Using chaos to improve measurement precision*

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Abstract: If the measuring signals were input to the chaotic dynamic system as initial parameters, the system outputs might be in steady state, periodic state or chaos state. If the chaotic dynamic system outputs controlled in the periodic states, the periodic numbers would be changed most with the signals. Our novel method is to add chaotic dynamic vibration to the measurement or sensor system. The sensor sensitivity and precision of a measurement system would be improved with this method. Chaotic dynamics measurement algorithms are given and their sensitivity to parameters are analyzed in this paper. The effects of noises on the system are discussed.

Key words: chaotic application, measurements precision, high sensitivity
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INTRODUCTION

For an industry control system, the high precision control depends mostly on sensors sensitivity and precision, so improving the sensors sensitivity and precision is a way to improve the control system precision. Our novel method to do this is to add chaotic dynamic vibration to the measurement or sensor system. As we know, a chaotic system has more than three kinds of steady states (attractors, periods or chaos) according to the initial parameters and is very sensitive to the initial parameters. For example, in the parabola mapping below:

$$X_{n+1} = 1 - A_1 A_2 X_n^2 \quad A_1 A_2 \in [0, 2], \ X \in [-1, 1] \tag{1}$$

Where X is an iterative vector, A_1, A_2 are control parameters. A_1, A_2 influence the parabola mapping outputs, when:

$A_1 A_2 \leqslant 0.75$, X is an attractor, a constant

$0.75 < A_1 A_2 \leqslant 1.40115\cdots$, X is in period state,

$1.401155\cdots < A_1 A_2 < 2\cdots$, X is in chaos state,

In period states, when the initial parameters (called control parameters here, $A_1 A_2$) change a little, the output of parabola mapping vector X may be change much, such as change from one periodic state to another periodic state. With this kind of special chaotic system characteristic in the measurement processing, we input the measuring signals into the chaotic dynamic system as the initial parameters to improve the sensor precision and sensitivity (Fig.1).

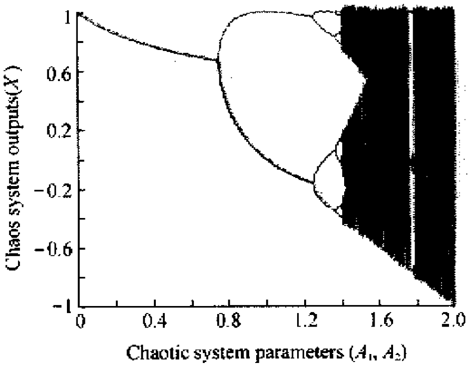

Fig.1 The chaos system states at different initial parameters

According to chaos theory, we can find the steady period when the parameter conditions are follows (Table 1):

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Table 1 The steady period of chaos theory

Period	$(A_1 A_2)$ value	Period	$(A_1 A_2)$ value
2	1	256	1.40114632...
4	1.31070264...	512	1.40115329...
8	1.38154748...	1024	1.40115478...
16	1.39694535...	2048	1.40115510...
32	1.40025308...	4096	1.40115517...
64	1.40096196...	8192	1.40115518...
128	1.40111380...	16384	1.40115518...

We can see in Table 1 that with increasing period, the difference between two $(A_1 A_2)$ values becomes smaller and smaller.

METHOD AND ALGORITHM

Our method is to put the measuring signals into the following chaos system, we designed:

$$X_{n+1} = 1 - AQX_n^2 \qquad (2)$$

Where Q is the value of the signal being measured, If $Q = 0$, an offset should be given to Q. A represents control parameter, used to control the whole chaos system work in some periodic states. X_n is the iterative vector. When the system works, at time t_0, take an initial X_0 and according to the Q_0 value, choose A to make the system outputs be in a periodic state and make the chaotic dynamic system oscillate; at time $t_0 + i\triangle t$ (i is from 1 to a predefined integer n, $\triangle t$ is calculating time of the step), get the Q_i according to Eq.(2), calculate X_i, (where Q includes the noise signal, we will discuss it in Section 4). When Q changes, we use the same system to sense it. we set A at a suitable value, a little change of Q value will change the dynamic systems outputs periods because the chaotic dynamic system Eq.(2) is very sensitive. If we get the periods, we can calculate the Q value. The measurement system to which such chaotic dynamic system is added should be adjusted before it works.

Calculating the periods of the given chaos dynamic mapping is needed before we give the measurement algorithm. We calculate the periods of a chaotic dynamic mapping at different parameters using the following algorithm (We call this algorithm A_1):

1. Initialize X_0, calculating precision, range of A, Q;
2. For the whole range of Q, according to the need for the precision P_1 (such as $P_1 = 10^{-5}$) can be divided into corresponding parts. According to Table 1, some calculations might be decreased.
3. For every Q, calculate X_n;
4. Get the period values, for example, if $n = 1000$, count the different value X_n from 200 to 1000 at some precision (such as $P_2 = 10^{-2}$), the result is periods value.

According the algorithm A_1, we can get the Table 2 ($X_0 = 0$, $P_1 = 10^{-5}$, $P_2 = 10^{-2}$):

Table 2 *AQ* value making the system outputs branch off

Period value	AQ	Period value	AQ
2	0.74707	3	1.24930
4	1.25012	5	1.36805
6	1.36856	7	1.36859
8	1.37153	9	1.39407
10	1.39465	11	1.39472
12	1.39479	13	1.39738
14	1.39812	15	1.39859
16	1.39981	17	1.40049
18	1.40052	19	1.40058
20	1.40063	21	1.40107

Because the chaos system is very sensitive to the parameter conditions, the results in Table 2 differ with different calculation precision. For the sake of the value is calculated before the testing process and the following algorithm is using the same precision, the values in Table 2 do not affect the testing results significantly. This point is dealt with in later discussions. Hao (1993) gave a theoretical method to get the period state values.

After (AQ) values leading to different period are given, in the testing process, the chaotic system can sense the changes of AQ from the system outputs. Because the chaos system is extremely sensitive and the range of (AQ) producing the system outputs is in a certain period and very small, parameter A should be adjusted in the measuring process to suit the (AQ) value. The chaotic measurement system algorithm is given as follows (we call this algorithm A_2):

(1) According to the measurement precision needed, choose three periods. For instance, we choose $(AQ)_1$ = at period 17, $(AQ)_2$ = at period 18, $(AQ)_3$ at period 19; and then set the calculating precision.

(2) Demarcation: Get the reference Q_0 and offset if needed, set the initial precision according to the algorithm A_1; $i = 0$;

(3) Calculate $A_i = (AQ)\,2/Q_i$, (make the chaotic measurement system outputs be in period 18 state with parameter A_i, Q_i);

(4) Calculate Eq.(2) using (A_iQ_i), if the periods value is not 18, increase the A_i precision and go to (3) and calculate A_i again, or else get the signal value Q, after filtering if needed, use A_i and Q to calculate the X according Eq.(2) until its period changes.

(5) If the period change to 17 or less than 17, let $Q_i + 1 = (AQ)1/Ai$; If the period changes to 19 or more than 19, let $Q_{i+1} = (AQ)_3/A_i$

(6) $i = i + 1$; go to (3)

(7) We then get the vector $\boldsymbol{Q_i}$, which is the signal we want to get in the system. In the more precise measurement system, this vector $\boldsymbol{Q_i}$ and $\boldsymbol{A_i}$ are source data to be dealt with later.

Precision analysis

The measure delicacy is decided by the following equation:

$$Q_2 - Q_1 = \frac{(AQ)_2}{A_1} - \frac{(AQ)_1}{A_1} = \frac{(AQ)_2 - (AQ)_1}{A_1} \qquad (3)$$

Relative precision is calculate by the following equation:

$$\frac{Q_2 - Q_1}{Q_1} = \frac{(AQ)_2 - (AQ)_1}{A_1 Q_1} = \frac{(AQ)_2 - (AQ)_1}{(AQ)_1} \qquad (4)$$

If we take the periods 17 and 18 as the system work periods, the ratio of the signal decrease should be:

$$\frac{Q_2 - Q_2}{Q_1} = \frac{(AQ)_2 - (AQ)_1)}{(AQ)_1} \approx \frac{1.40052 - 1.40049}{1.40049} = 0.002\% \qquad (5a)$$

The ratio of signal increase should be:

$$\frac{Q_3 - Q_2}{Q_2} = \frac{(AQ)_3 - (AQ)_2)}{(AQ)_2} \approx \frac{1.40058 - 1.40052}{1.40052} = 0.004\% \qquad (5b)$$

Because (AQ) is the full range of the parameters, the calculation and iterative process limit the system's long-term error.

Analysis of anti-disturbing

If the above algorithm is sensitive to noise, it will be inefficient in the measurement system. The effects of noise on the system are analyzed below theoretically and numerically.

First, we input white noise to the chaos measurement system, and suppose the maximal value of the noise is M. We use a randomizer for simulating noise at $[-0.5, 0.5]$, so the measurement system can be calculated as follows:

$$W_{n+1} = 1 - A(Q + M \times RAND)X_nX_n \qquad (6)$$

We can calculate for $M = 0.01, 0.001, 0.0001, 0.00001$, $A = 2$. The graphs are shown in Fig.2 to Fig.5.

From the figures, we can see that white noise affects the chaos system outputs value (vector $\boldsymbol{X}$), but cannot make the periods change, in the chaotic dynamic system. From Eq.(6), we have:

$$E(X_{n+1}) = 1 - (E(AQX_nX_n)) + E(AM \times RAND \times X_nX_n)) \qquad (7)$$

We write the white noise as $M(t)$, X_n as $X(t)$, so we have:

$$E(AM \times RAND \times X_nX_n)) = A \times R_{MX}(t_1, t_2) \qquad (8)$$

$R_{MX}(t_1, t_2)$ is the relation function of M and X, From Eq.(2), we have:

$$E(X_{n+1}) = 1 - E(AQX_nX_n) \qquad (9)$$

Compared with Eqs.(7) and (9), according to Eq.(8), we know that to make the measurement system unaffected by the noise, the noise should be limited as follows:

$$R_{MX}(t_1, t_2) = 0 \qquad (10)$$

For the measurement system, if Eq.(10) is valid, the system will have immune ability, and the ability to choose the measuring signal. If

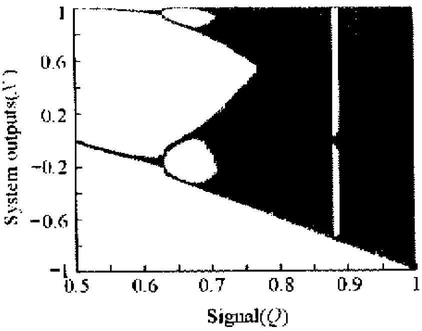

Fig.2 The chaos system outputs at $M = 0.01$ for different Q

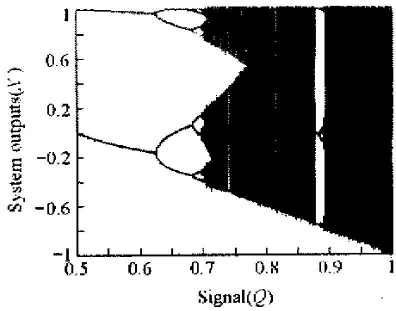

Fig.3 The chaos system outputs at $M = 0.001$ for different Q

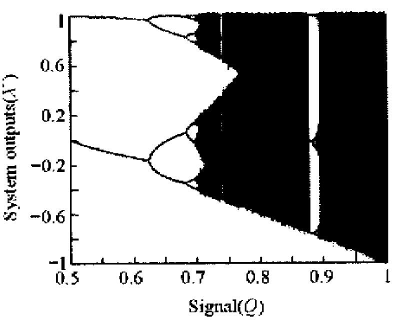

Fig.4 The chaos system outputs at $M = 0.0001$ for different Q

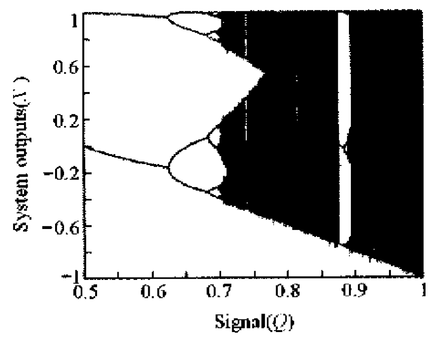

Fig.5 The chaos system outputs at $M = 0.00001$ for different Q

$$R_{MX}(t_1, t_2) \neq 0 \tag{11}$$

The chaotic measurement system will change its output states.

Test using the algorithm

Using the measurement algorithm A_2, for a feeble signal with stronger noise:

$$A(t) = A \times \sin(t) + M \times Rand$$

Where Rand is the random signal of the field $[-0.5, 0.5]$.

When $A = 10^{-3}$, $M = 10^{-2}$, use the A_2 algorithm to test it, the original signal is Fig.6, the result is 1 in Fig.7. Line 2 in Fig.7 is signal A sin(t).

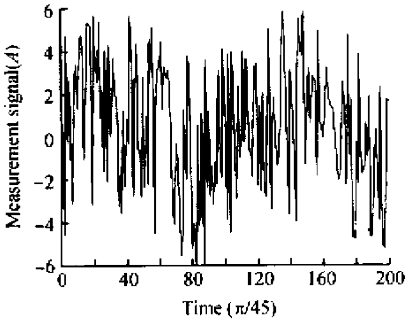

Fig.6 A feeble signal with stronger noise, $A(t)$

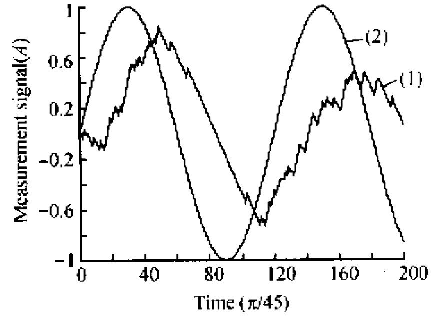

Fig.7 (1) the results of the chaos measurement system and (2) the feeble signal

The measurement error is includes two parts, the first one is the system error in that even with a pure signal (line 2 in Fig.8), and the output of the chaos measurement system is line 1 in Fig.8. The reason is in the choices of the periods as the sense period. Different sets of periods will lead to different test results and will give different system error. The second part is test error, resulting from miscalculation of the intercept, so that the random signal's effects cannot be removed completely.

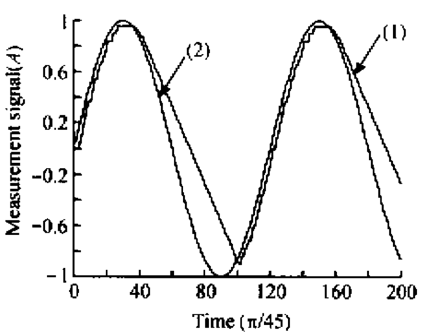

Fig.8 (1) the chaos measurement system output and (2) the pure signal 0.001 sin(t)

DISCUSSION

1. How to improve the precision

Because the chaotic measurement dynamic system is very sensitive to the parameters, if they are adjusted to make the system to work at the period states, little changes of the parameters may change the periods of the dynamic system, which is very sensitive, especially for measurement of feeble signals.

2. How to immunize the system from being adversely affect by stronger noise

When a noise signal was input into the chaos measurement dynamic system, after iterative calculation, the dynamic system outputs were not sensitive to the noise signal. In other words, the noise signal did not make the chaotic dynamic system change from one period state to another period state. To a measuring signal, the chaotic measurement system chooses the signal frequency. A practical system is very sensitive with some frequency according to Eq. (11). The measuring signal does not change rapidly at every step of the algorithm A_2.

3. The effect of parameters on the chaos measurement system

Because the chaos dynamic system is very sensitive to the parameters, the accuracy of parameters, the calculation precision and choices of work periods play important roles in the system.

CONCLUSIONS

Chaos dynamics is a novel theory, which can improve the measurement precision and sensor sensitivity. The measurement system is very sensitive to the input signal. If we choose suitable parameters for the system, it will work very well (He, 2000). It is a promising field for signal processing, although there is much work to do in this field. Our future work is to analyze the influences of every parameter and to design the chaos circuit.

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