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A mathematical model of symmetry based on mathematical definition*

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Abstract: Tolerance is imperative for seamless integration of CAD/CAM(Computer Aided Disign/Computer Aided Manufacture) which is just a text attribute and has no semantics in present CAD systems. There are many tolerance types, the relations between which are very complicated. In addition, the different principles of tolerance make study of tolerance difficult; and there may be various meanings or interpretation for the same type of tolerance because of the literal definition. In this work, latest unambiguous mathematical definition was applied to study, explain and clarify: (1) the formation and representation of tolerance zone, and (2) the formation and representation of variational elements; after which, the mathematical models of symmetry of different tolerance principles and different interpretations were derived. An example is given to illustrate the application of these models in tolerance analysis.

Key words: mathematical definition, symmetry tolerance, mathematical model, CAD/CAM, production engineering
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INTRODUCTION

The seamless integration of CAD and CAM is not easily achievable because of insufficient useful information on tolerance in CAD systems. How to interpret the semantics of tolerance in the modeling and representation of tolerance is becoming more and more important for the integration of tolerance in CAD/CAM. Research on tolerance had been carried out for many years (Bjorke, 1989, Wu et al., 1999, Ngoi et al, 1998), but the emphasis was on tolerance synthesis, tolerance analysis and optimal design of tolerance, not on tolerance modeling (Lee, 1998; Yang et al, 2000; Liu et al, 1999). Tolerance is defined exactly and unambiguously with expressive mathematical terms in ANSI Y 14.5.1.M (ASME, 1994), which eliminate the unambiguity of literal definition and interpret tolerance definitely. So the mathematical definition is very suitable for tolerance modeling and representation, whereas how to embody the semantics of tolerance and realize the application of mathematical definition in tolerance analysis should be researched deeply. Herein a mathematical model is needed which should appropriately express the semantics of tolerance.

In this study, taking the variations of DOFs (Degrees Of Freedom) as model variables, a mathematical model of symmetry is researched systematically based on the mathematical definition of tolerance, and its application in tolerance analysis is also discussed.

MATHEMATICAL REPRESENTATION OF A PLANE BASED ON THE VARIATION OF DOF

In general, objects are constructed by points, lines and faces according to a given way and have six DOFs at most, i.e., three transla-

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tional DOFs X, Y, Z and three rotational DOFs θ_x, θ_y, θ_z. The actual number of DOF is 6-n, where n is the number of invariation.

Fig. 1a shows an arbitrary planar surface with length $2a$ and width $2b$ in its local coordinate system(LCS), which has three DOFs θ_x, θ_y, Z. The enclosing rectangle ($2a \times 2b$) is referred to as an extent feature serving as the effective area of the tolerance zone. Fig. 1b shows the variational planar surface obstained by slightly varying the normal surface according to its DOF. For small variations of the model variables, the topology of the geometry remains unchanged (Roy, 1998).

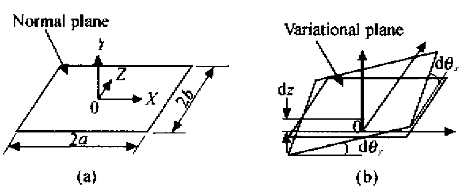

Fig.1 Normal and variational planar surface
(a) normal plane and its LCS; (b) variational plane

In Fig. 1a, the normal plane equation can be expressed as(Liu, 2000):

$$z = 0 \tag{1a}$$

Fig. 1b's variational plane equation can be given as:

$$z = dz + x \cdot d\theta_y + y \cdot d\theta_x \tag{1b}$$

That is,

$$Ax + By + Cz + D = 0 \tag{1c}$$

Where

$$A = d\theta_y;\ B = d\theta_x;\ C = -1;\ D = dz$$

$d\theta_x$, $d\theta_y$, dz are the variational values of model variables θ_x, θ_y, z respectively.

Obviously the variational planar equation can be represented by variation of DOF, so its variation can be referred to as model variables.

MATHEMATICAL MODEL OF SYMMETRY

Symmetry is used to control coaxiality or coplanarity of measured median elements with respect to datum elements. There are two kinds of interpretation in mathematical definition: either in terms of the surface of the actual feature or in terms of the resolved geometry (center point, axis, or center plane) of the applicable (mating or minimum material) actual envelope which are called "surface interpretation and resolved geometry interpretations" respectively. Moreover, because of different tolerancing principle, the mathematical model of symmetry will be very different. Here for the slot illustrated in Fig.2, the following four cases of mathematical models are researched.

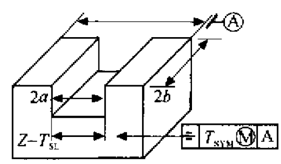

Fig.2 Part with requirement of symmetry

Case 1 Mathematical model of symmetry satisfying principle of independence and explained with resolved geometry interpretation.

Case 2 Mathematical model of symmetry satisfying principle of independence and explained with surface interpretation.

Case 3 Mathematical model of symmetry satisfying requirement of maximum material and explained with resolved geometry interpretation.

Case 4 Mathematical model of symmetry satisfying requirement of maximum material and explained with surface interpretation.

Other kinds of mathematical model can be derived similarly.

1. Mathematical model of symmetry satisfying principle of independence and explained with resolved geometry interpretation

When symmetry is given regardless of feature size (RFS), the value of symmetry is not relative to the size tolerance of the slot and principle of independence is satisfied. If symmetry is explained with resolved geometry interpretation, the median plane of the slot feature must lie within the zone bounded by a pair of parallel planes of size equal to the total allowable tolerance and whose position coincides with the true position. The LCS construction is shown in Fig.3.

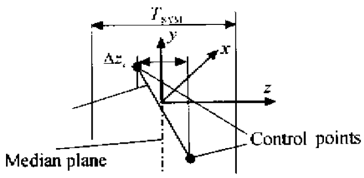

Fig.3 Symmetry tolerance zone when explained with resolved geometry

The median plane z_C is given by

$$z_C = dz_C + x \cdot d\theta_y + y \cdot d\theta_x \tag{2}$$

The requirement for symmetry can be given as

$$-\frac{T_{SYM}}{2} \leqslant z_C(x, y) \leqslant \frac{T_{SYM}}{2} \tag{3a}$$

or,

$$|\Delta z_C(x, y)| \leqslant T_{SYM} \tag{3b}$$

Where $\Delta z_C(x, y)$ is the z-value difference between any two of the four vertice of z_C i.e.,

$$\Delta z_C(x, y) = z(x_i, y_i) - z(x_j, y_j),$$
$$(x_i, y_i), (x_j, y_j) \in$$
$$\{(a, b), (a, -b), (-a, b), (-a, -b)\}$$

(Supposing the length and width of the rectangle plane is $2a$ and $2b$), T_{SYM} is the value of symmetry.

Substituting Eq.(2) into Eq.(3a) and Eq.(3b), the mathematical model of symmetry can be derived as follows.

Variations

$$-\frac{T_{SYM}}{2b} \leqslant d\theta_x \leqslant \frac{T_{SYM}}{2b} \tag{4a}$$

$$-\frac{T_{SYM}}{2a} \leqslant d\theta_y \leqslant \frac{T_{SYM}}{2a} \tag{4b}$$

$$-\frac{T_{SYM}}{2} \leqslant dz_C \leqslant \frac{T_{SYM}}{2} \tag{4c}$$

Constraints

$$-\frac{T_{SYM}}{2} \leqslant x \cdot d\theta_y + y \cdot d\theta_x \leqslant \frac{T_{SYM}}{2} \tag{4d}$$

$$-\frac{T_{SYM}}{2} \leqslant dz_C + x \cdot d\theta_y + y \cdot d\theta_x \leqslant \frac{T_{SYM}}{2} \tag{4e}$$

2. Mathematical model of symmetry satisfying principle of independence and explained with surface interpretation

In this case, the two parallel planar surfaces of the slot must lie within the symmetry tolerance zone(SYTZ) bounded by a pair of parallel planes of size equal to half of the total allowable tolerance T_{SYM} respectively and the distance between the median plane of SYTZ and that of the slot feature is equal to half of the actual width W_{AM} of the slot. There are three cases according to W_{AM}.

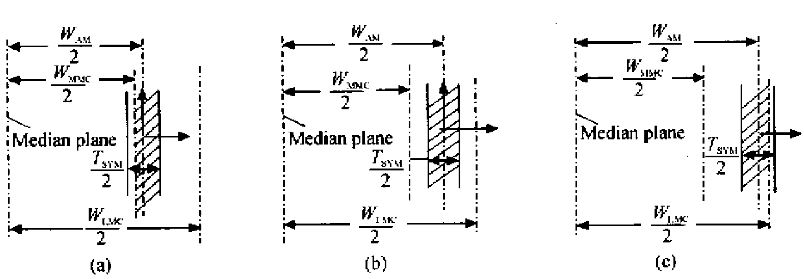

Fig.4 Different positions of symmetry tolerance when explained with surface omterpretation and given regardless of feature size

(a) when the lower bound of SYTZ is outside SITZ
(b) when both bounds of SYTZ is with SITZ
(c) when the upper bound of SYTZ is outside SITZ

Case 2.1 The lower bound of SYTZ is located outside SITZ(Size Tolerance Zone) in Fig. 4a. Therefore the lower bound of SITZ is regarded as the that of RTZ, whereas the upper limit of SYTZ is regarded as that of RTZ(Result Tolerance Zone).

Case 2.2 The lower and upper bound of SYTZ are all located within SITZ in Fig. 4b.

Therefore the upper and lower limit of SYTZ is regarded as those of RTZ respectively.

Case 2.3 The upper bound of SYTZ is located outside of SITZ in Fig. 4c. Therefore the upper limit of SITZ is regarded as that of RTZ, whereas the lower limit of SYTZ is regarded as that of RTZ.

The LCS construction is shown in Fig. 4a. The mathematical model of symmetry of this case can be defined as follows:

Variations

$$-\frac{T_{\mathrm{up}}-T_{\mathrm{btm}}}{2b} \leqslant \mathrm{d}\theta_x \leqslant \frac{T_{\mathrm{up}}-T_{\mathrm{btm}}}{2b} \quad (5a)$$

$$-\frac{T_{\mathrm{up}}-T_{\mathrm{btm}}}{2a} \leqslant \mathrm{d}\theta_y \leqslant \frac{T_{\mathrm{up}}-T_{\mathrm{btm}}}{2a} \quad (5b)$$

$$T_{\mathrm{btm}} \leqslant \mathrm{d}z \leqslant T_{\mathrm{up}} \quad (5c)$$

Constraints

$$-\frac{T_{\mathrm{up}}-T_{\mathrm{btm}}}{2} \leqslant x\cdot\mathrm{d}\theta_y + y\cdot\mathrm{d}\theta_x \leqslant \frac{T_{\mathrm{up}}-T_{\mathrm{btm}}}{2} \quad (5d)$$

$$T_{\mathrm{btm}} \leqslant \mathrm{d}z_C + x\cdot\mathrm{d}\theta_y + y\cdot\mathrm{d}\theta_x \leqslant T_{\mathrm{up}} \quad (5e)$$

Where T_{up} and T_{btm} are the value of the upper and lower bound of RTZ respectively which will be substituted with different value according to different position of symmetry according to Fig. 4, i.e. for case 2.1, T_{up} is equal to $T_{\mathrm{SYM}}/2$, T_{btm} is equal to T_{SL}. for case 2.2, T_{up} is equal to $T_{\mathrm{SYM}}/2$, T_{btm} is equal to $-T_{\mathrm{SYM}}/2$. for case 2.3, T_{up} is equal to T_{SU}, T_{btm} is equal to $-T_{\mathrm{SYM}}/2$.

3. Mathematical model of symmetry satisfying requirement of maximum material and explained with resolved geometry interpretation

The mathematical model of this case is similar to that of Case I mentioned in Section 3.1, except that the value of symmetry can be compensated by the difference of actual size and MMC size of the slot feature. The mathematical model of this case can be obtained by changing inequations (4a – e).

Variations

$$-\frac{T_{\mathrm{SYM}}+T_C}{2b} \leqslant \mathrm{d}\theta_x \leqslant \frac{T_{\mathrm{SYM}}+T_C}{2b} \quad (6a)$$

$$-\frac{T_{\mathrm{SYM}}+T_C}{2a} \leqslant \mathrm{d}\theta_y \leqslant \frac{T_{\mathrm{SYM}}+T_C}{2a} \quad (6b)$$

$$-\frac{T_{\mathrm{SYM}}+T_C}{2} \leqslant \mathrm{d}z_C \leqslant \frac{T_{\mathrm{SYM}}+T_C}{2} \quad (6c)$$

Constraints

$$-\frac{T_{\mathrm{SYM}}+T_C}{2} \leqslant x\cdot\mathrm{d}\theta_y + y\cdot\mathrm{d}\theta_x \leqslant \frac{T_{\mathrm{SYM}}+T_C}{2} \quad (6d)$$

$$-\frac{T_{\mathrm{SYM}}+T_C}{2} \leqslant \mathrm{d}z_C + x\cdot\mathrm{d}\theta_y + y\cdot\mathrm{d}\theta_x \leqslant \frac{T_{\mathrm{SYM}}+T_C}{2} \quad (6e)$$

where T_C is the compensation value.

4. Mathematical model of symmetry satisfying requirement of maximum material and explained with surface interpretation

Compared with the mathematical model of Case II mentioned in Section 3.2, the boundaries of RTZ for each surface of the slot in Fig.5 are definitive, that is, the distance between the upper boundary and the median plane of the slot feature is equal to the sum of the upper bound value of size tolerance and the normal width of the slot, whereas the distance between the lower boundary and the median of the slot feature is equal to the difference of the normal width of the slot minus the lower bound value of size tolerance and half of the value of symmetry tolerance. Using similar method, the mathematical model can be represented as follows.

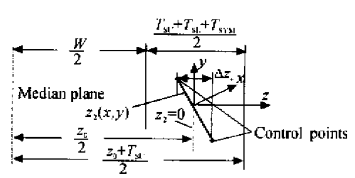

Fig.5 Tolerance zone of symmetry given RFS and explained with surface interpretation

Variations

$$-\frac{T_S+T_{\mathrm{SYM}}}{4b} \leqslant \mathrm{d}\theta_x \leqslant \frac{T_S+T_{\mathrm{SYM}}}{4b} \quad (7a)$$

$$-\frac{T_S+T_{\mathrm{SYM}}}{4a} \leqslant \mathrm{d}\theta_y \leqslant \frac{T_S+T_{\mathrm{SYM}}}{4a} \quad (7b)$$

$$-\frac{T_{\mathrm{SL}}+T_{\mathrm{SYM}}}{2} \leqslant \mathrm{d}z \leqslant \frac{T_{\mathrm{SU}}}{2} \quad (7c)$$

Constraints

$$-\frac{T_S + T_{SYM}}{4} \leqslant x \cdot d\theta_y + y \cdot d\theta_x \leqslant \frac{T_S + T_{SYM}}{4} \tag{7d}$$

$$-\frac{T_{SL} + T_{SYM}}{2} \leqslant dz + x \cdot d\theta_y + y \cdot d\theta_x \leqslant \frac{T_{SU}}{2} \tag{7e}$$

Where T_S is the value of size tolerance, $T_S = T_{SU} + T_{SL}$.

APPLICATION

In this section, the above-mentioned mathematical models are used in assembly interference analysis, which is really a special kind of tolerance analysis, that is, interference occurs when there exists negative space between any parts of assembly. Assembly interference examination is usually checked just for assembly of normal dimension which can't include tolerance information, whereas tolerance can be taken into consideration conveniently by using the above mathematical models.

Fig.6 is a slot and a key for assembly together (1983). Part M will be inserted in the slot of part N as a whole key, whereas the middle part of the part N will be inserted as a key in the slot of part M. There are two fittings: one happens at width 30 mm and the other happens at width 10 mm. To simplify the problem, the sizes 30 mm of two parts are all supposed normal values which are fitted just right and can locate the z-axis position of part M in part N, the necessary tolerances are showed in Fig. 6, which satisfy β-distribution of $\alpha = 2$, $\beta = 2$. The problem is to analyze the probability of interference between the above two parts.

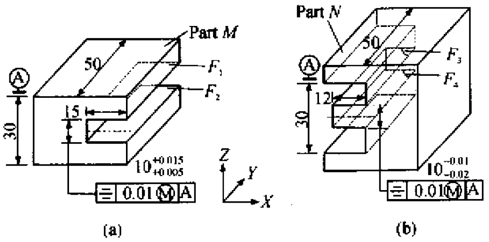

Fig.6 A slot and a key to be assembled together
(a) a part with slot feature; (b) a part with key feature

In Fig.6, the value of symmetry of the slot and the key are given at MMC. When the actual condition is not MMC, the value of symmetry can be compensated.

Virtual size of the key is equal to $10 - 0.01 + 0.01 = 10.00$ mm

Virtual size of the slot is equal to $10 + 0.005 - 0.01 = 9.995$ mm

So interference should occur under extreme conditions. By using the above mathematical models, the parts with tolerance information can be obstained. After that, they are assembled together to verify whether interference exists. 1000, 3000, 10000 times of variational assemblies are simulated respectively with Monte Carlo simulation (Xu, 1982) and the result of probability of interference is given in table 1. Fig.7 is a typical case of interference of the variational slot and key.

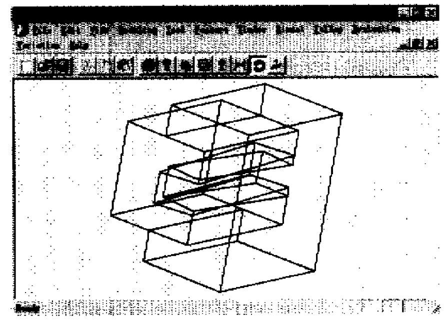

Fig.7 Inference between the slot and key

If the symmetry tolerance is not given at MMC, but at RFS, the principle of independence is satisfied. The probability of interference is also got by Monte Carlo method similarly and the result is given in Table 1.

This example illustrates that tolerance analysis can be conveniently conducted with different tolerance principles and interpretations by using the proposed mathematical model.

CONCLUSIONS

The work for the first time researched deeply and systematically the modeling method of symmetry with the use of different tolerancing princi-

Table 1 The probability of interference between the slot and the key

	Satisfy the requirement of maximum material[2]			Satisfying the principle of independence[2]		
Times	$N = 1000$	$N = 3000$	$N = 10000$	$N = 1000$	$N = 3000$	$N = 10000$
RGI[1]	5.52%	5.64%	5.56%	2.16%	2.43%	2.31%
SI[1]	4.93%	4.99%	5.05%	2.02%	2.28%	2.16%

[1] RGI means resolved geometry interpretation, SI means surface interpretation.

[2] All variables satisfy β-distribution of $\alpha = 2$, $\beta = 2$

ples and different interpretations based on the new mathematical definition. A series of new mathematical models of symmetry were obstained with variations of DOF as model variables representing the semantics of tolerance expressively and unambiguously. The result have very important applications for seamless integration of CAD and CAM.

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