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Chaotic phenomenon and the maximum predictable time scale of observation series of urban hourly water consumption*

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Abstract: The chaotic characteristics and maximum predictable time scale of the observation series of hourly water consumption in Hangzhou were investigated using the advanced algorithm presented here is based on the conventional Wolf's algorithm for the largest Lyapunov exponent. For comparison, the largest Lyapunov exponents of water consumption series with one-hour and 24-hour intervals were calculated respectively. The results indicated that chaotic characteristics obviously exist in the hourly water consumption system; and that observation series with 24-hour interval have longer maximum predictable scale than hourly series. These findings could have significant practical application for better prediction of urban hourly water consumption.

Key words: Hourly water consumption series, Lyapunov exponent, Chaos, Maximum predictable time scale
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INTRODUCTION

The prediction of hourly water consumption is of crucial importance to optimal operation of urban water supply systems, and is always at the top of the agendas of urban water supply authorities. Although various methods have been produced over a long period of theoretical study (Lü *et al.*, 1998; Yang *et al.*, 2002; Zhou *et al.*, 2002), very few practical methods for prediction have yet been found. There are many reasons for the difficulty in application of practical predictive methods, but of particular importance is a lack of understanding of the characteristics of the observed series of hourly water consumption. Since the change of urban hourly water consumption is influenced by many factors such as economic activities, holidays and climate conditions (e.g. rainfall, sunshine/cloud time, temperature etc.), it is a nonlinear dynamic problem. It is widely accepted that the development of hourly water consumption time series consists of four components, i.e. a periodic component, a component containing intrinsic trends, a climatically determined component and a random component (Lü *et al.*, 1998; Zhou *et al.*, 2002). It was confirmed that the climate system is a kind of chaotic system (Tsonis and Elsner, 1988). Moreover the existence of climate influenced chaotic characteristics of other observed data series such as mine water discharge and dam monitoring data had also been confirmed (Han, 2001; Wang *et al.*, 1999). With regard to hourly water consumption time series influenced by climate; however, no study was carried out to verify the existence of chaotic characteristics.

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In this study, observed series of urban hourly water consumption were firstly confirmed as having chaotic characteristics. Due to the lack of long-term predictability of chaotic systems, the maximum predictable time scale for observed series of urban hourly water consumption was then identified to provide new insight into the prediction of hourly water consumption.

THEORY OF CHAOTIC IDENTIFICATION

Chaos and Lyapunov exponent

Chaotic phenomena are common in nonlinear dynamic systems, and were deterministic and very sensitive to initial conditions. Considering the sensitivity towards initial conditions, a chaotic system is short of long-term predictability, while short-time predictability does exist because of the system's determinism. A chaotic system was mainly identified by: testing Lyapunov exponents, evaluating fractal dimensions and analyzing Kolmogorov power spectra. Because of the explicit physical significance of Lyapunov exponents, in this study the chaotic characteristics of hourly water consumption series were identified using its largest Lyapunov exponent.

Lyapunov exponents measure the rate of divergence or convergence of two nearby initial points of a dynamical system. A positive Lyapunov exponent measures the average exponential divergence of two nearby trajectories whereas a negative Lyapunov exponent measures the average exponential convergence of two nearby trajectories. If a discrete nonlinear system is dissipative, a positive Lyapunov exponent confirms the existence of chaos. Table 1 gives the relation between Lyapunov exponents and system motion properties.

Table 1 Relation between Lyapunov exponent and system motion properties

Motion property	Lyapunov exponents
Constant	$L_{E_i} < 0,\ i = 1,2,\cdots,n$
Periodic	$L_{E_1} = 0, L_{E_i} < 0, i = 2,3,\cdots,n$
Quasi-periodic	$L_{E_1} = L_{E_2} = 0, L_{E_i} < 0, i = 3,4,\cdots,n$
Chaotic	$L_{E_1} > 0, L_{E_i}$ can be positive, negative or zero, i=2,3,…,n
Random	$L_{E_1} \to \infty, L_{E_i}$ can be arbitrarily assumed number, i=2,3,…,n

L_{E_1} represents the Lyapunov exponent of distance expansion rate, L_{E_2} represents the Lyapunov exponent of area expansion rate, L_{E_i} represents the Lyapunov exponent of i-dimensional expansion rate, and the unit is bits/time. L_{E_1} is the largest Lyapunov exponent to be discussed

Reconstruction of phase space

Reconstruction of phase space is a necessary precondition for calculating Lyapunov exponents of time series. For time series with a single variable, Packard *et al.*(1980) proposed a method to reconstruct phase space. A stagnancy time τ (an integer multiple of Δt) is selected by uniformly sampling from an observed time series where $x_i=x(t_i)$ with $t_i=t_0+i\Delta t$ and i=1,2,…,n. And an m-dimensional phase space ($m>d$, d is attractor dimension) can be constructed as follows:

$$\boldsymbol{X}=\begin{bmatrix}\boldsymbol{X}(t_1)\\ \boldsymbol{X}(t_2)\\ \vdots\\ \boldsymbol{X}(t_i)\\ \vdots\\ \boldsymbol{X}(t_{n'})\end{bmatrix}=\begin{bmatrix}x(t_1) & x(t_1+\tau) & \cdots & x(t_1+(m-1)\tau)\\ x(t_2) & x(t_2+\tau) & \cdots & x(t_2+(m-1)\tau)\\ \vdots & \vdots & \vdots & \vdots\\ x(t_i) & x(t_i+\tau) & \cdots & x(t_i+(m-1)\tau)\\ \vdots & \vdots & \vdots & \vdots\\ x(t_n-(m-1)\tau) & x(t_n-(m-2)\tau) & \cdots & x(t_n)\end{bmatrix} \tag{1}$$

where $\tau=k\Delta t$ (k=1,2,…) is the stagnancy time; $x(t_i)$ (i=1,2,…,n') represents a phase point in the m-dimensional phase space; $n'=n-(m-1)k$ and m is the embedding dimension.

According to Takens's (1981) embedding theory, when a d-dimensional attractor can be embedded in an m-dimensional ($m\geq 2d+1$) phase space, the geometric characteristics of the original attractor can be reconstructed, and the system's evolution rules can be studied.

During the reconstruction of phase space, the selection of stagnancy time τ has to ensure the relative independence of its sub-values, i.e. the autocorrelation coefficients of coordinates of phase space are chosen to be as small as possible. In the current calculation, the stagnancy time τ is selected when the first negative autocorrelation coefficients are reached.

IMPROVED ALGORITHM FOR THE LARGEST LYAPUNOV EXPONENT

There are many methods for calculating the largest Lyapunov exponent of chaotic systems, among which Wolf's method is frequently adopted because of its explicit physical meaning. However in this study it was found that there exist two main practical problems associated with the conventional Wolf method. The first one is that, during the conversion of old to new vectors, the new vectors have to fulfill two strict conditions: smaller vector length and smaller angle between the new and old vector. No specific ranges for the vector length and angle between the new and old vector are available in the literature. The other problem is that the stability of calculated results depends heavily on the quantity of data. Considering that the Lyapunov exponent tends to be stable when m is greater than or equal to the minimum embedding dimension, in this paper an improved algorithm for largest Lyapunov exponent is put forward to overcome the above problems. The optimal variables in the algorithm are the weight α applied to the length of new vector and the angle between new and old vectors; and the optimal objective is the stability of Lyapunov exponent within certain ranges. The weight α varies from 0 to 1 with an increment 0.05 for each search step. The approaches for the improved algorithm are given below:

(1) Reconstruct the phase space using the stagnancy time τ determined by minimizing the autocorrelation coefficients of phase space (Li and Liu, 2000) and using the minimum embedding dimension m_0 obtained by calculation of the fractal dimensions in the chaotic system (Wang and Chao, 1995);

(2) Take the original phase point $x(t_1)$ as the initial point, then select the point $x(t_1)'$ nearest to $x(t_1)$ from the rest; and the two points form the original vector denoted as $\boldsymbol{V}_1$ with length $L_1(t_1)$;

(3) Suppose the evolution time of the initial vector on the trajectories of the system as τ; the evolution vector $\boldsymbol{V}_1'$ is derived, and the corresponding end points are $x(t_1+\tau)$ and $x(t_1+\tau)'$. Then calculate the length of $\boldsymbol{V}_1'$ which is marked as $L_1'\cdot(t_1+\tau)$, hence obtaining $\lambda_1=\frac{1}{\tau}\log_2(L_1'/L_1)$;

(4) Set $x(t_1+\tau)$ as a new basic point; select a new vector denoted as $\boldsymbol{V}_2$ to replace $\boldsymbol{V}_1$ and calculate its length as L_2. Considering the requirements that $\boldsymbol{V}_2$ must have a shorter length and a smaller angle θ with $\boldsymbol{V}_1'$, $L_2=\alpha L_2\sin\theta+(1-\alpha)L_2\cos\theta$ is deduced with α as the weight;

(5) Set $\boldsymbol{V}_2$ as a new initial vector, repeat (3) to get $\lambda_2=\frac{1}{\tau}\log_2(L_2'/L_2)$;

(6) Continue the above process until the end point of $\{x(t_i)\}$. Take a mathematical average of the exponential growth rates λ_i (i=1,2,…,N) as the estimated value of the Lyapunov exponent, i.e. $L_{E_1}=\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\tau}\log_2(L_i'/L_i)$, where N represents the total step number of evolution, and L_E represents the changes of information per unit time;

(7) Increase the embedding dimension from m to m_0+5, and repeat (3)–(5);

(8) Take 0.05 as the step length of search, and search α within the range [0,1]. Repeat (2)–(7), examine the stability of the largest Lyapunov exponent within $m\in[m_0, m_0+5]$ with different α. The standard deviation *std* is considered as an indicator of the stability of the results, and the mathematical average of the Lyapunov exponents corresponding to various minimum standard deviations min(*std*) is taken as the largest Lyapunov exponent of the time series.

MAXIMUM PREDICTABLE TIME SCALE

From Table 1, a positive largest Lyapunov exponent of time series indicates dissipative trajectories with bifurcation and multi-periodicity so that the time series cannot be predicted in the long-term. Its maximum predictable time scale T_f, however, can be estimated using the following relationship with the maximum Lyapunov exponent:

$$T_f = \frac{1}{L_{E_1}} \tag{2}$$

Here T_f has the same unit as Δt.

Huang (2000) pointed out that within the range of the maximum predictable time scale T_f, the systematic error of prediction is insensitive to the variations of prediction step length; while outside the range, the systematic error can be greatly magnified. Therefore T_f is defined as an index for measuring the predictability of chaotic systems (Huang, 2000).

APPLICATION

Basic data

Using the above method, analysis and calculation of the largest Lyapunov exponent were carried out for the hourly water consumption series (8760 observation points) in 1997 and daily maximum and minimum hourly water consumption series from 1997 to 1998 in Hangzhou (from the averaged hourly water consumption in Fig.2, we can find that the maximum water consumption time in Hangzhou is at 9:00 am, and that the minimum water consumption time is at 4:00 am). Hangzhou lies in the southern part of the Changjiang River Delta and the lower reach of the Qiantang River, the climate here belongs to the Semitropics Wet Season Wind Climate Area with four distinct seasons. Hangzhou is warm and damp and has plenty of rain and average annual rainfall of 1453 mm. Its average annual temperature is 28.6 °C in summer, and 3.8 °C in winter. The period without frost lasts 230 to 260 days. At the end of 1999, the downtown area was 683 km^2, the urban population had reached 1752700, and the number of tourists was 250 000 000 person-time per year. Groundwater resource is little exploited, and the city is supplied with surface water. Main urban water consumers include the industrial, domestic, municipal engineering and agricultural sectors (although agriculture water consumption is comparatively small). Domestic usage accounts for the greatest overall consumption, amounting to 55% of the total. The observed series of water consumption are shown in Figs.1, 3, 4.

Lyapunov exponents of observed series of hourly water consumption and the maximum predictable time scale

MATLAB language was adopted for the calculation program. The Lyapunov exponents of the hourly water consumption series in 1997, the observed series of water consumption at 9:00 am and 4:00 am and the calculated results of the maximum mum predictable time scale are shown in Tables 2, 3, showing that:

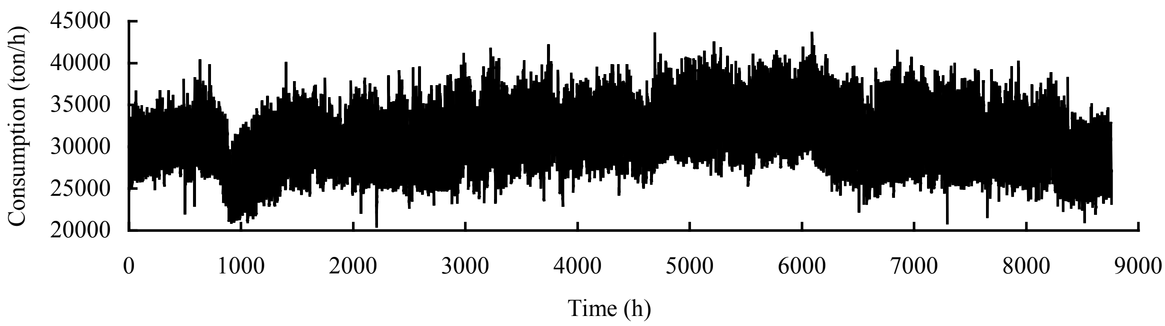

Fig.1 Time series of hourly water consumption in 1997

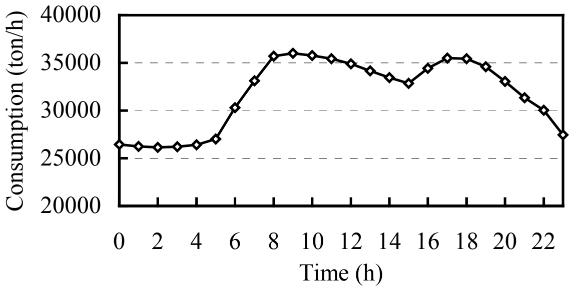

Fig.2 Hourly water consumption profile (averaged from January 1997 to December 1998)

(1) When the embedding dimension m is greater than the minimum embedding dimension m_0, the Lyapunov exponent of each series all demonstrate good stability. The maximum range of exponents is within 0.003, and the data accuracy can meet the requirement of chaos identification and determination of the maximum predictable time scale;

(2) The calculated Lyapunov exponents of the observed series of hourly water consumption were

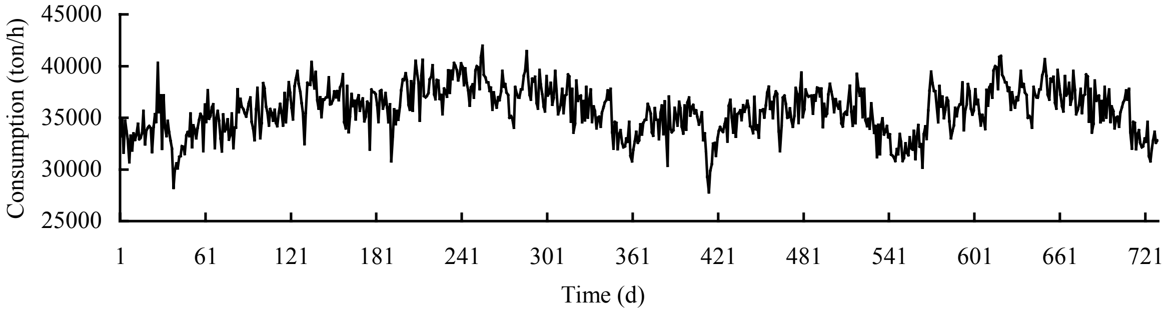

Fig.3 Time series of water consumption observed at 9:00 am

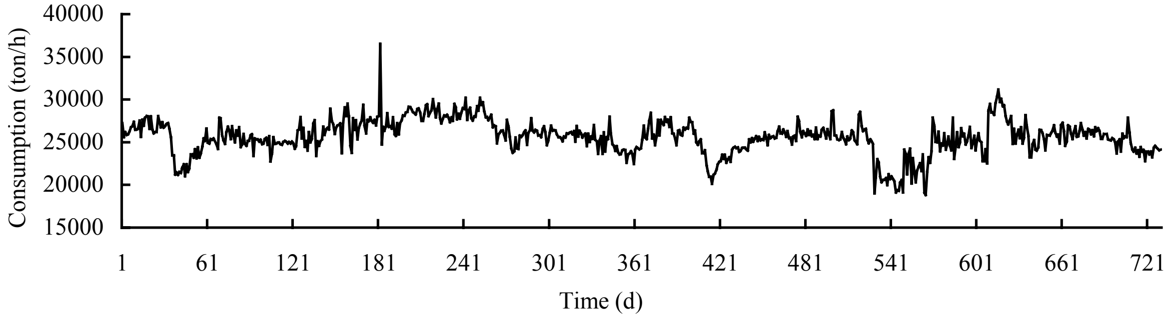

Fig.4 Time series of water consumption observed at 4:00 am

Table 2 Lyapunov exponent of each observed time series of hourly water consumption

Observed time series of hourly water consumption			Observed time series of water consumption at 9:00 am each day			Observed time series of water consumption at 4:00 am each day		
m	N	Lyapunov exponent	m	N	Lyapunov exponent	m	N	Lyapunov exponent
12	1232	0.0097	12	70	0.02167	10	370	0.00935
13	345	0.0127	13	259	0.01521	11	220	0.01495
14	465	0.0127	14	160	0.01613	12	552	0.01302
15	2724	0.0127	15	179	0.01431	13	192	0.01279
16	675	0.0127	16	217	0.01503	14	178	0.01456
17	2119	0.0127	17	203	0.01527	15	411	0.01486
18	798	0.0127	18	189	0.01543	16	397	0.01294

Note: N represents the total steps of phase space evolution. Due to the limitation of space, the above table only provides the Lyapunov exponents corresponding to the embedding dimensions m from m_0-1 to m_0+5

more stable than those at 4:00 am and 9:00 am each day, which shows that the improved algorithm still has certain degree of data dependency, but was a significant improvement on Wolf (1985)'s method. The authors could not get stable results for the above hourly water consumption series at 4:00 am and 9:00 am using Wolf's method;

(3) The observed series with the same properties such as at 4:00 am and 9:00 am had approximate Lyapunov exponent values. According to chaotic theory, the magnitude of the Lyapunov exponent reflects the degree of chaos within a certain time scale. It can be estimated that the chaos of the observed time series of water consumption at 9:00 am was greater than that at 4:00 am. On the contrary, the maximum predictable time scale of the 4:00 am time series (72.2 days) was greater than that of the 9:00 am series (65.7 days). In addition, the Lyapunov exponent of the observed time series of hourly water consumption with one-hour intervals was smaller than the results of that with 24-hour intervals. Meanwhile, because the interval of series sampling (1 hour) is shorter, the corresponding maximum predictable time scale (3.28 days) was not very long.

Figs.5a–5c display the trajectories in three-di-

Table 3 Chaotic parameter of each observed time series of hourly water consumption

Observed time series	Relevant parameters	Largest Lyapunov exponent	Maximum predictable scale
Observed time series of hourly water consumption	τ= 7, m_0=13, α=0.60	0.0127	78.7 hours (3.28 days)
Observed time series of water consumption at 9:00 am each day	τ= 5, m_0=13, α=0.65	0.01523	65.7 days
Observed time series of water consumption at 4:00 am each day	τ= 6, m_0=11, α=0.65	0.01385	72.2 days

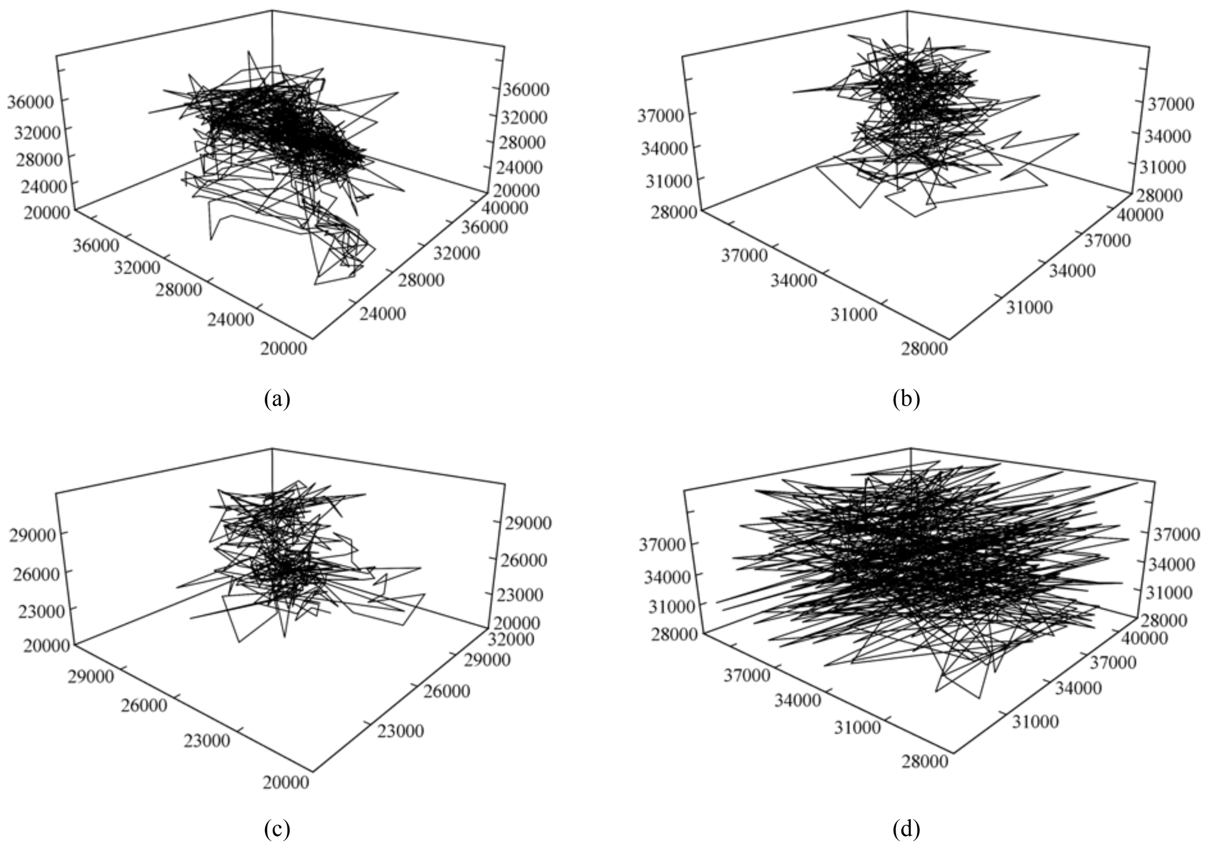

Fig.5 Trajectories in three-dimensional phase space

mensional phase space representing respectively the observed data of continuous hourly water consumption, and the observed data of water consumption at 9:00 am and at 4:00 am. For comparison, the trajectory in three-dimensional phase space of the white noise sequence is presented in Fig.5d. Figs.5c and 5d clearly show differences between the two trajectories.

CONCLUSION

1. Along with the components of periodicity, trend and randomness known as usual, the calculated largest positive Lyapunov exponents show that a distinct chaotic component exists in the observed time series of hourly water consumption.

2. Lack of long-term predictability of the chaotic system determines that urban hourly water consumption cannot be predicted for a long period. Different hourly water consumption time series have different maximum predictable scales. It is fairly obvious that the water consumption time series with 24-hour intervals have longer maximum predictable scale than that of one-hour intervals, which has practical significance for better prediction of urban hourly water consumption.

3. Because of the existence of a chaotic component, the research for prediction of urban hourly water consumption cannot only be limited to a consideration of its intrinsic trend and periodicity. It is a meaningful and feasible initiative to introduce the relevant theories of nonlinear dynamic systems to the field of hourly water consumption prediction, and there is considerable scope for further research work in this area.

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