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Axisymmetric fundamental solutions for a finite layer with impeded boundaries*

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Abstract: Axisymmetric fundamental solutions that are applied in the consolidation calculations of a finite clay layer with impeded boundaries were derived. Laplace and Hankel integral transforms were utilized with respect to time and radial coordinates, respectively in the analysis. The derivation of fundamental solutions considers two boundary-value problems involving unit point loading and ring loading in the vertical. The solutions are extended to circular distributed and strip distributed normal load. The computation and analysis of settlements, vertical total stress and excess pore pressure in the consolidation layer subject to circular loading are presented.

Key words: Consolidation, Integral transform, Finite layer, Impeded boundaries
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INTRODUCTION

Biot's consolidation theory was proposed in 1941 from continuous medium fundamental equations. The theory taking into account the coupling between the solid strains and pore pressure dissipation is called "coupled consolidation theory". The solution of governing equations of a porous solid is more complicated than those involving ideal elastic solids. McNamee *et al*. (1960a) presented a solution for plane and axially symmetric problems in terms of two displacement functions. The general solution for displacement functions is obtained through the application of Laplace and Fourier integral transforms for plane problems and Laplace and Hankel integral transforms for axisymmetric problems. Gibson *et al*. (1970) obtained a solution for plane strain and axially symmetric consolidation of a clay layer on a smooth impervious base. Puswewala *et al*. (1988) presented axisymmetric fundamental solutions for a completely saturated porous elastic solid. Gu *et al*. (1992) presented a solution for an axisymmetric vertical loaded multi-layer base. Huang *et al*. (1996) also obtained an analytical solution for finite layer through expanding the field of solution.

All the above investigations assumed that the consolidation layer surface is fully permeable or impermeable, but that the surface might be affected by geotechnical engineering impediments such as sand cushion in preloading, replacement layer of embankment impeding permeation of the consolidation layer. The impeded layer can be simplified impeded boundaries of the sub-consolidation layer. Xie (1996) studied one dimensional consolidation of layered soils with impeded boundaries. However results for two-dimensional problem with impeded boundaries have not been reported.

This paper deals with the derivation of axisymmetric fundamental solutions that are applied in the consolidation calculations of a finite layer with impeded boundaries. The derivation of fundamental solutions considers two boundary-value problems involving unit point loading and ring loading in the vertical direction. The solutions are extended to circular loading and strip loading. The calculation and analysis of settlements, vertical total stress and excess pore pressure in

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the consolidation layer subject to circular loading are presented.

GOVERNING EQUATIONS AND GENERAL SOLUTION

Following Biot (1941), the displacement equations for axisymmetric deformations of a completely saturated isotropic and homogeneous poroelastic solid in a cylindrical coordinate system are expressed in the following:

$$\left(\nabla^2 - \frac{1}{r^2}\right)u_r + (2\eta - 1)\frac{\partial e}{\partial r} + \frac{1}{G}\frac{\partial p_f}{\partial r} = 0 \tag{1}$$

$$\nabla^2 u_r + (2\eta - 1)\frac{\partial e}{\partial z} + \frac{1}{G}\frac{\partial p_f}{\partial z} = 0 \tag{2}$$

$$c\nabla^2 e = \frac{\partial e}{\partial t} \tag{3}$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

$$e = \frac{1}{r}\left[\frac{\partial}{\partial r}(ru_r) + r\frac{\partial u_z}{\partial z}\right]$$

$$\eta = (1 - \nu)/(1 - 2\nu)$$

$$c = 2G\eta k$$

In the above equations, $u_r(r, z, t)$ and $u_z(r, z, t)$ denote displacements in r and z directions, respectively; $p_f(r, z, t)$ is the excess pore pressure; G and ν denote shear modulus and Poisson's ratio of the bulk material; k is the coefficient of permeability of the medium; e is the dilatation of the bulk material.

The stress-strain relations are expressed as

$$\frac{\sigma_{rr}}{2G} = \varepsilon_{rr} + (\eta - 1)e + \frac{p_f}{2G} \tag{4}$$

$$\frac{\sigma_{\theta\theta}}{2G} = \varepsilon_{\theta\theta} + (\eta - 1)e + \frac{p_f}{2G} \tag{5}$$

$$\frac{\sigma_{zz}}{2G} = \varepsilon_{zz} + (\eta - 1)e + \frac{p_f}{2G} \tag{6}$$

$$\frac{\sigma_{zr}}{2G} = \varepsilon_{zr} \tag{7}$$

Where $\sigma_{rr}(r, z, t)$, $\sigma_{\theta\theta}(r, z, t)$, $\sigma_{zz}(r, z, t)$ and $\sigma_{zr}(r, z, t)$ are non-zero stress components of the bulk material; $\varepsilon_{rr}(r, z, t)$, $\varepsilon_{\theta\theta}(r, z, t)$, $\varepsilon_{zz}(r, z, t)$ and $\varepsilon_{zr}(r, z, t)$ are bulk strain components.

Following McNamee *et al*. (1960b), displacements $u_r(r, z, t)$ and $u_z(r, z, t)$ and pore pressure $p_f(r, z, t)$ are expressed in terms of two functions ϕ and ψ in the following form:

$$u_r = -\frac{\partial \psi}{\partial r} + z\frac{\partial \phi}{\partial r} \tag{8}$$

$$u_z = -\frac{\partial \psi}{\partial r} + z\frac{\partial \phi}{\partial r} - \phi \tag{9}$$

$$p_f = -2G\left(\frac{\partial \phi}{\partial z} - \eta\nabla^2\psi\right) \tag{10}$$

It is convenient to dedimensionalize all quantities with respect to length and time by selecting a certain length "R" as unity, and "R^2/c" as a unit of time, respectively. The substitution of Eqs. (8) – (10) in Eqs. (1) – (3) yield the following governing equations for functions ϕ and ψ:

$$\nabla^4\psi = \nabla^2\frac{\partial \psi}{\partial t} \tag{11}$$

$$\nabla^2\phi = 0 \tag{12}$$

Laplace and Hankel transforms of Eqs. (11) and (12) result in the following two ordinary differential equations. Define ϕ_{hl} and ψ_{hl} as zero-order Hankel transform and Laplace transform of functions ϕ and ψ, respectively.

$$\left(\frac{\partial^2}{\partial z^2} - \zeta^2 - p\right)\left(\frac{\partial^2}{\partial z^2} - \zeta^2\right)\phi_{hl} = 0 \tag{13}$$

$$\left(\frac{\partial^2}{\partial z^2} - \zeta^2\right)\psi_{hl} = 0 \tag{14}$$

where p is Laplace transform parameter; ζ is Hankel transform parameter; $\gamma = \sqrt{\zeta^2 + p}$.

Solution of the above two equations are

$$\psi_{hl} = A(\zeta, p)e^{-z\zeta} + B(\zeta, p)e^{-z\gamma} + C(\zeta, p)e^{z\zeta} + D(\zeta, p)e^{z\gamma} \tag{15}$$

$$\phi_{hl} = E(\zeta)e^{-z\zeta} + F(\zeta)e^{z\zeta} \tag{16}$$

Based on Eqs. (4) – (16), general solutions for displacements, excess pore pressure and stresses for axisymmetric deformations of a completely saturated porous elastic solid can be written as follows:

$$\bar{u}_z = \int_0^\infty \zeta J_0(r\zeta)[\zeta A e^{-z\zeta} + \gamma B e^{-z\gamma} - \zeta C e^{z\zeta} -$$

$$\gamma D e^{z\gamma} - (z\zeta + 1) E e^{-z\zeta} + (z\zeta - 1) F e^{z\zeta}] d\zeta \tag{17}$$

$$\bar{u}_r = \int_0^\infty \zeta^2 J_1(r\zeta)[A e^{-z\zeta} + B e^{-z\gamma} + C e^{z\zeta} + D e^{z\gamma} - zE e^{-z\zeta} - zF e^{z\zeta}] d\zeta \tag{18}$$

$$\bar{p}_f = 2G\int_0^\infty \zeta J_0(r\zeta)[\eta p B e^{-z\gamma} + \eta p D e^{z\gamma} + \zeta E e^{-z\zeta} - \zeta F e^{z\zeta}] d\zeta \tag{19}$$

$$\bar{\sigma}_{zz} = 2G\int_0^\infty \zeta J_0(r\zeta)[-\zeta A e^{-z\zeta} - \zeta B e^{-z\gamma} - \zeta C e^{z\zeta} - \zeta D e^{z\gamma} + (z\zeta + 1) E e^{-z\zeta} + (z\zeta - 1) F e^{z\zeta}] d\zeta \tag{20}$$

$$\bar{\sigma}_{zr} = 2G\int_0^\infty \zeta^2 J_1(r\zeta)[-\zeta A e^{-z\zeta} - \gamma B e^{-z\gamma} + \zeta C e^{z\gamma} + \gamma D e^{z\gamma} + z\zeta E e^{-z\zeta} - z\zeta F e^{z\zeta}] d\zeta \tag{21}$$

where the superposed bar denotes the Laplace transform of the relevant quantity.

Combining the above general solutions with different boundary conditions, fundamental solutions can be obtained.

FUNDAMENTAL SOLUTIONS

1. Fundamental solutions subject to point loading

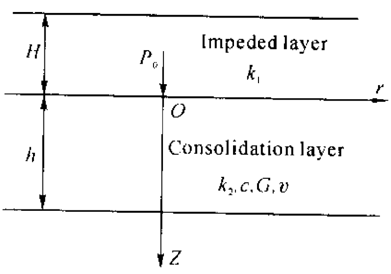

Fig.1 Axisymmetric point loading

In Fig.1, H denotes the thickness of the impeded layer; h is the thickness of the consolidation layer; k_1 denotes the coefficient of permeability of the impeded layer; k_2 is the coefficient of permeability of the consolidation. It is assumed that permeation only takes place vertically in the impeded layer not considering the vertical deformation of the impeded layer and shearing stress over the plane ($z = 0$). Following Terzaghi one dimensional consolidation theory, the boundary conditions for the surface of the consolidation layer are derived in the following forms.

$$k_2 \frac{\partial p_f}{\partial z} = k_1 \frac{p_f}{H} \quad (z = 0) \tag{22}$$

$$\sigma_{rz} = 0 \quad (z = 0) \tag{23}$$

$$\sigma_{zz} = p_0 \delta(r)/(2\pi r) \quad (z = 0) \tag{24}$$

In the meantime, it is assumed that the base of the consolidation layer rests on a smooth, rigid and impervious medium, so that, over this plane,

$$\sigma_{rz} = 0 \quad (z = h) \tag{25}$$

$$\frac{\partial p_f}{\partial z} = 0 \quad (z = h) \tag{26}$$

$$u_{zz} = 0 \quad (z = h) \tag{27}$$

By Laplace transform and the zero order Hankel transform, the boundary conditions are as follows:

$$k_2 \frac{\partial \bar{p}_f}{\partial z} = k_1 \frac{\bar{p}_f}{H} \quad (z = 0) \tag{28}$$

$$\bar{\sigma}_{rz} = 0 \quad (z = 0) \tag{29}$$

$$\bar{\sigma}_{zz} = p_0/(2\pi p) \quad (z = 0) \tag{30}$$

$$\bar{\sigma}_{rz} = 0 \quad (z = h) \tag{31}$$

$$\frac{\partial \bar{p}_f}{\partial z} = 0 \quad (z = h) \tag{32}$$

$$\bar{u}_{zz} = 0 \quad (z = h) \tag{33}$$

Base on Eqs. (17) – (33), the fundamental solutions in the Laplace domain can be obtained in the following form.

(1) Displacement in the radial direction:

$$\bar{u}_r = -\frac{p_0}{16Gp\pi}\cdot \int_0^\infty \zeta J_1(r\zeta) e^{-z(\gamma+\zeta)-h(\gamma+2\zeta)} (\zeta e^{z\zeta}(-2e^{2h\zeta}Hk_2\zeta + \alpha_1 + e^{4h\zeta}\alpha_2)(e^{2z\gamma} + e^{2h\gamma}) - e^{z\gamma+2h\zeta}(\gamma\alpha_1 + \alpha_3\beta_1) + e^{z\gamma+2(h+z)\zeta}(\gamma\alpha_2 - \alpha_3\beta_1) + e^{z\gamma+2h(\gamma+\zeta)}(\gamma\alpha_1 + \alpha_4\beta_1) + e^{2h(\gamma+\zeta)+z(\gamma+2\zeta)}(-\gamma\alpha_2 + \alpha_4\beta_1) + e^{2h\gamma+z\gamma+4h\zeta}(-\gamma\alpha_2 + pz\eta\alpha_4) + e^{z\gamma+4h\zeta}(\gamma\alpha_2 - pz\eta\alpha_3) - e^{z(\gamma+2\zeta)}(\gamma\alpha_1 + pz\eta\alpha_3) + e^{2h\gamma+z\gamma+2z\zeta}\cdot(\gamma\alpha_1 + pz\eta\alpha_4))/(\cosh(h\gamma)(-Hk_2\zeta^3 + 2hk_1p\zeta\eta + Hk_2\zeta^3\cosh(2h\zeta) + k_1(\zeta^2 + p\eta)\cdot\sinh(2h\zeta)) + \gamma\sinh(h\gamma)(-2k_1\zeta\cosh(h\zeta)^2 + Hk_2(2hp\zeta\eta + (-\zeta^2 + p\eta)\sinh(2h\zeta))))d\zeta \tag{34}$$

2) Displacement in the vertical direction:

$$\bar{u}_z = \frac{p_0}{4Gp\pi}\int_0^\infty J_0(r\zeta)\, e^{-z(\gamma+\zeta)}(\gamma\zeta e^{z\zeta}(e^{2z\gamma} - e^{2h\gamma})(2Hk_2\zeta e^{2h\zeta\zeta} - \alpha_1 - \alpha_2 e^{4h\zeta}) + e^{z\gamma+2h\zeta}\cdot(-\gamma\zeta\alpha_1 + p\eta\alpha_3 - \alpha_3\beta_1\zeta) + e^{z\gamma+2h(\gamma+\zeta)}(\gamma\zeta\alpha_1 - p\eta\alpha_4 + \alpha_4\beta_1\zeta) - e^{z\gamma+2(h+z)\zeta}(\gamma\zeta\alpha_2 - p\eta\alpha_3 - \alpha_3\beta_1\zeta) + e^{2h(\gamma+\zeta)+z(\gamma+2\zeta)}(\gamma\zeta\alpha_2 - p\eta\alpha_4 - \alpha_4\beta_1\zeta) + e^{z(\gamma+2\zeta)}(\gamma\zeta\alpha_1 + p\alpha_3(-1+z\zeta)\eta) + e^{2h\gamma+z\gamma+2z\zeta}(-\gamma\zeta\alpha_1 - p\alpha_4(-1+z\zeta)\eta) + e^{z\gamma+4h\zeta}(\gamma\zeta\alpha_2 - p\alpha_3(1+z\zeta)\eta) + e^{2h\gamma+z\gamma+4h\zeta}\cdot(-\gamma\zeta\alpha_2 + p\alpha_4(1+z\zeta)\eta))/(Hk_2((1+e^{2h\gamma})\cdot(-1+e^{2h\zeta})^2\zeta^3 + 4e^{h(\gamma+2\zeta)}\gamma\sinh(h\gamma)(2hp\zeta\eta + (-\zeta^2+p\eta)\sinh(2h\zeta))) + k_1((1-e^{2h\gamma})(1+e^{2h\zeta})^2\gamma\zeta + 4e^{h(\gamma+2\zeta)}\cosh(h\gamma)(2hp\zeta\eta + (\zeta^2+p\eta)\sinh(2h\zeta))))d\zeta \qquad (35)$$

3) Vertical total compressive stress:

$$\bar{\sigma}_{zz} = \frac{p_0}{2p\pi}\int_0^\infty \zeta J_0(r\zeta)\, e^{-z(\gamma+\zeta)}(\zeta^2 e^{z\zeta}(e^{2z\gamma} + e^{2h\gamma})(-2Hk_2\zeta e^{2h\zeta} + \alpha_1 + \alpha_2 e^{4h\zeta}) - e^{z\gamma+2h\zeta}\cdot(\gamma\zeta\alpha_1 - p\eta\alpha_3 + \alpha_3\beta_1\zeta) + e^{z\gamma+2h(\gamma+\zeta)}(\gamma\zeta\alpha_1 - p\eta\alpha_4 + \alpha_4\beta_1\zeta) + e^{z\gamma+2(h+z)\zeta}(\gamma\zeta\alpha_2 - p\eta\alpha_3 - \alpha_3\beta_1\zeta) + e^{2h(\gamma+\zeta)+z(\gamma+2\zeta)}(-\gamma\zeta\alpha_2 + p\eta\alpha_4 + \alpha_4\beta_1\zeta) - e^{z(\gamma+2\zeta)}(\gamma\zeta\alpha_1 + p\alpha_3(-1+z\zeta)\eta) + e^{2h\gamma+z\gamma+2z\zeta}(\gamma\zeta\alpha_1 + p\alpha_4(-1+z\zeta)\eta) + e^{z\gamma+4h\zeta}\cdot(\gamma\zeta\alpha_2 - p\alpha_3(1+z\zeta)\eta) + e^{2h\gamma+z\gamma+4h\zeta}(-\gamma\zeta\alpha_2 + p\alpha_4(1+z\zeta)\eta))/(Hk_2((1+e^{2h\gamma})(-1+e^{2h\zeta})^2\zeta^3 + 4e^{h(\gamma+2\zeta)}\gamma\sinh(h\gamma)(2hp\zeta\eta + (-\zeta^2+p\eta)\sinh(2h\zeta))) + k_1((1-e^{2h\gamma})(1+e^{2h\zeta})^2\gamma\zeta + 4e^{h(\gamma+2\zeta)}\cosh(h\gamma)(2hp\zeta\eta + (\zeta^2+p\eta)\sinh(2h\zeta))))d\zeta \qquad (36)$$

4) Shearing stress:

$$\bar{\sigma}_{rz} = -\frac{p_0}{8p\pi}\int_0^\infty \zeta^2 J_1(r\zeta)\, e^{-z(\gamma+\zeta)-h(\gamma+2\zeta)}\cdot(\gamma e^{z\zeta}(e^{2z\gamma} - e^{2h\gamma})(-2Hk_2\zeta e^{2h\zeta} + \alpha_1 + \alpha_2 e^{4h\zeta}) + e^{z\gamma+2h\zeta}(\gamma\alpha_1 + \alpha_3\beta_1) + e^{z\gamma+2(h+z)\zeta}(\gamma\alpha_2 - \alpha_3\beta_1) - e^{z\gamma+2h(\gamma+\zeta)}\cdot(\gamma\alpha_1 + \alpha_4\beta_1) + e^{2h(\gamma+\zeta)+z(\gamma+2\zeta)}\cdot(-\gamma\alpha_2 + \alpha_4\beta_1) + e^{2h\gamma+z\gamma+4h\zeta}(\gamma\alpha_2 - pz\alpha_4\eta) - e^{z\gamma+4h\zeta}(\gamma\alpha_2 - pz\eta\alpha_3) - e^{z(\gamma+2\zeta)}(\gamma\alpha_1 + pz\eta\alpha_3) + e^{2h\gamma+z\gamma+2z\zeta}(\gamma\alpha_1 + pz\eta\alpha_4))/(\cosh(h\gamma)\cdot(-Hk_2\zeta^3 + 2hk_1p\zeta\eta + Hk_2\zeta^3\cosh(2h\zeta) + k_1(\zeta^2+p\eta)\sinh(2h\zeta)) + \gamma\sinh(h\gamma)\cdot(-2k_1\zeta\cosh(h\zeta)^2 + Hk_2(2hp\zeta\eta + (-\zeta^2+p\eta)\sinh(2h\zeta))))d\zeta \qquad (37)$$

5) Excess pore-water pressure:

$$\bar{p}_f = -\frac{p_0}{8\pi}\int_0^\infty \eta\zeta J_0(r\zeta) e^{-z(\gamma+\zeta)-h(\gamma+2\zeta)}(-1 + e^{2h\zeta})(\alpha_3 e^{z\gamma}(e^{2z\zeta} + e^{2h\zeta}) - \alpha_4 e^{z\gamma+2h\gamma}(e^{2z\zeta} + e^{2h\zeta}) - \alpha_1 e^{z\zeta}(e^{2z\zeta} + e^{2h\gamma}) + \alpha_2 e^{z\zeta+2h\zeta}(e^{2z\gamma} + e^{2h\gamma}))/(\cosh(h\gamma)(-Hk_2\zeta^3 + 2hk_1p\zeta\eta + Hk_2\zeta^3\cosh(2h\zeta) + k_1(\zeta^2+p\eta)\sinh(2h\zeta)) + \gamma\sinh(h\gamma)(-2k_1\zeta\cosh(h\zeta)^2 + Hk_2(2hp\zeta\eta + (-\zeta^2+p\eta)\sinh(2h\zeta))))d\zeta \qquad (38)$$

where

$$\alpha_1 = -k_1 + Hk_2\zeta$$
$$\alpha_2 = k_1 + Hk_2\zeta$$
$$\alpha_3 = -k_1 + Hk_2\gamma$$
$$\alpha_4 = k_1 + Hk_2\gamma$$
$$\beta_1 = p\eta(2h - z)$$

2. Fundamental solution subject to ring loading

The system under consideration is shown in Fig. 2. Application of Laplace transform and Hankel transform yields the following boundary conditions over the plane ($z = 0$):

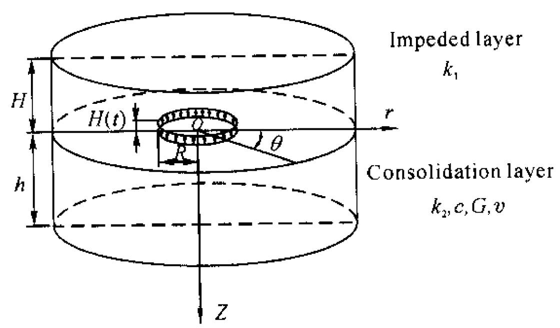

Fig. 2 Axisymmetric ring loading

$$k_2\frac{\partial \bar{p}_f}{\partial z} = k_1\frac{\bar{p}_f}{H} \qquad (39)$$

$$\bar{\sigma}_{rz} = 0 \qquad (40)$$

$$\bar{\sigma}_{zz} = RJ_1(R\zeta) \qquad (41)$$

Only change $P_0/2\pi$ in the fundamental solution for point load into $RJ_0(R\zeta)$, and the fundamental solution for ring load can be obtained.

SOLUTIONS FOR OTHER DISTRIBUTED LOAD

1. Solutions for circular loading

The system under consideration is shown in Fig. 3. Use of Laplace transform and Hankel transform yields the following boundary conditions over ($z = 0$):

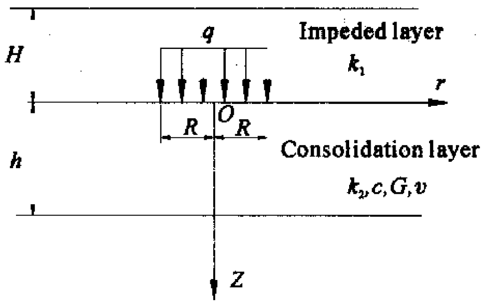

Fig.3 Axiymmetric circular loading

$$k_2 \frac{\partial \bar{p}_f}{\partial z} = k_1 \frac{\bar{p}_f}{H} \quad (42)$$

$$\bar{\sigma}_{rz} = 0 \quad (43)$$

$$\bar{\sigma}_{zz} = qRJ_1(R\zeta)/(p\zeta) \quad (44)$$

Only change $P_0/2\pi$ in the fundamental solution for point loading into $qRJ_1(R\zeta)/\zeta$, and the fundamental solution for ring loading can be obtained.

2. Solutions for strip loading

The system subject to strip loading is shown in Fig. 4. Based on the fundamental solutions for point loading, the infinite integration in the y direction and finite integration in the x direction of the fundamental solutions yields the solution of point M in the medium for strip loading.

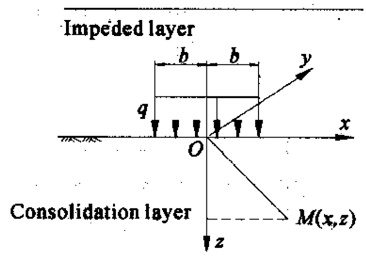

Fig.4 Strip loading

NUMERICAL COMPUTATION AND ANALYSIS (CIRCULAR LOADING)

All the above solutions should be evaluated numerically. In the numerical quadrature scheme, all infinite integrals with respect to the Hankel transform parameter ζ are evaluated using the Simpson rule ; and time domain solutions are computed using the approximate Laplace inverse formula suggested by Schapery (1962).

1. The settlement at the center of the loaded area

As shown in Fig. 3, for $k1/k2 = 1$, $R = h = 1$m, $q = 1\text{N/m}^2$, $\nu = 0$, H from 0 m to 5 m, the effect of the impeded layer thickness on the progress of settlement at the center of the loaded area is shown in Fig. 5. The results have been plotted in the form of curves of $2Gu_{zz}/qR$ against ct/R^2. It is apparent that the impeded layer thickness has an important influence. When the impeded layer thickness is zero, the boundary is fully permeable.

2. The top surface settlement of consolidation layer

For $k1/k2 = 1$, $R = h = 1$m, $H = 1$m, $q = 1\text{N/m}^2$, $\nu = 0$, when the impeded layer thickness is 1m, the top surface settlement of the consoli-

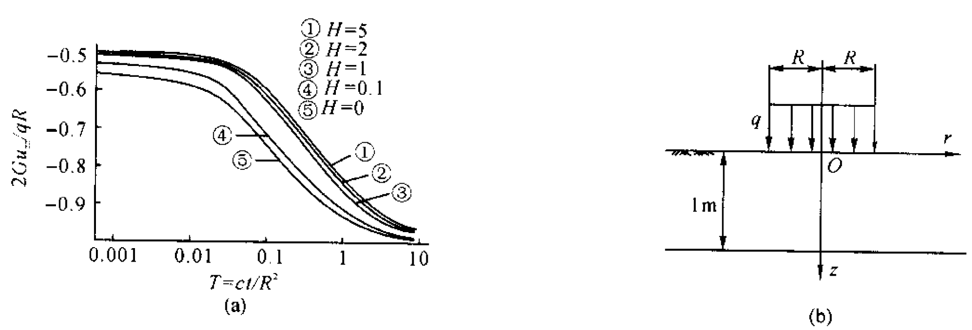

Fig.5 (a) Effect of the impeded layer thickness on settlement at the center of the loaded area; (b) sketch map for the position of calculation point

dation layer develops with time as shown in Fig. 6. There are some slight upheavals beyond the loaded area.

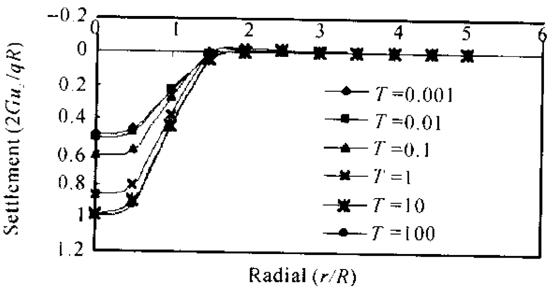

Fig.6 The development with time of the top surface settlement of the consolidation layer

3. Total compressive stress at 0.5m under the center of the loaded area

For $k1/k2 = 1$, $R = h = 1\text{m}$, $q = 1\text{N/m}^2$, $\nu = 0$, the changing with time of vertical total compressive stress at 0.5m under the center of the loaded area during consolidation is shown in the Fig.7. The results have been plotted in the form of curves of σ_{zz}/q against ct/R^2. The thicker the impeded layer, the smaller the change range of the total stress. It is clearly shown in Fig.8 that the change of vertical total stress is related to the change of excess pore pressure.

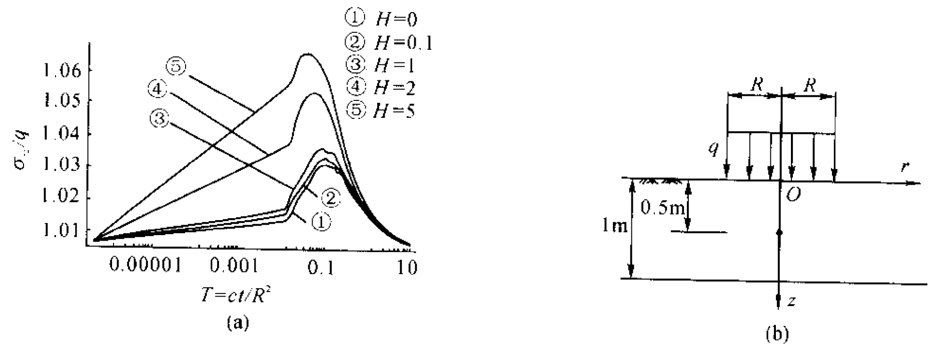

Fig.7 (a) Effect of the impeded layer thickness and time on vertical total compressive stress; (b) sketch map for the position of calculation point

4. Excess pore pressure

For $k1/k2 = 1$, $R = h = 1\text{m}$, $q = 1\text{N/m}^2$, $\nu = 0$, the changing with time of excess pore pressure at 0.5 m under the center of the loaded area is shown in the Fig. 8. The results have been plotted in the form of curves of p_f/q against ct/R^2. Mandel-Cryer effect is obvious for $H = 0$. Mandel-Cryer effect is weakened with thickening of the impeded layer.

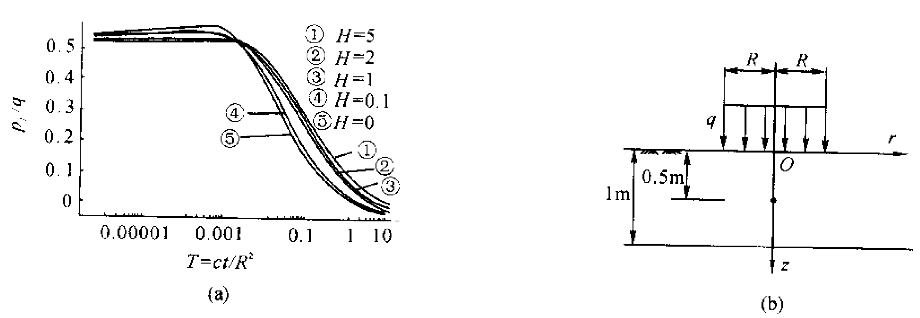

Fig.8 (a) Effect of the impeded layer thickness and time on excess pore pressure; (b) sketch map for the position of calculation point

CONCLUSIONS

Explicit solutions for Laplace transforms of displacements, tractions, pore pressure ware derived for a finite layer with impeded boundaries and subjected to axisymmetric point loads and ring loads. The Laplace and Hankel transforms utilized in this paper are efficient. The solutions are extended to circular distributed and strip distributed normal loadings. The result is convergent and satisfactory.

As a result of the calculation and analysis, it is apparent that the thickness of the impeded layer has an important influence on settlement, vertical total stress and pore pressure of the consolidation layer. The vertical total stress is time-dependant during consolidation. The thicker the impeded layer is, the smaller the changing range of the vertical total stress and the weaker the Mandel-Cryer effect. The impeded layer delays the settlement progress of the consolidation layer.

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