Application of stochastic method to optimum design of energy-efficient induction motors with a target of LCC*

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Received Jan.18，2002；revision accepted June 6，2002

Abstract: For an energy-efficient induction machine，the life-cycle cost（LCC）usually is the most important index to the consumer. With this target，the optimization design of a motor is a complex nonlinear problem with constraints. To solve the problem，the authors introduce a united random algorithm. At first，the problem is divided into two parts，the optimal rotor slots and the optimization of other dimensions. Before optimizing the rotor slots with genetic algorithm（GA），the second part is solved with TABU algorithm to simplify the problem. The numerical results showed that this method is better than the method using a traditional algorithm.

Key words: Induction motor，Global optimization，Life-cycle cost，Energy-efficient，Genetic algorithm，TABU algorithm

Document: A **CLC number:** TM343.2

INTRODUCTION

Most countries have implemented new policies on the energy saving and environment protection. American President Bush signed "The Comprehensive National Energy Policy Act" in 1992 stating that no general-purpose，three-phase induction motors will be allowed into United States except energy-efficient motors. As a result，almost all motor manufacturers produced "High-efficiency" or "Super High-efficiency" motors.

One of the most important indexes to users is the life-cycle cost（LCC）of an energy-efficient induction motor.

The optimization design of squirrel-cage induction motors is a complex nonlinear problem with constraints in the following aspects：（1）there are too many variables involved in the problem，and the many variables are coupled with each other，so that the state spaces of the variables are very huge，and their state combinations exceed 10^{24}（Madescu *et al*.，1998），so it is difficult to ransack the spaces by small steps.（2）The problem has strong nonconvexity；and the property of the function will deteriorate when the rotor slots dimensions are optimized，etc. Therefore，it is apt that achievement of a local optimized solution will result in loss of the global one when parameters are optimized with a deterministic algorithm.

With the development of the modern mathematics，many scholars introduced the random approach into optimization algorithm and advanced some new and powerful algorithmic models for nonlinear problems. Among them，the genetic algorithm is especially applicable for solving the global optimization of a complex function. However，the genetic algorithm also has some defects：the amount of calculation is very huge；the convergence speed is slow，and so on. To improve the algorithm mentioned above，the authors firstly break down the problem into some small steps；then optimizes the convex variables with TABU algorithm that has high convergence speed to reduce the dimension of the problem. There-

* Project（No.601299）supported by the Science Foundation of Hebei Province，China.

fore the amount of calculation will decrease enormously and the requirements of practical designs are met.

FUNDAMENTAL ALGORITHM

The global optimization of electrical machines can be by the following nonlinear programming problem:

Problem 1: $\max f(x) \mid g_i(x) \geqslant 0 \ (i = 1, 2, \cdots, m)$

where, $x \in X_n$ the optimized variable in the field of definitions X_n;

$f(x)$, the objective function;

$g_i(x) \geqslant 0$ the constraint conditions.

In this paper, $f(x)$ is the life-cycle cost (LCC) as described by Hamer *et al*. (1997):

$$LCC = PP + EF(KW) \quad (1)$$

where, *PP*, motor purchase (RMB ¥);

EF, evaluation factor (¥/kW);

KW, power loss (kW).

$$EF = C(N)(PWF) \quad (2)$$

$$KW = hp \times 0.746(1/\eta - 1) \quad (3)$$

where, *C*, power cost (¥/kW·h);

N, operating time each year (h/year);

PFW, cumulative present worth factor (year).

It can be made as sophisticated as you want to make it, to account for the time value of money, tax deductions, depreciation, etc. For most economic comparisons, the assumption that a three-year payback for energy efficiency is acceptable; setting it equal to four is an adequate simplification.

hp, motor nameplate horsepower (hp);

η, motor efficiency;

To determine the role of the efficiency in the LCC, take an example of a 25 hp, 4 poles, energy-efficient motor. The data are as follows:

PP of ¥ 2650;

C of ¥ 0.45/(kW·h);

N of 7000h/year;

PFW of 4 year;

η of 93%.

By Eqs(1) – (3),

$EF = 0.45 \times 7000 \times 4 = 12600$

$KW = 25 \times 0.746 \times (100/93 - 1) = 1.404$

$EF(\mathrm{KW}) = 12600 \times 1.404 = 17687 >> 2650$

This shows that for a long operate-time motor, the LCC is determined by the efficiency mainly. The higher the η, the lower is the LCC, so that the optimization of motor with a $f(x)$ of LCC is equivalent to that of $\eta(x)$ approximately.

In this paper, the constraint conditions include: (1) the rotor-locked torque of the motor T_{st} and the maximum torque T_m are not less than the standard value, T_{st}' and T_m'; (2) the rotor-locked current I_{st} is not greater than the standard value I_{st}'; (3) the length of core L is not longer than L_{max}; (4) the current density J, the magnetic flux density B and the thermal load index AJ are not greater than the threshold values J_{max}, B_{max} and AJ_{max} separately.

In practical designs, to ensure successful manufacturability of the designs, reductions in electromagnetic noise, stray losses, and harmonic torque, some values of parameters of motors, such as the slot combination, air gap length, diameter of core, winding pitch, and rotor end-ring dimension are usually prescribed in advance. Consequently, in this paper, the optimized variable x is mainly composed of the following parameters: (1) The length of core L; (2) Turns of the windings n and the winding-wire cross section s or its gauge d; and (3) The dimensions of the slots (the dimension of stator slot and four types of rotor slots commonly used are shown in Fig.1).

In the above three groups of variables, the optimization of the dimensions of rotor-slots is usually considered the most difficult and complex. This is because: (1) T_{st} and I_{st} are usually the main constraint conditions that influence the improvement of efficiency of induction motors and, T_{st} and I_{st} are usually obtained through the optimization design of rotor slots in practical projects. (2) To obtain the highest efficiency under the constraint conditions of rotor-locked performance, in modern squirrel-cage induction motors, various kinds of special-shaped rotor slots, such as convex-slot (No.3 and No.4 of Fig.1), are adopted. Due to the saturation and skin effect, the function of the rotor-locked performance vs. parameters of rotor slots is extremely complex; is usually strongly nonconvex and nonlinear, even discontinuous somewhere. At the same time, the dimensions of slots are very closely coupled to each other.

In the derivation of the optimization design of the discontinuous, anomalous and nonconvex function like this, the genetic algorithm is widely applied.

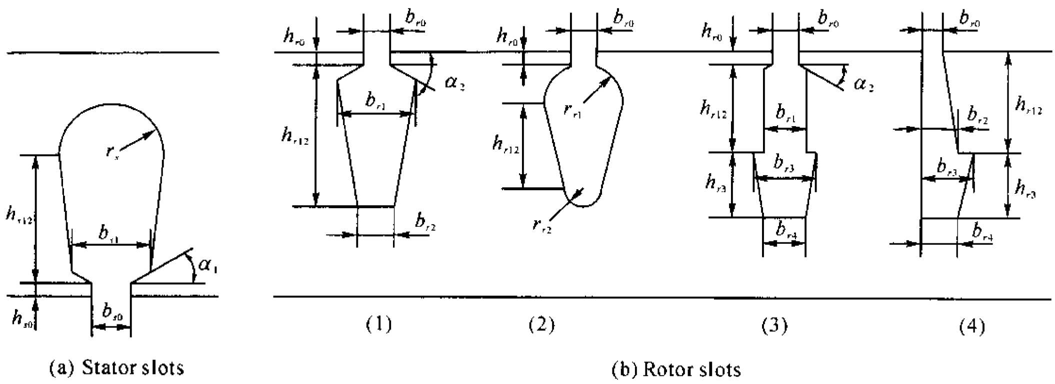

Fig.1 Dimension of stator slot (a) and four types of rotor slots commonly used (b)

Genetic Algorithm

The genetic algorithm was derived from the simulative tests in which some biologists simulated the biological evolution process with computer. In 1960, Professor Holland from Michigan University first-used it in practical optimization problems. It is an algorithm that simulates the natural rule of survival of the fittest and genetic evolution process with coding technology of variables, and is accomplished through message of adaptive values. The detailed contents of the algorithm follows (Ma *et al*., 1998; Liu *et al*., 1995; Goldberg, 1988):

1. Select the parameters of algorithm: the quantity of all individuals N, the probability of crossover P_c, the probability of mutation P_m, and the generation gap G.

2. Start the populations, compute the flexibility F_i, and decide the number of overlapped individuals between two generations N_{overlap}.

3. Propagate this generation by Reproduction, Crossover and Mutation.

4. Form the next generation by N_{overlap} of best individuals selected from this generation and ($N-N_{\text{overlap}}$) of new individuals produced from step 3.

5. Judge the results, if it reaches the scheduled generations of propagating, go to step 6; otherwise go to step 3.

6. Output the present best individuals.

From the above statements, it can be seen that the genetic algorithm does not rely on the mathematical models of problems, therefore it is particularly suitable for solving the complex problem with extremely bad function-property. But its defect is that it always converges slowly (especially when its mutation probability P_{m} is big). The length of the binary code needed can be estimated as follows: Assume that the definition domain of all dimensions is $[a_i, b_i]$, the step is h, and the number of variables for coding is m, then the length of the code will be:

$$N = \sum_{i=1}^{m} \text{INT}\left[\log_2\left(\frac{b_i - a_i}{h} + 1\right)\right] \quad (4)$$

To shorten the length of codes can accelerate convergence enormously, so we first reduce the dimension of the problem with a TABU algorithm before the practical utilization of the genetic algorithm.

TABU Algorithm

TABU algorithm is a kind of heuristic random searching algorithm (Glover, 1989) whose convergence speed is faster than that of other random algorithms commonly used. An improved TABU algorithm is shown as follows (Yang *et al*., 1998; Hu *et al*., 1990; Bland *et al*., 1991):

1. Initialization: to select x arbitrarily; set $f_{\text{opt}} = f(x)$, to calculate the step vector h_i, set $K = 0$.

2. Set $j = 1$.

3. Produce a new feasible state along the coordinate direction j in every neighborhood $N(x, h_i)(i = 1, 2, \cdots, r)$ of the present state x at random; compute the value of the objective function, and set the x^* to be the state whose value

of the objective function is maximum.

4. If $f(x^*) > f_{opt}$, set $x_{opt} = x^*$, $f_{opt} = f(x^*)$.

5. If $K > K_{hold}$, go to step 9; otherwise set $K = K + 1$.

6. If the searching for N coordinate directions does not finish, set $j = j + 1$, $x = x^*$, then go to step 3;

7. If the end condition is satisfied, go to step 9;

8. If the continuous circulation reaches N_{start} cycles, set $x = x_{opt}$; then go to step 2.

9) Output the optimal solution.

From the above explanation, the algorithm is different from normal brachistochrone methods; its transition of state possesses is a "climbing" ability; which means that in this algorithm it is allowed to return to the previous states, and then reach the objective point through a different direction. Therefore, it is possible to find the global optimum from the local extreme points. So that, this algorithm is also useful for solving the optimization design of continuous function with multi-extreme points, despite its being saddled with some nonconvex "noises".

SOLUTION OF GLOBAL OPTIMIZATION

1 Simplification of problem

First, set the objective function to efficiently divide Problem 1 into the following two problems:

Problem 2: max $\eta(x) \mid T_m - T_m' \geqslant 0$, $J_{max} - J \geqslant 0$, $AJ_{max} - AJ \geqslant 0$ $(x \in X_n)$

Problem 3: max $\eta(x) \mid y = y^*$, $T_{st} - T_{st}' \geqslant 0$, $I_{st}' - I_{st} \geqslant 0$

In Problem 3, $x \in X_m$ is the dimension of rotor slots; $y = [L, n, d, b_{so}, b_{s1}, r_s, h_{so}, h_{s12}]^T \in X_{n-m}$, $y^* = [L^*, n^*, d^*, b_{so}^*, b_{s1}^*, r_s^*, h_{so}^*, h_{s12}^*]^T \in X_{n-m}$ is the solution of Problem 2, and $x^* \in X_n$ is a vector composed of relevant parameters.

Second, assume the following:

(1) Conditional optimization design of Problem 3 equals to Problem 1. That is to say, the dimensions of rotor slots are independent of other optimized variables.

(2) The solution of optimization problem without starting constraints can be obtained from the design in which rotors adopt the type 1 slots (see Fig.1) and the teeth of stator and rotor are all parallel.

A great quantity of checking computations and experiential data confirms that the above assumption is feasible in practical projects.

Dimensions of the slot opening b_{s0}, h_{s0} and b_{r0}, h_{s1}, and the slot-filling factor (the total conductor cross-section per effective stator-slot area) are given according to experiences and the demands of technology. Then, with a linear predigest, the objective function of Problem 2 $\eta(x)$ can be simplified as the following equation:

$$\eta(x) = \eta(n, b_{T1}, b_{T2}, h_{s12}, h_{r12}) \tag{5}$$

where b_{T1} and b_{T2} are the teeth-width of stator and rotor.

Third, introduce a penalty function to turn the objective function into an augmented objective function as follow:

$$f'(x) = f(x) + \sum_{i=1}^{m} a_i \varphi_i(x) \tag{6}$$

$$\varphi_i(x) = \begin{cases} 0 & (g_i(x) \geqslant 0) \\ g_i(x) & (g_i(x) < 0) \end{cases} \tag{7}$$

where, a_i is penalty factor that is normally set up to be a very large positive number;

Then the optimization design of Problem 2 and 3 with constraints are transformed into equivalent optimization designs of Problem 2′ and 3′ without constraints.

Fourth, to determinate the adaptive function of Problem 3′, take an example of a type of slot with complex structure, type 3. According to experiences, schedule the dimensions of slot opening, b_{s0}, h_{s0} and b_{r0}, h_{s1}, and assume the teeth parallel, thus there will be the following relation:

$$b_{r3} = b_{r4} + 2\pi h_{r3} / Q_2 \tag{8}$$

where, Q_2 is the number of rotor slots.

Now, assume the value of h_{r12}, h_{r3} and b_{r4} in their fields of definitions at random. If the solution of η (h_{r12}, h_{r3}, b_{r4}, b_{r1}) exists, it can be proved that for variable b_{r1}, the augmented objective function of Problem 3 $f_3'(b_{r1})$ is a one-dimensional upper unimodal function in the field of definitions. With the 0.618 Algorithm (Cheng, 1993), it is easy to obtain the solution of the equation: $F(h_{r12}, h_{r3}, b_{r4}) = \max f_3'(b_{r1})$. For type 3 slots, it is proposed that the nonnegative function $F'(h_{r12}, h_{r3}, b_{r4}) = \max\{F(h_{r12}, h_{r3}, b_{r4}), 0\}$ is the adaptive function of Problem 3′. Then the independent variables in the genetic algorithm will be reduced to three (whose feasible states of solution space are smaller than 10^7).

2 Procedure

First, solve Problem 2′. Because the property of function is better, the optimization design adopts TABU algorithm. To ensure the multiformity of solution, we define the length of core L to be the major variable. Then we make L to be accumulated from $L_{\min}$ to $L_{\max}$ by a step of h_L and optimize other variables for every L. After comparing and sorting the solutions with different sizes of length of core, to select a certain quantity of optimized solutions for the next optimization according to the demands of designs.

Second, solve Problem 3′ with a genetic algorithm. (1) Code the variables and define the domain for searching. (2) Start the populations, solve $F_i'(x)$ and optimize it. To ensure the stability of the algorithm, we introduce a "big mutation operation" (Ma, 1998) into the algorithm.

Third, to make the solution accurate, we optimize all the variables using TABU algorithm after the second optimization. Sequentially we obtain a matrix of solution for different combinations of slot type and core-length.

Finally, the difference of the LCC of all the solutions of the matrix are calculated and designers can select the final plan by judging the LCC and the manufacturability.

It should be noted that the solution obtained by the optimization design always concentrates on the edge of constraint conditions. The discretization of the dimensions of parts easily causes productions problems. To make sure there is a reliable margin of technology, we still verified the rotor-locked property with a probabilistic method for the selected final plan.

3 Example

With this algorithm, we designed a series of energy-efficient induction motors that comply with NEMA standards (NEMA, 1993). Take an example of a 25hp, four poles, 60Hz motors. The results of optimization are shown in Table 1 as follows:

Table 1 Comparison of different optimization algorithms

No.	Algorthms	Results of Optimization	
		η(%)	LCC(¥)
1	Hook-Jeeves Algorithm	92.7	21182
2	TABU Algorithm	92.9	20648
3	Algorithm used in this paper	93.4	19261

From the above results, it can be concluded that the solution obtained by the algorithm used in this paper is obviously better than that obtained by algorithm 1 and algorithm 2. At the same time, we also tried to optimize all variables with a genetic algorithm (Algorithm 4). However, because the coding was too long and it was difficult to determine the feasible region of search of optimized variables, the variables often exceeded the feasible value region, so that the time of iterative computation for analyzing programs is lengthened and the convergence speed of Algorithm 4 was extremely slow, and it was hard to get the solution similar to that of algorithm 3 even using ten times the operation time of algorithm 3.

CONCLUSIONS

The LCC is the most important index of energy-efficient induction motors. But the optimization of motor parameters aimed at achieving minimum LCC is very difficult. A new combined random method for the global optimization of complex function is introduced in this paper.

The genetic algorithm is preponderantly used in the global optimization design of complex function. However, because its convergence speed is too slow, it can hardly meet the demands of engineering designers.

To balance the operation time and the quality of optimization, in this paper, we divide the problem into two lower-dimension problems and then simplify them. The optimization is achieved with a stochastic method that is a combination of a genetic algorithm and a TABU algorithm. This method takes full advantage of the high convergence speed of the TABU algorithm without losing the ergodicity of the genetic algorithm; thereby shortening the operation time by 90% compared with the genetic algorithm. The result is satisfactory and can meet the demands for the design of energy-efficient induction machines.

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