Journal of Zhejiang University SCIENCE ISSN 1009-3095 http://www.zju.edu.cn/jzus E-mail: jzus@zju.edu.cn



# A matching algorithm based on hybrid matrices consisting of reference differences and disparities

GUAN Ye-peng (管业鹏)<sup>†</sup>, GU Wei-kang (顾伟康)

(Dept. of Information Science and Electronics Engineering, Zhejiang University, Hangzhou 310027, China) <sup>†</sup>E-mail: seugyp@sohu.com; zjuypguan@126.com Received Sept. 28, 2003; revision accepted Dec. 5, 2003

**Abstract:** Unique correct correspondence cannot be obtained only by use of gray correlation technique, which describes gray similar degree of feature points between the left and right images too unilaterally. The gray correlation technique is adopted to extract gray correlation peaks as a coarse matching set called multi-peak set. The disparity gradient limited constraint is utilized to optimize the multi-peak set. Unique match will be obtained by calculating the correlation of hybrid matrices consisting of reference differences and disparities from the multi-peak set. Two of the known corresponding points in the left and right images, respectively, are set as a pair of reference points to determine search direction and search scope at first. After the unique correspondence is obtained by calculating the correlation of the hybrid matrices from the multi-peak set, the obtained match is regarded as a new reference point till all feature points in the left (or right) image have been processed. Experimental results proved that the proposed algorithm was feasible and accurate.

Key words: Gray correlation, Multi-peak set, Reference difference, Disparity, Hybrid matrixDocument code:ACLC number:TP391.41

### INTRODUCTION

A common method for extracting depth information from intensity images is based on the principles of stereovision. In this method, a pair of images is obtained using two cameras displaced from each other by a known distance. After determining the stereo correspondence, the distance of various points is computed using triangulation (Jeon *et al.*, 2001). Stereo correspondence is the problem of finding corresponding points in two or more images of the same scene, usually assuming known camera geometries.

Various algorithms with certain constraints and matching strategies (Akutsu, 1994; Alex *et al.*, 2003; Ho and Yip, 1996; Kanade and Okutomi, 1994; Kittler *et al.*, 1993; Vasilescu and Terzopoulos, 2002; Wei *et al.*, 1998; Lu *et al.*, 1997; Zitnick and Kanade, 2000) proposed reducing the possibility of false matches, but many problems still remain in stereo correspondence.

To solve the correspondence, one basic idea is based on comparability measurement and continuity hypothesis. When left and right cameras observe the same spatial point P, the gray intensity of the corresponding point  $p_1$  and  $p_2$  in left and right images at P should be nearly equal depending on the comparability measurement. Because noise always exists in actual images, the gray correlation value Cwithin a window W is usually used to express the gray comparability

$$C = \frac{1}{M} \sum_{(i,j) \in W} [I_1(u_1 - i, v_1 - j) - \overline{I_1}] \cdot [I_2(u_2 - i, v_2 - j) - \overline{I_2}]$$
(1)

796

where  $(u_1, v_1)$  are the coordinates of  $p_1$  in the left image and  $(u_2, v_2)$  are the coordinates of  $p_2$  in the right image; W is a window centered at  $p_1$  (or  $p_2$ ); M is the number of points within window W;  $\overline{I}_1$  and  $\overline{I}_2$ are the average gray intensity over a window W on the left and right images, respectively.

Greater correlation coefficient C denotes more similarity between  $p_1$  and  $p_2$ . Obviously, maximum gray correlation is not the sufficient condition of  $p_1$ corresponding to  $p_2$ . However, it is one of the most simple and easiest ways to find the matching candidates.

Correct correspondence cannot be obtained only by use of gray correlation, which describes gray similar degree of points between the left and right images too unilaterally.

However, gray correlation technique can be used to extract multi-peak feature points with gray correlation coefficients less than certain range of maximum coefficient as a coarse matching set (Guan *et al.*, 2003), which is called multi-peak set (MPS).

In this paper, we present a novel stereo algorithm. Utilizing a pair of known corresponding reference points, we acquire unique matching point by calculating the correlation of hybrid matrices (HM) consisting of reference differences and disparities from MPS. The uniqueness and continuity assumptions by Marr and Poggio are adopted in this paper.

Dynamic limit search method is introduced to reduce the computational complexity. A pair of known points in the left and right images, respectively, is set as corresponding reference points before stereo matching at first. One of feature points with Euclidean distance nearest to the corresponding reference point in the left (or right) image is regarded as a matching point. From the horizontal difference between the matching point and its corresponding reference point, we determine search direction and scope. The unique correspondence with the matching point is obtained by calculating the correlation of HM from MPS. The conjugated points are taken as a new pair of corresponding reference points.

Experimental results of 3-D reconstruction on

different real testing objects proved that the introduced algorithm was effective.

The paper is organized as follows. In Section 2, we analyze gray correlation matching technique essential for the development of the proposed method. We adopt this matching technique to get MPS, and utilize a pair of known corresponding points to determine the unique correspondence by computing the correlation of HM from MPS in Section 3. The experimental results are given in Section 4. Finally, conclusions are given in Section 5.

## VIEW OF GRAY CORRELATION MATCHING

It is known from the binocular stereo parallel photography system that the disparity d is related to the depth z by

$$d = Bf/z \tag{2}$$

where B and f are baseline and focal length, respectively.

#### Directivity

Eq.(2) shows that the location of the perspective projection in the left image will shift a distance d horizontally relative to that of in the right image for the same scene point; in other words, if the positive horizontal direction is towards the right, the location of the perspective projection in the right image will deviate a distance d towards the left. Based on this directivity, while seeking correspondence of feature point in the right (or left) image with a point in the left (or right) image, we can only search within a certain scope towards the right (or left) along the corresponding epipolar line in the left (or right) image.

In practice, the candidate feature points may not lie exactly on the epipolar line due to noise and perspective distortion, and as a result, we search for these points within an epipolar band in the neighborhood of the original epipolar line.

### Analysis of gray correlation matching technique

As we know, unique correct correspondence

cannot be obtained only by use of the gray correlation between the left and right images.

Take a pair of actual parallel stereo images for example. Horizontal pixel coordinates of seven pairs of feature points in the left and right images, respectively, are shown in Table 1.

 Table 1 Horizontal pixel coordinates for stereo images (pixels)

Left image	343	423	504	586	668	754	839
Right image	205	268	331	396	462	529	598

Based on the directivity of stereo imaging, we adopt gray correlation matching technique to determine matches of two feature points in the left image, x=423 and 586 respectively, across the right image. Assume that the deviation towards the left is 300 pixels, namely the search scope is [123–423] and [286–586] in the right image, respectively. The maps of gray correlation are shown in Fig.1 and Fig.2, respectively.

If the criterion of maximum gray correlation is adopted, it is easily deduced that corresponding horizontal pixel coordinates would be x'=205 and x'=462, respectively, from Fig.1 and Fig.2. In fact, however, the actual horizontal pixel coordinates should be x'=268 and x'=396, respectively.

It is obvious that correct correspondence cannot be achieved when gray correlation matching algorithm is used alone.



Since gray correlation describes the gray similarity of the feature points region between the left and right images unilaterally, false matches will happen inevitably. Gray correlation matching technique, however, can be adopted to determine gray correlation coefficients  $C_{ij}$  between feature points in the left and right images. For confirming match candidates in the next step, multi-peak feature points with gray correlation coefficients satisfying  $C_{ij} \ge \max C - \varepsilon$  (where  $\max C = \max(C_{ij})$ ,  $\varepsilon$  is a small value) are taken as a coarse matching set, which is called gray correlation multi-peak set.

In this paper, we present a novel stereo algorithm for determining matches from the MPS.

A coarse-to-fine control strategy to reduce the probability of error matches was proposed by Grimson (1985). When there is iteration pattern, however, in a "biggish" scene, error matches are not always removed by low resolution. Okutomi and Kanade (1990) proposed a matching algorithm that can select an appropriate window size adaptively. Hu and Ahuja (1994) proposed a matching algorithm involving geometry, rigidity and disparity constraints. Lew *et al.*(1994) proposed a feature-matching algorithm obtained by computing the similarity of gray intensity, orientation gradient, etc. Other matching algorithms had also been recomm-



Fig.1 The gray correlation between the feature point (*x*=423) in the left image and points in the right image



Fig.2 The gray correlation between the feature point (x=586) in the left image and points in the right image

ended (Atallah, 2001; Brünig and Niehsen, 2001; Kanade and Yamada, 2003; Kim and Choi, 2000).

The stereo algorithm proposed in this paper differs from existing matching algorithms. Multipeak feature points are extracted as a coarse matching set called MPS after gray correlation matching technology was adopted. MPS is optimized by disparity gradient limit constraint. We utilize a pair of known corresponding reference points to determine unique match by calculating the correlation of HM from MPS. The proposed approach is easy and simple.

The spatial relativity of the matching points set should be considered to get the unique correct match. Ideal matching should be done in the 3-D space. However, in the beginning, the depths cannot be computed and the computational complexity forbids us to consider all combinations of points simultaneously since N points yield  $2^N$  combinations (Hu and Ahuja, 1994). When the spatial point set, however, projects into image plane point set, the geometrical shape exhibits consistency, so correlation matching can be done in the 2-D image. One simple way to test for local geometric consistency is to consider only triples of points simultaneously and update the set of matched pairs (the rigid set) recursively (Hu and Ahuja, 1994). These tests aimed at identifying points that have correct matches by considering groups of points together. It is obvious that the calculation is time-consuming.

Our algorithm is as follows:

A pair of known corresponding points is used to confirm the search direction of the match before stereo matching. The corresponding points (p, p'vector) are regarded as a pair of reference points. One of the feature points (q vector) with Euclidean distance nearest to the corresponding reference point in the left (or right) image is taken as a matching point.

According to the definition of disparity gradient (Burt and Julesz, 1980)

$$G_{d} = 2 \frac{\|(\boldsymbol{p} - \boldsymbol{p}') - (\boldsymbol{q} - \boldsymbol{q}')\|}{\|(\boldsymbol{p} - \boldsymbol{p}') + (\boldsymbol{q} - \boldsymbol{q}')\|}$$
(3)

The scope of vector q' can be ascertained by

Eq.(3).

### (1) Disparity

Assume vector p and p' are a pair of corresponding reference points in the left and right images, respectively.

$$\Delta \boldsymbol{D} = \boldsymbol{p} - \boldsymbol{p}' \tag{4}$$

where  $\Delta \boldsymbol{D}$  is disparity.

(2) Reference difference

Assume vector q is a matching point; vector p is its corresponding reference point.

$$\Delta \boldsymbol{R} = \boldsymbol{q} - \boldsymbol{p}. \tag{5}$$

where  $\Delta \mathbf{R}$  is reference difference.

(3) The hybrid matrices consisting of reference differences and disparities

$$\boldsymbol{M}_{1} = [\boldsymbol{P} \quad \boldsymbol{P}' \quad \Delta \boldsymbol{R} \quad \Delta \boldsymbol{D}] \tag{6}$$

$$\boldsymbol{M}_2 = [\boldsymbol{Q} \ \boldsymbol{Q}' \ \Delta \boldsymbol{R}' \ \Delta \boldsymbol{D}'] \tag{7}$$

where  $\Delta \mathbf{R}' = \mathbf{q}' - \mathbf{p}'$  and  $\Delta \mathbf{D}' = \mathbf{q} - \mathbf{q}'$ .  $\mathbf{q}'$  is a vector to be solved for matching point  $\mathbf{q}$ .  $M_1$  and  $M_2$  are called hybrid matrices.

(4) Calculation of correlation between hybrid matrices  $M_1$  and  $M_2$ 

$$C(q') = \frac{\sum [M_1 - \bar{M}_1] \cdot [M_2 - \bar{M}_2]}{\left\{ \sum [M_1 - \bar{M}_1]^2 \sum [M_2 - \bar{M}_2]^2 \right\}^{\frac{1}{2}}}$$
(8)

The vector q' is a optimal value as C(q') is maximum.

Assuming that a pair of corresponding reference points are x=343 and x'=205, we confirm the unique match of x=423 within the MPS (x'=205, 268, 331 and 396) in our approach. The map of HM correlation is shown in Fig.3

It is easily deduced that the match is x'=268 from Fig.3. The result is consistent with fact.

Next, within the MPS (x'=331, 396, 462 and 529), assuming that the corresponding reference points are x=504 and x'=331, we ascertain the match of x=586 by calculating the HM correlation.

The map of HM correlation is shown in Fig.4.

We can easily determine the match of x=586 is x'=396 from Fig.4. The result is also consistent with fact.

In order to reduce the computational complexity, dynamic limit search is adopted in the paper. Take matching point q and the obtained corresponding point q' as a new pair of reference points. Determine one of the feature points nearest to the new corresponding reference point in the left (or right) image among the residual feature points. Take this feature point as the next matching point. Adopt the introduced matching strategies until all feature points in the left (or right) image have been processed.



Fig.3 HM correlation between the feature point (x=423) in the left image and points in the right image



Fig.4 HM correlation between the feature point (*x*=586) in the left image and points in the right image

# EXPERIMENTAL RESULTS

Two pairs of actual stereo images (plaster lotus flower model and deflection yoke) are used for matching and 3-D restoration to test whether the algorithm developed is valid or not. The stereo images were captured by Nikon (Coolpix950) digital camera (1600×1200 pixels). The stereo images of the plaster lotus flower model are shown in Fig.5.

Adopt the algorithm recommended to ascertain correspondences. 3-D reconstruction is done by matching results with linear imaging geometrical model. The result of restoration is shown in Fig.6.

The shape of the 3-D reconstruction being coincident with that of the actual plaster lotus flower model from Fig.6 proved that the algorithm was valid. Next, stereo images of deflection yoke with irregular shape are shown in Fig.7.

In the same way, we use the matching strategies proposed to ascertain the correspondences of



Fig.5 Stereo images of plaster lotus flower model (a) Left image; (b) Right image



Fig.6 Result of 3-D restoration on plaster lotus flower model

stereo images. 3-D reconstruction was done by matching results with the linear imaging geometrical model. The result of restoration is shown in Fig.8.

The shape of the 3-D reconstruction being also coincided with that of the deflection yoke from Fig.8 proved that the matching algorithm was valid, too.

A standard piece (SP) with given 3-D coordinates (size:  $50 \times 50 \times 50 \text{ mm} \pm 10 \mu \text{m}$ ) was used for matching and 3-D reconstruction to further test the accuracy of the algorithm. Nikon camera was utilized to shoot a pair of images in two different locations. The captured images are shown in Fig.9. The distance between the camera and the SP was about 350 mm; the inclination of optic axes was about 18°.

Determine matches of stereo images with our approach. 3-D reconstruction was also done with the matching results with the linear imaging geometrical model. The absolute differences between the result of restoration and the actual 3-D coordinates of the SP is shown in Table 2.



**Fig.7** Stereo images of deflection yoke (a) Left image; (b) Right image



Fig.8 Result of 3-D reconstruction on deflection yoke

The result of restoration being almost identical with the actual coordinates proved that the algorithm was credible.

#### CONCLUSIONS

Since gray correlation describes gray similarity of the feature points region between the left and right images unilaterally, unique correct correspondence cannot be obtained only by use of gray correlation matching technique. However, gray correlation matching technique can be used to extract multi-peak feature points with gray correlation coefficients less than certain range of maximum coefficient as a coarse matching set, which is called MPS.

Utilizing a pair of known corresponding reference points, we could easily get unique correspondence by calculating the correlation of HM from the MPS.

Taking matching point and its corresponding point as a new pair of reference points, we could easily determine the reference difference and the HM. The matching efficiency is improved greatly.

It was proved that the proposed algorithm is efficacious and precise by 3-D reconstruction on two pairs of actual natural images and an object with given 3-D coordinates.



(a) Left image; (b) Right image

 Table 2 Absolute differences of 3-D coordinates between

 the SP and the result of 3-D reconstruction (mm)

	x	у	Ζ
Max.	0.2321	0.0763	0.3236
Min.	$6.8467 \times 10^{-5}$	$1.0180 \times 10^{-4}$	$9.3265 \times 10^{-4}$
Average	$6.3531 \times 10^{-6}$	$-5.4660 \times 10^{-6}$	$8.468 \times 10^{-6}$

#### References

- Akutsu, T., 1994. Efficient and Robust Three-dimensional Pattern Matching Algorithm Using Hashing and Dynamic Programming Techniques. Proceedings of the Twenty-Seventh Annual Hawaii International Conference on System Sciences, Wailea, USA, 5:225-234.
- Alex, M., Vasilescu, O., Terzopoulos, D., 2003. Multilinear Subspace Analysis of Image Ensembles. Proceedings of 2003 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 2:93-99.
- Atallah, M.J., 2001. Faster image template matching in the sum of the absolute value of differences measure. *IEEE Trans. on Image Processing*, **10**(4):659-663.
- Brünig, M., Niehsen, W., 2001. Fast full-search block matching. *IEEE Trans. on Circuits and Systems for Video Technology*, **11**(2):241-247.
- Burt, P., Julesz, B., 1980. A disparity gradient limit for binocular fusion. *Science*, **208**:615-617.
- Grimson, W.E.L., 1985. Computational experiments with a feature based stereo algorithm. *IEEE Trans. on PAMI*, **7**(1):17-34.
- Guan, Y.P., Tong, L.S., Chen, N., 2003. A study of measurement on deflection yoke based on binocular stereo vision. ACTA ELECTRONICA SINCA, 31(9): 1382-1385 (in Chinese).
- Ho, W.P., Yip, R.K.K., 1996. A Dynamic Programming Approach for Stereo Line Matching with Structural Information. Proceedings of the 13th International Conference on Pattern Recognition, Vienna, Austria, 1: 791-794.
- Hu., X.P., Ahuja, N., 1994. Matching point features with ordered geometric, rigidity, and disparity constrains. *IEEE Trans. on PAMI*, **16**(10):1041-1049.
- Jeon, J., Kim, K., Kim, C., Ho, Y.S., 2001. A Robust Stereo-matching Algorithm using Multiple-baseline Cameras. IEEE Pacific Rim Conference on Communications, Computers and signal Processing, 1:263-266.
- Kanade, T., Okutomi, M., 1994. A stereo matching algorithm with an adaptive window: Theory and experime-

nt. IEEE Trans. on PAMI, 16(9):920-932.

- Kanade, T., Yamada, A., 2003. Multi-subregion Based Probabilistic Approach toward Pose-invariant Face Recognition. Proceedings of 2003 IEEE International Symposium on Computational Intelligence in Robotics Automation, Kobe, Japan, 2:954-959.
- Kim, J.N., Choi, T.S., 2000. A fast full-search motion-estimation algorithm using representative pixels and adaptive matching scan. *IEEE Trans. on Circuits* and Systems for Video Technology, **10**(7):1040-1048.
- Kittler, J., Christmas, W.J., Petrou, M., 1993. Probabilistic Relaxation for Matching Problems in Computer Vision. Proceedings of Fourth International Conference on Computer Vision, Berlin, Germany, p.666-673.
- Lew, M.S., Huang, T.S., Wong, K., 1994. Learning and feature selection in stereo matching. *IEEE Trans. on PAMI*, 16(9):869-875.
- Lu, Y.H., Kubik, K., Bennamoun, M., 1997. Stereo Image Matching Based on Probability Relaxation. Proceedings of IEEE on Speech and Image Technologies for Computing and Telecommunications, Brisbane, Australia, 1:315-318.
- Okutomi, M., Kanade, T., 1990. A Locally Adaptive Window for Signal Matching. Proceedings of the Third International Conference on Computer Vision, Osaka, Japan, p.190-199.
- Vasilescu, M.A.O., Terzopoulos, D., 2002. Multilinear Image Analysis for Facial Recognition. Proceedings of 16th International Conference on Pattern Recognition, 2:511-514.
- Wei, G.Q., Brauer, W., Hirzinger, G., 1998. Intensity- and gradient-based stereo matching using hierarchical Gaussian basis functions. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **20**(11): 1143-1160.
- Zitnick, C.L., Kanade, T., 2000. A cooperative algorithm for stereo matching and occlusion detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **22**(7):675-684.

Welcome visiting our journal website: *http://www.zju.edu.cn/jzus*Welcome contributions & subscription from all over the world
The editor would welcome your view or comments on any item in the journal, or related matters
Please write to: Helen Zhang, Managing Editor of JZUS
E-mail: jzus@zju.edu.cn Tel/Fax: 86-571-87952276