# Fault location of two-parallel transmission line for double phase-to-earth fault using one-terminal data

ZHANG Qing-chao(张庆超)<sup>†</sup>, DUAN Hui(段晖), GENG Chao(耿超), SONG Wen-nan(宋文南)

(School of Electrical Engineering and Energy, Tianjin University, Tianjin 300072, China)

†E-mail: qczhang@tju.edu.cn

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**Abstract:** An accurate algorithm for fault location of double phase-to-earth fault on transmission line of direct ground neutral system is presented. The algorithm, which employs the faulted phase network and zero-sequence network as fault-location model in which the source impedance at the remote end is not involved, effectively eliminates the effect of load flow and fault resistance on the accuracy of fault location. The algorithm achieves accurate location by measuring only one local end data and is used in a procedure that provides automatic determination of faulted types and phases, and does not require the engineer to specify them. Simulation results showed the effectiveness of the algorithm under the condition of double phase-to-earth fault.

**Key works:** Fault location, Transmission line, Double phase-to-earth fault **Document code:** A **CLC number:** TM77, F540.3

#### INTRODUCTION

Transmission lines of a power system are subject to many kinds of faults. The principal types are: phase-to-earth; double phase-to-earth (phase-phase-earth); phase-to-phase; three-phase. Following the occurrence of a transmission line fault, the maintenance crew must find and fix the problem to restore the service as quickly as possible. Rapid restoration of the service reduces outage time and loss of revenue. Therefore, accurate fault location under a variety of fault conditions is an important requirement.

A variety of fault location algorithms have been developed in recent years. Most accurate results were obtained using algorithms that considered the fault data from two terminals of the line together (Kezunovic *et al.*, 1995; 1996; Novosel *et al.*, 1996). However, two-terminal data are not widely available. From the practical viewpoint, it is desirable for equipment to use only one-terminal data (Zamore *et al.*, 1996; Thomas *et al.*, 2001; Bo, 2002; Eriksson *et al.*, 1985; Djuric *et al.*, 1998; Zhang *et al.*, 1998; 1999).

### PROPOSED FAULT LOCATION ALGORITHM

The load is not generally contained in the zero-sequence circuit of a faulted power network. Therefore, we use the zero-sequence circuit as the model of fault location to eliminate the effect of load disturbance on the accuracy of fault location. The circuit of a transmission line with double phase-to-earth fault is shown in Fig.1.

To simplify the introduction of the algorithm, one model of two-circuit transmission line with-

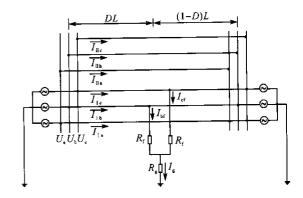


Fig. 1 Double phase-to-earth fault

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out shunt capacitance is considered. In terms of the superposition principle in linear network, the faulted network is broken into three sequence networks, namely, positive, negative and zero sequence network.

They are given in Figs. 2, 3 and 4 respectively. Thus, the relationship between the voltage  $U_{\rm b}$  and its sequence components  $U_{\rm b1}$ ,  $U_{\rm b2}$  and  $U_{\rm b0}$  can be expressed as

$$U_{\rm b} = U_{\rm b1} + U_{\rm b2} + U_{\rm b0} \tag{1}$$

Applying KVL in Figs. 2, 3 and 4 respectively, sequence component  $U_{\rm b1}$ ,  $U_{\rm b2}$  and  $U_{\rm b0}$  are

$$U_{\rm b1} = DZ_{11}I_{1\rm b1} + R_{\rm f}I_{\rm bf1} + E_{\rm bf1} + R_{\rm g}I_{\rm g1} \tag{2}$$

$$U_{b2} = DZ_{12}I_{1b2} + R_{f}I_{bf2} + E_{bf2} + R_{g}I_{g2}$$
 (3)

$$U_{b0} = DZ_{10}I_{1b0} + R_{f}I_{bf0} + E_{bf0} + R_{g}I_{g0} + DZ_{m0}I_{1lb0}$$
(4)

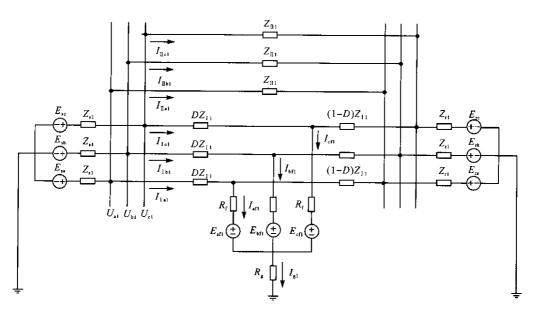


Fig.2 Positive-sequence network

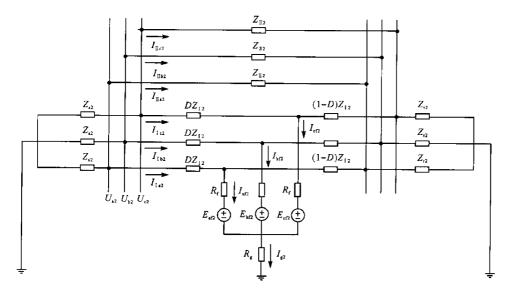


Fig.3 Negative-sequence network

Of a transmission line in a three-phase power system, the positive and negative sequence impedance  $Z_1$  and  $Z_2$  are always equal. Sequence impedance's subscripts are deleted to simplify the wording and notation Z' represents positive-sequence impedance of line, that is

$$Z_{11} = Z_{12} = Z_C I \tag{5}$$

where subscript I denotes first circuit line. Substituting parameters from Eq.(5) into Eqs. (2), (3) and (4); then parameters from Eqs. (2), (3) and (4) into Eq.(1), we get

$$U_{\rm b} = DZ_1I_{\rm lb} + (Z_{\rm l0} - Z_1)DI_{\rm lb0} + R_{\rm f}I_{\rm bf} + R_{\rm g}I_{\rm g} + DZ_{\rm m0}I_{\rm lb0}$$
(6)

where

$$I_{1b} = I_{1b1} + I_{1b2} + I_{1b0}$$

$$I_{bf} = I_{bf1} + I_{bf2} + I_{bf0}$$

$$I_{g} = I_{g1} + I_{g2} + I_{g0}$$

$$0 = E_{bf1} + E_{bf2} + E_{bf0}$$

Similarly, we get

$$U_{c} = DZ_{1}I_{Ic} + (Z_{10} - Z_{1})DI_{Ic0} + R_{f}I_{cf} + R_{g}I_{g} + DZ_{m0}I_{IIc0}$$
(7)

Adding Eq.(6) and Eq.(7) together, we get  $U_{\rm b} + U_{\rm c} = DZ_{\rm I}(I_{\rm lb} + I_{\rm lc}) + (Z_{\rm l0} - Z_{\rm I})D(I_{\rm lb0} + I_{\rm lc0}) + R_d(I_{\rm bf} + I_{\rm cf}) + DZ_{\rm m0}(I_{\rm lb0} + I_{\rm lc0})$ (8)

$$I_{\mathrm{g}} = I_{\mathrm{bf}} + I_{\mathrm{cf}}$$
  
 $R_{\mathrm{d}} = R_{\mathrm{f}} + 2R_{\mathrm{g}}$ 

In Eq.(8), parameters  $Z_{\rm I}$ ,  $Z_{\rm m0}$  and  $Z_{\rm I0}$  are known, and the voltage  $U_{\rm b}$  and  $U_{\rm c}$  and the current  $I_{\rm Ib}$ ,  $I_{\rm Ic}$ ,  $I_{\rm Ib0}$ ,  $I_{\rm Ic0}$ ,  $I_{\rm Ib0}$  and  $I_{\rm Ilc0}$  are easily obtained from data provided by a register device at the monitored point. In order to eliminate unknown  $I_{\rm bf}$  and  $I_{\rm cf}$ , the relation between the zero-sequence current  $I_{\rm af0}$  and the currents  $I_{\rm af}$ ,  $I_{\rm bf}$  and  $I_{\rm cf}$  are considered.

$$3I_{\text{af0}} = I_{\text{af}} + I_{\text{bf}} + I_{\text{cf}} = I_{\text{bf}} + I_{\text{cf}}$$
 (9)

where

$$I_{\rm af} = 0$$

Substituting parameters from Eq. (9) into Eq. (8), we get

$$U_{\rm b} + U_{\rm c} = DZ_{\rm I}(I_{\rm lb} + I_{\rm Ic}) + (Z_{\rm l0} - Z_{\rm I})D(I_{\rm lb0} + I_{\rm Ic0}) + 3R_{\rm d}I_{\rm af0} + DZ_{\rm m0}(I_{\rm llb0} + I_{\rm Ilc0})$$
(10)

At the fault point in Fig. 4, the current  $I_{\rm al0}$  can be expressed as

$$I_{\text{af0}} = I_{\text{Ia0}}^1 - I \tag{11}$$

 $I_{\rm la0}^1$  is still a quantity that cannot be determined at the local end. In order to remove it, applying KVL in Loop a-b-c-d-a in Fig. 4, we get

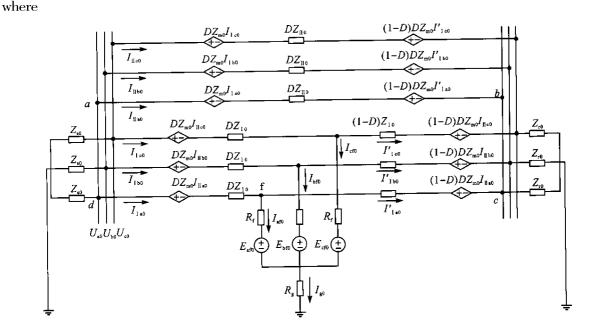


Fig.4 Zero-sequence network

$$DZ_{m0}I_{1a0} + Z_{110}I_{11a0} + (1 - D)Z_{m0}I_{1a0}^{1} - DZ_{m0}I_{1a0} - DZ_{m0}I_{1a0} - DZ_{m0}I_{1a0} - (1 - D)Z_{m0}I_{11a0} - (1 - D)Z_{m0}I_{1a0} - (1 - D)Z_{m0}I_{1a0}$$

from which

$$I_{\text{la0}}^{1} = \frac{-D(Z_{\text{m0}} - Z_{\text{l0}})I_{\text{la0}} + (Z_{\text{m0}} - Z_{\text{l0}})I_{\text{lla0}}}{(1 - D)(Z_{\text{m0}} - Z_{\text{l0}})}$$
(12)

Substituting parameters from Eq.(12) into Eq. (11), the resulting equation is

$$I_{\text{af0}} = \frac{(Z_{\text{m0}} - Z_{\text{10}})I_{\text{la0}} - (Z_{\text{m0}} - Z_{\text{110}})I_{\text{Ila0}}}{(1 - D)(Z_{\text{m0}} - Z_{\text{10}})}$$
(13)

Then substituting parameters from Eq. (13) into Eq. (10), the following equation is obtained without  $I_{a0}$ .

$$U_{b} + U_{c} = DZ_{I}(I_{lb} + I_{Ic}) + (Z_{I0} - Z_{I})D(I_{lb0} + I_{lc0}) + 3R_{d} + \frac{(Z_{m0} - Z_{I0})I_{Ia0} - (Z_{m0} - Z_{I0})I_{Ila0}}{(1 - D)(Z_{m0} - Z_{I0})} + DZ_{m0}(I_{Ilb0} + I_{Ilc0})$$
(14)

Eq.(14) does not involve the load  $Z_{\rm L}$  and remote end source impedance  $Z_{\rm r}$ . This means that the algorithm can locate a fault without being influenced by the load flow and the change of the remote source.

Rewriting the equation in terms of the sequence component of the reference phase (phase a), we get

$$U_{\rm b} + U_{\rm c} = DZ_{\rm 1}(I_{\rm lb} + I_{\rm lc}) + 2(Z_{\rm 10} - Z_{\rm 1})DI_{\rm 10} + 3R_{\rm d} \frac{(Z_{\rm m0} - Z_{\rm 10})I_{\rm 10} - (Z_{\rm m0} - Z_{\rm 10})I_{\rm 110}}{(1 - D)(Z_{\rm m0} - Z_{\rm 10})} + 2DZ_{\rm m0}I_{\rm 110},$$
(15)

where the subscripts of phase b and phase c are omitted from the zero sequence components.

The zero-sequence currents can be determined in terms of three phase currents.

$$I_{10} = (I_{1a} + I_{1b} + I_{1c})/3$$
  
 $I_{110} = (I_{11a} + I_{11b} + I_{11c})/3$ 

Rewriting Eq. (15), we get

$$a_1(j\omega)R_d + a_2(j\omega)D + a_3(j\omega)D^2 + a_4(j\omega) = 0$$
 (16)

The unknowns in Eq.(16) are the fault resistance  $R_{\rm d}$  and the fault location D. The coefficients are defined as

$$\begin{aligned} a_{1}(j\omega) &= -3 \big[ (Z_{10} - Z_{m0}) I_{10} - (Z_{110} - Z_{m0}) I_{110} \big] \\ a_{2}(j\omega) &= (Z_{m0} - Z_{10}) \big[ Z_{I} (I_{1b} + I_{1c}) + \\ 2(Z_{10} - Z_{1}) I_{10} + 2Z_{m0} I_{110} + (U_{b} + U_{c}) \big] \\ a_{3}(j\omega) &= -(Z_{m0} - Z_{10}) \big[ Z_{I} (I_{1b} + I_{1c}) + \\ 2(Z_{10} - Z_{1}) I_{10} + 2Z_{m0} I_{110} \big] \\ a_{4}(j\omega) &= -(Z_{m0} - Z_{10}) (U_{b} + U_{c}) \end{aligned}$$

Z-transform theory gives a method to locate transmission line fault with sampling data of faulted power network.

Rewriting Eq. (16) in L-transform domain, we get

$$a_1(s)R_d + a_2(s)D + a_3(s)D^2 + a_4(s) = 0$$
(17)

If the sample interval T is chosen to be sufficiently small compared to the system's time constants, Eq. (17) can be transformed from s-domain to z-domain.

$$a_{1}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)R_{d} + a_{2}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)D + a_{3}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)D^{2} + a_{4}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right) = 0 \quad (18)$$

where

z = z-transformation operator

Rearranging Eq. (18), we get

$$b_1(z)R_d + b_2(z)D + b_3(z)D^2 + b_4(z) = 0$$
(19)

where

$$z = \frac{2 - Ts}{2 + Ts}$$

For a series of samples  $U_{\rm b}$  ( $t_0-nT$ ),  $U_{\rm c}$ ( $t_0-nT$ ),  $I_{\rm lb}$ ( $t_0-nT$ ),  $I_{\rm lc}$ ( $t_0-nT$ ),  $I_{\rm lc}$ ( $t_0-nT$ ), and  $I_{\rm l0}$ ( $t_0-nT$ )(n=N,N-1), ..., 1, 0), the z-transform domain and the time domain are related as follows.

$$\begin{split} Z^{-1} & \left[ z^{-n} U_{\rm b}(z) \right] = U_{\rm b}(t_0 - nT) \\ Z^{-1} & \left[ z^{-n} U_{\rm c}(z) \right] = U_{\rm c}(t_0 - nT) \\ Z^{-1} & \left[ z^{-n} I_{\rm b}(z) \right] = I_{\rm b}(t_0 - nT) \\ Z^{-1} & \left[ z^{-n} I_{\rm c}(z) \right] = I_{\rm c}(t_0 - nT) \\ Z^{-1} & \left[ z^{-n} I_{\rm l0}(z) \right] = I_{\rm l0}(t_0 - nT) \\ Z^{-1} & \left[ z^{-n} I_{\rm l0}(z) \right] = I_{\rm l0}(t_0 - nT) \end{split}$$

where

$$Z^{-1}$$
 = reciprocal z-transformation

At  $t = t_0$ , we get

$$b_1(t_0)R_d + b_2(t_0)D + b_3(t_0)D^2 + b_4(t_0) = 0$$
(20)

And at  $t = t_0 - T$ , we get

$$b_1(t_0 - T)R_d + b_2(t_0 - T)D + b_3(t_0 - T)D^2 + b_4(t_0 - T) = 0$$
(21)

Eliminating  $R_d$  from Eqs. (20) and (21), we get

$$B_1 D + B_2 D^2 + B_3 = 0 (22)$$

where

$$B_1 = b_2(t_0 - T) - \frac{b_1(t_0 - T)b_2(t_0)}{b_1(t_0)}$$

$$b_1(t_0 - T)b_1(t_0)$$

$$B_2 = b_3 (t_0 - T) - \frac{b_1 (t_0 - T) b_3 (t_0)}{b_1 (t_0)}$$

$$B_3 = b_4(t_0 - T) - \frac{b_1(t_0 - T)b_4(t_0)}{b_1(t_0)}$$

The fault distance D is the unknown to be determined. It can be obtained by solving Eq. (22) with iterative Newton-Raphson method.

## PROPOSED ALGORITHM FOR IDENTIFYING FAULTED TYPES

Under normal operating condition or during the occurrence of a non-earth fault, zero-sequence voltages or currents will not appear in power systems. They can, however, be picked up at the monitored point when an earth fault, phase-to-earth or double phase-to-earth, occurs on transmission lines.

The identifications of faulted phases are based on comparisons between the amplitudes of current vectors. First, choose the largest from the phase currents. Then, each phase current is divided by the largest. If the ratio is greater than a setting-value (usually 0.7), this phase will be regarded as a faulted one. If the ratio is less than another setting-value (usually 0.3), this phase is a "healthy" one.

#### SIMULATION RESULTS

We start to operate the algorithm while detecting the signal of the zero-sequence voltage. First, the sampling data of voltage and currents from the lines are stored in a data file for further processing. Then, the voltage and current vectors are extracted from the data. Third, the faulted types and phases are determined by a computer program that can analyze voltage and current vectors. Finally, Eq. (22) is solved with the Newton-Raphson method.

A model system and its line parameters for computer simulation are shown in Fig.5 and Table 1, respectively. The double phase-to-earth faults are considered. The faults are applied on the line from D=0 through D=1 with fault resistance from 0  $\Omega$  to 500  $\Omega$ . The interval T of data sampling is 0.1 ms. To estimate the error in calculation, the algorithm is tested under various conditions. The simulations are made with accurate line impedance and sampling data of the voltages and currents. In all cases, the results showed that the error in locating the fault was less than 1%.



Fig. 5 Model system with two-circuit line for simulation

Table 1 Line parameters of model system

| Circuit    | $L_1/\mathrm{mH}$ | $L_0/\mathrm{mH}$ | $L_{ m m0}/{ m mH}$ | $R_1/\Omega$ | $R_0/\Omega$ | $C_1/\mu { m F}$ | $C_0/\mu { m F}$ | l/km |
|------------|-------------------|-------------------|---------------------|--------------|--------------|------------------|------------------|------|
| Circuit-I  | 153.1             | 549.0             | 351.4               | 31.20        | 49.49        | 1.060            | 0.5765           | 120  |
| Circuit-II | 159.1             | 551.4             | 351.4               | 39.80        | 57.80        | 1.043            | 0.5714           | 120  |

### SENSITIVITY STUDIES

Simulation on a system model with distributed-parameters non-transposed line was done. The results showed that the estimated fault locations were close to actual ones and that errors in fault location decision were not more than 1% when the algorithms was used on systems with non-transposed line.

Current transformer (CT) will operate at the limit of field saturation in first period just after a line fault occurs because of the line current surges. In this case, the accuracy of locating a line fault will be affected significantly if the sampling data in the period are directly employed in the proposed algorithm. Further simulation results showed that the error of fault location will reduce and in most cases be less than 5% if the sampling data was filtered through the Fast Fourier Transformation (FFT) program before computing with the algorithm of fault location.

In the simulation results described above, it was assumed that the line parameters and sampling data were accurately known. However, in practice, these parameters and data may be inaccurate. Therefore, it is necessary to study the sensitivity of the algorithm to the error of line parameters and sampling data. The following general conclusions were obtained from additional simulations.

- 1. Ten-percent error of transmission line resistance or inductance will result in 10% error in fault location.
- 2. Ten-percent error of the sampling data of voltage and current will result in 10 % error in fault location.

#### CONCLUSIONS

This paper presents a novel method of locating transmission line fault using one terminalda-

ta. The main characteristics of the method are that: (1) the faulted phase and zero-sequence circuit are used for modeling; (2) the model of fault location does not involve the source impedance of the remote end resource; (3) the accuracy of fault location is not affected by load flow and fault resistance; (4) the short data window is used.

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