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Hydrostatic settling velocity of sediments*

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Abstract: The settling of sediments is an accelerative process in which the concentration of the main sediments zone will heavily influence settling velocity, but the explicit relationship between the concentration and settling velocity has not been reported in literature. Here a theoretical function was built for the time dependent concentration and time dependent settling velocity of sediments; then the entire settling process reflecting concentration was shown on the basis of sediments instant-settlement theory and mathematical method. Agreement of computed results and experimental data was found. Several governing parameters, including particle size, particle density, initial suspended sediments concentration and suspension height, were discussed with a series of calculated velocity curves. The research indicated that (1) the presented concentration-velocity time relationship is rational, (2) settling process of the sediments group with variation of concentration consists of acceleration stage, uniform motion stage and deceleration stage, and (3) particle size, particle density and initial suspended sediments concentration have more influence on the settling velocity than the suspension height and water temperature.

Key words: hydrostatic settling velocity, suspension concentration, settling in group, sediments
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INTRODUCTION

Knowledge of sediments settlement is often applied to many engineering fields such as hydraulic engineering, environmental engineering and chemical engineering. In fact, hydrostatic settling velocity is a elementary factor affecting settlement of sediment groups.

The settling velocity of sediment groups is exponentially accelerated during the initial stage and rapidly approaches a terminal value (Qian et al., 1983; Wu et al., 2000). To simplify practical problems, the actual settlement velocity in water is always replaced by that terminal value and the effect of variation of the concentration in the main sediments zone is ignored completely. In other words, the problem is regarded as a simple problem with a infinite bottom. But every actual problem has finite physical boundary. The suspension-concentration of the main sediments zone will increase with time if there is little or no flocculation in the suspension.

The concentration of the main sediments zone will increase nonlinearly with time, and its distribution along depth is also a nonlinear problem. But the settlement of the whole sediments body is more of concern in practical engineering. In this paper, an explicit relationship between time-dependent concentration and time-dependent settling velocity will be built for integrated sediments, then the entire settling process reflecting concentration will be simulated with mathematical method, the computed results will be compared with experimental data. At last, several influential parameters will be discussed.

GOVERNING EQUATION OF HYDROSTATIC SETTLEMENT OF SEDIMENTS

1. The governing equation of sediment particle

According to the common ideal, the sediment particle is simplified as a spherical entity. The general equation of the movement of a sediment particle with mass M can be expressed as

$$F = M\frac{\mathrm{d}u_s}{\mathrm{d}t} \tag{1}$$

where, u_s is the particle's instantaneous velocity and t is time span; F is the total force acting on the particle and is decomposed to:

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gravity force + buoyancy force + drag force + added-mass force + Basset force + Magnus force + Saffman force (2)

where, the last two items are termed lift force; it is caused by an object's rotation in fluid medium and is always ignored as it is of so small order of magnitude. According to Wood and Jenkins (1973), the Basset force can be neglected because of its short lasting-time and small influence range. With these assumptions and the condition of small Reynolds number, the hydrostatic movement equation of a sediment particle can take the form

$$M\frac{du_s}{dt} = (M - m)g - 3\pi d\mu u_s - \frac{1}{2}m\frac{du_s}{dt} \quad (3)$$

where, $M = \frac{1}{6}\pi d^2\rho_s$, $m = \frac{1}{6}\pi d^3\rho_w$, ρ_s and d are density and size of sediment particle respectively, ρ_w is the density of water, μ is dynamic viscosity of water, g is acceleration of gravity.

2. The governing equation of sediments

Considering the settlement of sediments in group, there is non-slipping phenomenon on the boundary of settling solid for the viscosity of fluid. So, the particle will settle down attached to some water and other water will flow upwards at the same time. Moreover, the density and viscosity of the suspension will be changed, so the submerged weight and drag force should be modified. The modified equation of motion takes the form (Wu et al., 1996) below.

$$M\frac{du_s}{dt} = (M - m)g(1 - S_v) - 3\pi d\mu_m \cdot \left(\frac{1}{S_{vy}}u_s\right) - \frac{1}{2}m\frac{du_s}{dt}\left(\frac{1}{S_{vy}}\right) \quad (4)$$

where, S_v is volume content of sediments, μ_m and μ_0 are the coefficients of dynamic viscosity of the suspension and clear water respectively, which has the form (Fei, 1993)

$$\mu_m = \left(1 - \frac{S_v}{S_m}\right)^{-2.0}\mu_0 \quad (5)$$

where, S_{vm} is the ultimate volume content of the suspension which takes the form:

$$S_{vm} = 0.91 - 0.2\log(1/d) \quad (6)$$

In Eq. (4), S_{vy} is the volume of up-flowing water due to the settlement of sediments among unit volume water (Wu et al., 1996). S_{vy} can be calculated from the equation below.

$$S_{vy} = 1 - (1 + 2\delta_0/d)^3 S_v \quad (7)$$

δ_0 in Eq. (7) is a function of concentration and particle size and can be calculated by an explicit formula (Wu et al., 1996).

The first term on the right of Eq.(4) is the effective weight of particles in suspension, the following two terms are fluid drag force and added-mass force jointly affecting up-flowing current.

With the initial conditions: $t = 0$ and $u_{s0} = 0$, the settling velocity solved from Eq. (4) takes the form:

$$u_s = u_{se}(1 - e^{\zeta \cdot t}) \quad (8)$$

where

$$u_{se} = \frac{\mu_0}{\mu_m}(1 - s_v)S_{vy}\frac{\rho_s - \rho_w}{18v}d^2 \quad (9)$$

$$\zeta = \frac{18\mu_m}{d^2(S_{vy}\rho_s + \rho_w/2)} \quad (10)$$

where, v is coefficient of kinematic viscosity.

RELATIONSHIP OF VELOCITY AND CONCENTRATION

According Eqs. (8) – (10), settling velocity u_s is only a function of time t and concentration S_v; and for constant size d and density ρ_s of the sediment particle, the concentration with passage of time will be affected by the prior settlement process. If S_{v0} is the initial volume concentration of the suspension, H is the total height of the sample, h_s is distance of sediments from the upper surface of the water body, the averaged volume concentration of the suspension zone (the shaded area in Fig.1) at t_s can be determined from:

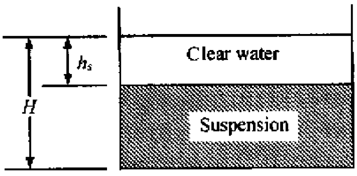

Fig.1 Model of sediments settlement

$$S_v = S_{v0} \frac{1}{1 - h_s / H} \quad (11)$$

where

$$h_s = \int_0^{t_s} u_s \mathrm{d}t \quad (12)$$

A coupling relationship between u_s and S_v can be found in th above two expressions, u_s can be computed with mathematical method. At a given time, the last concentration can be used to calculate settlement velocity u_s first; then the concentration S_v at this time can deduced from the last u_s. With new S_v, a new u_s can be computed easily. The above steps should be iterated until the error reaches a given small value.

SIMULATION OF SEDIMENTS SETTLEMENT

1. Several parameters for calculation

The following numerical simulation carried out based on a sediments settlement experiment on a freshwater lake. In a large-scale lake dredging project launched 1999's year-end, the dredged sediments were piped to a sludge-reservoir. Several hydraulic parameters of the piped sludge suspension determined at the laboratory were[①]: average size $d_{50} = 0.0125$ mm; the main content of particles was silty clay; the density of particle $\rho_s = 2.288 \times 10^3$ kg/m^3, the initial volume concentration of suspension was 10.1%. The test suspension sample: $H = 55$ cm, $\rho_w = 1.0 \times 10^3$ kg/m^3; $v = 1.41 \times 10^{-6}$ m^2/s, $\mu_0 = 1.140 \times 10^{-3}$ N·s/m^2.

2. Calculated results and their analysis

A computing program was compiled based on obtained u_s-S_v relationship. The above parameters were used to compute the entire settlement process as plotted in Fig.2.

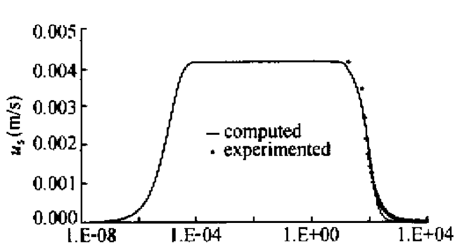

Fig.2 Computed and experimental settling velocity

The velocity of the sediments settling process had three different stages:

1) Accelerated stage: The time span of this stage was small, $t_1 = 1.4 \times 10^{-4}$ s. This was a varying accelerated stage where the acceleration was small (initially but increased rapidly), then reduced to zero. In other words, the velocity approaches a terminal velocity. In fact, this stage can not be observed with conventional method in laboratory.

2) Uniform-motion stage: After the acceleration reached zero, the settlement velocity would keep the terminal value use for about 60 s. If the test suspension has infinite bottom ($H = \infty$), the terminal velocity will be maintained forever. But accelerated period was so short that the concentration barely changed, so the terminal velocity with varying concentration was almost equal to the value with constant concentration.

3) Decelerated stage: The velocity decelerated during this period, which took about 60 s, till the terminal value decreased to 2% of the initial velocity.

3. Comparison of experimental results and computed results

Suspension samples were taken randomly from the entrance of a dredged-sludge reservoir and delivered to laboratory, where they were poured into a 30 cm diameter, 70 cm height glasss settlement cylinder. The elevation of sediments upper surface was monitored by naked eyes after the sample with 55 cm height was dispersed with a muddler. The observed velocities are marked in Fig.2.

Because the initial period of settlement was completed very rapidly, the elevation of sediments upper surface varied so lightly that it could not be monitored exactly. The good agreement between calculated results and observed results in Fig. 2 verified completely the reliability of these data. With regard to the later period, several observed values exceeded the computed curve lightly. Because the averaged concentration within the main sediments zone replaced the actual concentration on the upper surface of the sediments when the u_s-S_y explicit relationship was built, the concentration would actually in-

① Municipal Engineering Institute of Zhejiang University. Experimental report of dredged sediments of West Lake. 2000,4.

crease along depth in main sediments zone during the whole settling process in fact, in other words, the concentration for calculation was larger than the actual value. For this reason, the theoretical velocity would be decelerated rapidly during the later period, so the velocity curve would accordingly be under the observed data.

DISCUSSION OF GOVERNING PARAMETERS

The hydrostatic settling velocity of sediments is governed by many parameters, such as water-temperature, particle size, particle density, initial concentration of suspension and depth of suspension.

1. Water temperature

Study of the effect of temperature variation on settlement velocity showed that the settling velocity was inversely related to temperature (Vanoni, 1981). Although the average size varied with temperature, the variation range was small. To calculate the settling velocity, a constant particle size can be used if the temperature does not vary largely. It can not generate obvious error. For example, the error in the calculated velocity can not exceed 2% with the temperature varying from 20 – 30℃.

2. Particle size

According Eq. (9), the terminal velocity varies with particle size in geometrical manner. Keeping other conditions constant, the settling velocities of sediments with different particle size are plotted in Fig.3.

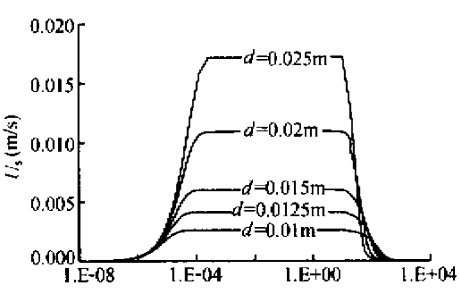

Fig.3 Settling velocity with different particle size

The time span of the accelerated stage, uniform motion stage and decelerated stage are denoted with ΔT_1, ΔT_2 and ΔT_3 respectively. Based on these curves, ΔT_1 and ΔT_3 will be lengthened with increasing d, but ΔT_2 will be shortened. The extent of acceleration during the initial stage and deceleration during the final stage have increasing trends, and usually do not decrease with lengthened time span. The rising initial velocity of a particle in the initial stage accelerated with increase in particle size; the high velcity during the decelerated stage accelerates the accretion of sediment concentration and decelerates the termiral velocity.

3. Particle density

The particle density (or proportion) is also an important parameter of settling velocity. Particle density can not change the time span of settling stages like particle size d although u_{se} will increase with increasing particle density. According to Fig.4, the density did not vary with ΔT_1, ΔT_2 and ΔT_3, but the extent of acceleration during the initial stage and deceleration during the final stage are increased with increased u_{se}.

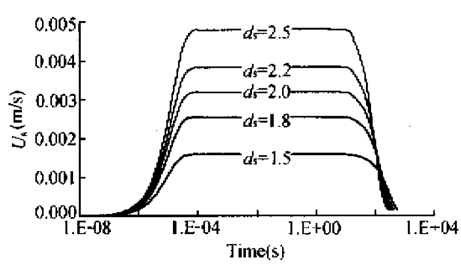

Fig.4 Settling velocity with different sediment density

4. Initial concentration of suspension

If several particles aggregate in fluid, they will fall into a group; the falling velocity will be larger that of one single particle. On the other hand, if the particles are dispersed in fluid, the falling velocity will be decelerated due to the mutual interference of the particles.

This interference affects the concentration of the suspension, but as the concentration varies with time, the initial value must be taken into account. The present numerical model takes into account the above-mentioned effect. The settling velocities of sediments with different initial concentration are shown in Fig.5.

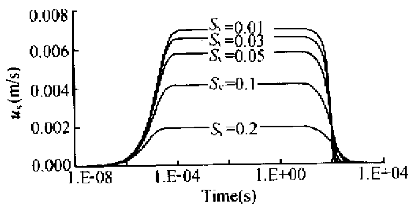

Fig.5 Settling velocity with different initial concentration

These curves indicate that the terminal velocity decrease with increasing initial concentration. The accretion of initial concentration shortened ΔT_1 and lengthened ΔT_2, but had no effect on ΔT_2. The velocity curves during the initial and final stages became plain with the decreased terminal velocity.

5. Height of suspension

The settlement of sediment with infinite bottom was the main concern in previous literature, so the effect of suspension depth on velocity was not considered. But actual engineering practice indicated that finite physical bottom was in every sediment settlement project and that the depth of bottom must be taken into account. Eq. (11) also indicated that suspension height had more effect on concentration with time. The increase of height will slow the increase concentration and decelerate the attenuation of velocity. The terminal velocity will be maintained forever with $H = \infty$. The settling velocities of sediments with different suspension height are shown in Fig.6.

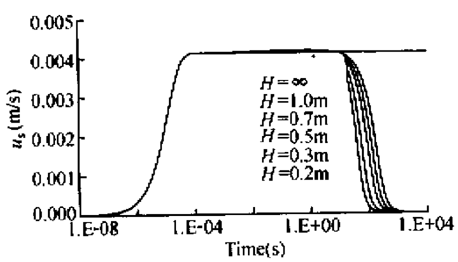

Fig.6 Settling velocity with different suspension height

In the above figure, the all velocity curves coincide during the first two stages, spread apart in the last stage. Decrease of height hastened deceleration rapidly. Because the two former stages' time span was small, the concentration varied slightly with small h_s, the velocities with varying-concentration were nearly equal to those with constant concentration. During the last stage with longer time span, the concentration varied when h_s reached a sizable value; the variation of H affected the suspended sediment concentration. These conclusions agree with actual observations.

CONCLUSIONS

1. The basis u_s-S_v relationship and numerical methods were used to study in detail the entire settling process of sediments in group. The calculated results were verified with observed data.

2. The settling process of sediments in group is a varying-acceleration process, considering of accelerated stage, uniform-motion stage and decelerated stage.

3. Particle size, particle density and initial concentration have more effect on settling velocity than water-temperature and suspension height.

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