.92) \times \exp(i.\beta 720)] \quad (12)$$

Fig. 8 shows the simulation frequency response of One-to-Three SAW filter design.

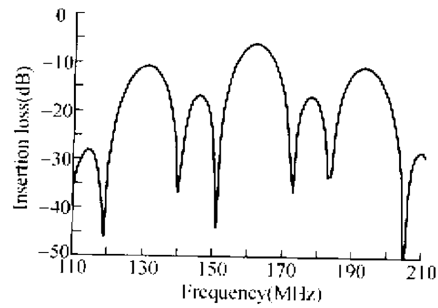

Fig.8 The frequency response of One-to- Three SAW filter design, the whole center frequency (f_0) = 162 MHz, IL = − 6.05 dB and 3-dB bandwidth ≈ 10 MHz

3. Multi-to-Multi SAW filter design

In this case, we will use a SAW filter design of ten input rungs and also ten output rungs, both have the same center frequency (80 MHz) and also same number of electrodes (five electrodes). The separation distance between every two adjacent rungs is equal to 388 μm, for more details see Fig.9.

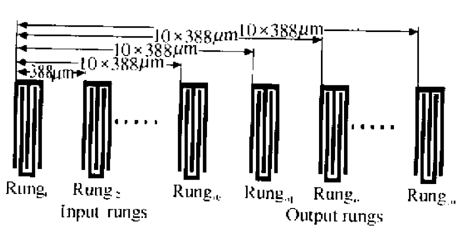

Fig.9 The SAW filter design for Multi-to-Multi (ten-to-ten) case

The mathematical model for this SAW filter design will take the form:

$$H(f) = ([F.R.\ Rung]_{i1} + [F.R.\ Rung]_{i2} + \cdots + [F.R.\ Rung]_{i10}) \times ([F.R.\ Rung]_{o1} + [F.R.\ Rung]_{o2} + \cdots + [F.R.\ Rung]_{o10}) \quad (13)$$

Now, by using MATLAB, we can find the simulation frequency response (Fig.10).

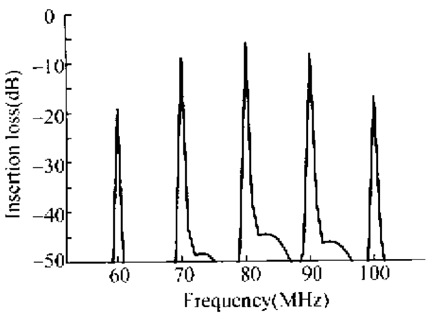

Fig.10 Frequency response Multi-to-Multi (ten-to-ten) SAW filter design, the whole center frequency = 80 MHz, IL = −6 dB and 3-dB = 0.72 MHz

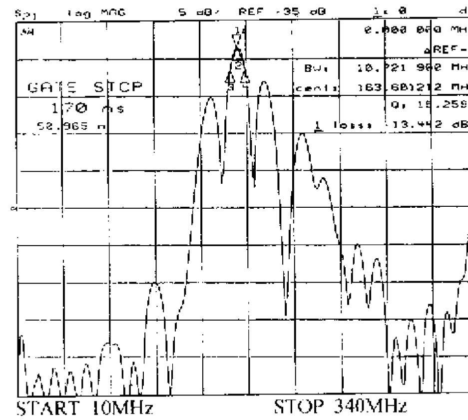

Fig.12 Frequency response of One-to-One SAW filter design f_0 = 163.6 MHz, 3-db BW = 10.321 MHz and IL = −13.442 dB (Issa Haitham, 1999).

EXPERIMENTAL RESULTS

One-to-One & One-to-Two SAW filter designs

One-to-One and One-to-Two designs had been done practically using the testing equipment HP8753D network analyzer, Cascade Microtech 9100 Probe Station and Microtech Air Coplanar Probe (for more details about the test results, refer to reference (Issa Haitham, 1999)). Fig.11 and 12 show the frequency response of One-to-One and One-to-Two SAW filters respectively.

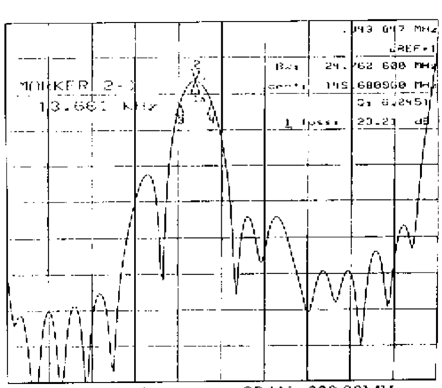

Fig.11 Frequency response of One-to-One SAW filter design f_0 = 149.68 MHz, 3-db BW = 24.76 MHz and IL = −23.21 dB (Issa Haitham, 1999).

One-to-Three & Multi-to-Multi SAW filter designs

To prove the correctness of our simulation program for those two designs, we will mention two similar SAW filter designs (Colin K. Campbell, 1998).

Fig. 13 shows the predicted frequency response (in general) for One-to-Three SAW filter design. Fig. 14 shows the simulation and the practical results for the Multi-to-Multi (ten-to-ten) SAW filter design.

From those two figures, we notice that such kind of designs will produce what we call Comb Filter.

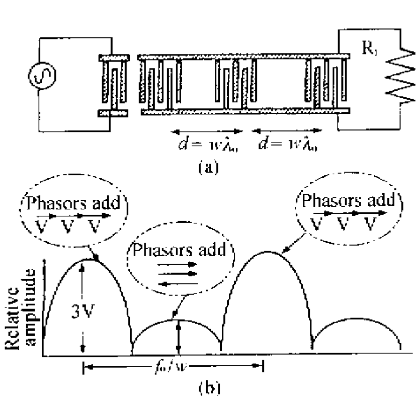

Fig.13 SAW comb filter design and its voltage phasor
(a) SAW comb filter using a three-rung tapped delay line;
(b) Use of voltage phasor to illustrate the comb pattern.

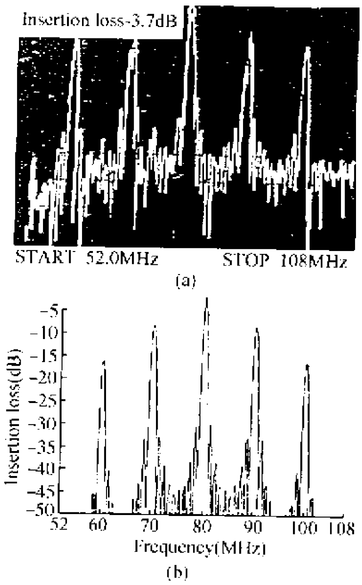

Fig.14 Experimental and predicted frequency response for Ten - to - Ten SAW filter

(a) experimental frequency response center frequency = 80 MHz, IL = 3.7 dB and 3-dB BW = 0.66;
(b) Predicted frequency response, center frequency = 80 MHz, IL = 3.7 dB and 3-dB BW = 0.66 MHz

COMPARISON BETWEEN SIMULATION AND TEST RESULTS

For One-to-One and One-to-Two designs, the frequency response of both simulation and test results are similar in center frequency and 3-dB bandwidth, but different in insertion loss, because of some second order effects (Issa Haitham, 1999). This kind of design can find applications in mobile and wireless communications (Campbell, 1998).

For One-to-Three and Multi-to-Multi designs, the center frequency and the 3-dB BW of simulation results are similar to that of the test ones, but different in IL values. These designs have many applications; here, we have selected the SAW filter designs that can give the Comb filter features that can be used for many applications. The standard condition for the Comb filter will be satisfied if the separation distance between every two adjacent rungs is equal to a multiple of wavelength ($w\lambda_0$) (Campbell, 1998). Table (1) shows the comparison between simulation and test results.

Table 1 Comparison of simulation and test results

SAW filter design case	Center frequency (f_0) MHz		3-dB bandwidth MHz		Insertion loss (dB)	
	Simulation	Test	Simulation	Test	Simulation	Test
One-to-One	150.5	149.68	23	24.76	− 10.29	− 23.2
One-to-Two	164	163.60	10.35	10.321	− 6.23	− 13.4
One-to-Three	162	−	10	−	− 6.05	−
Ten-to-Ten	80	80	0.72	0.66	− 6	− 3.7

CONCLUSIONS

In this work, we derived a general mathematical model (GMM) using the superposition process. This model can simulate the frequency response of all possible combinations of SAW filter designs. The frequency response of the practical results is similar to the simulation results in center frequency and 3-dB bandwidth, but different in insertion loss values, due to some second order effects and the time gated method (this method will produce an extra insertion loss) (Issa Haitham, 1999).

Finally, this model can be considered as a promising one for resolving the frequency response of SAW filter designs.

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