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Robust H_∞ output feedback control for a class of uncertain Lur'e systems with time-delays*

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Abstract: In this work, the analysis of robust stability and design of robust H_∞ output feedback controllers for a class of Lur'e systems with both time-delays and parameter uncertainties were studied. A robust H_∞ output feedback controller based on Linear Matrix Inequalities (LMIs) was developed to guarantee the robust stability and H_∞ performance of the resultant closed-loop system. The presented design approach is based on the application of descriptor model transformation and Park's inequality for the bounding of cross terms and is expected to be less conservative compared to reported design methods. Finally, illustrative examples are advanced to demonstrate the superiority of the obtained method.

Key words: Lur'e systems, Robust H_∞ control, Linear Matrix Inequality (LMI)
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INTRODUCTION

Control of delay systems has been a topic of recurring interest over the past decades since time delays are often the main causes for instability and poor performance of systems and encountered in various engineering systems such as chemical process, long transmission lines in pneumatic systems, and so on (Hale, 1977; 1993). During recent years, a large amount of attention has been paid to the problem of stabilization of linear systems and nonlinear systems with time-delays. For the case of uncertain systems with time-delays, the method based on the concepts of quadratic stability and quadratic stabilizability has been shown to be effective in dealing with these problems in both continuous and discrete contexts, some sufficient conditions in the form of the GBRL (generalized bounded real lemma) are derived (Yu and Chen, 1997; Yu and Chu, 1999; Su *et al*., 1997).

On the other hand, H_∞ control problem has attracted much interest in the past decades. One of its main advantages is that it is insensitive to exact knowledge of the statistical characteristics of noise signals. Choi and Chung (1997) developed controller design method for linear systems with time-variant and time-invariant state delays, respectively, both based on the LMI approach. Guo (2002) studied the problem of H_∞ output feedback control for time-delay systems with nonlinear and parametric uncertainties and derived some sufficient conditions based on GBRL and LMI technology. Unfortunately, all the proposed methods of

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robust H_∞ control for time-delay systems are conservative. The main source of conservatism, on the one hand, that is caused by the distributed nature of delay which has not been successfully tackled; on the other hand, the treatment of norm-bounded uncertainties as an additional disturbance (Fu *et al.*, 1992) or the polytopic uncertainty via a single Lyapurov function (Choi and Chung, 1997; Guo, 2002) leads to conservative results. Recently, a new approach to H_∞ filtering was introduced (Fridman and Shaked, 2001; Fridman *et al.*, 2003). This approach applies a Lyapurov-Krasovskii functional and is based on an equivalent descriptor model and deriving a bounded-real lemma (BRL) for the corresponding adjoint system; the derived results have less conservatism. However, due to the difficulty in dealing with the problem of H_∞ output feedback control, to the best of our knowledge, the problem of H_∞ output feedback control for a class of uncertain Lur'e systems with time-delays has not been fully investigated yet.

In this work, the problem of H_∞ output feedback control was studied for a class of uncertain Lur'e systems with time-delays based on the idea of Fridman *et al.*(2003). The nonlinear terms appearing in the uncertain Lur'e delay system lead to difficulty in designing a robust H_∞ output feedback controller. For simplicity, if some assumptions are made on the nonlinear terms, the sufficient conditions for the existence of delay dependent robust H_∞ output feedback control in terms of LMIs can be obtained; which guarantees the H_∞ performance of the resultant closed-loop system, and the H_∞ output feedback controllers, can be easily obtained by using LMI Toolbox. Compared with the results (Guo, 2002), the conservatism is obviously lessened. Finally, illustrative examples are advanced to demonstrate the superiority of the obtained method.

Throughout this note, for real symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, the notation $\boldsymbol{X}\geq\boldsymbol{Y}$ (respectively, $\boldsymbol{X}>\boldsymbol{Y}$) means that the matrix $\boldsymbol{X}-\boldsymbol{Y}$ is positive-semidefinite (respectively, positive-definite). $\boldsymbol{A}^{\mathrm{T}}$ denotes the transpose of $\boldsymbol{A}$. $\boldsymbol{A}(\cdot)$ denotes time-variant matrix. $(\cdot)$ denotes the variable of the time-variant matrix. $L_2[0, \infty)$ is the space of square integrable functions over $[0,\infty)$. $C_\tau=C([-\tau,0], \mathrm{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau,0]$ into R^n with topology of uniform convergence. $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm.

SYSTEM DESCRIPTION AND DEFINITIONS

Consider the following uncertain Lur'e systems with time-delays described by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= (\boldsymbol{A}_0+\Delta\boldsymbol{A}_0(x,t))\boldsymbol{x}(t)+\sum_{i=1}^{k}(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i(x,t))\cdot \\ &\quad \boldsymbol{x}(t-h_i(t))+\boldsymbol{E}_{10}f_1(\sigma(t))+\sum_{i=1}^{k}\boldsymbol{E}_{1i}f_{1i}(\sigma(t-h_i(t))) \\ &\quad +(\boldsymbol{B}_{10}+\Delta\boldsymbol{B}_{10}(x,t))\boldsymbol{w}(t)+\sum_{i=1}^{k}(\boldsymbol{B}_{1i}+\Delta\boldsymbol{B}_{1i}(x,t))\cdot \\ &\quad \boldsymbol{w}(t-h_i(t))+(\boldsymbol{B}_{20}+\Delta\boldsymbol{B}_{20}(x,t))\boldsymbol{u}(t) \\ &\quad +\sum_{i=1}^{k}(\boldsymbol{B}_{2i}+\Delta\boldsymbol{B}_{2i}(x,t))\boldsymbol{u}(t-g_i(t))\end{aligned} \tag{1}$$

$$\begin{aligned}\boldsymbol{z}(t) &= (\boldsymbol{C}_{10}+\Delta\boldsymbol{C}_{10}(x,t))x(t)+\sum_{i=1}^{k}(\boldsymbol{C}_{1i}+\Delta\boldsymbol{C}_{1i}(x,t))\cdot \\ &\quad \boldsymbol{x}(t-h_i(t))+\boldsymbol{E}_{20}f_2(\sigma(t))+\sum_{i=1}^{k}\boldsymbol{E}_{2i}f_{2i}(\sigma(t-h_i(t))) \\ &\quad +(\boldsymbol{D}_{10}+\Delta\boldsymbol{D}_{10}(x,t))\boldsymbol{w}(t)+\sum_{i=1}^{k}(\boldsymbol{D}_{1i}+\Delta\boldsymbol{D}_{1i}(x,t))\cdot \\ &\quad \boldsymbol{w}(t-h_i(t))+(\boldsymbol{D}_{20}+\Delta\boldsymbol{D}_{20}(x,t))\boldsymbol{u}(t) \\ &\quad +\sum_{i=1}^{k}(\boldsymbol{D}_{2i}+\Delta\boldsymbol{D}_{2i}(x,t))\boldsymbol{u}(t-g_i(t))\end{aligned} \tag{2}$$

$$\begin{aligned}\boldsymbol{y}(t) &= (\boldsymbol{C}_{20}+\Delta\boldsymbol{C}_{20}(x,t))\boldsymbol{x}(t)+\sum_{i=1}^{k}(\boldsymbol{C}_{2i}+\Delta\boldsymbol{C}_{2i}(x,t))\cdot \\ &\quad \boldsymbol{x}(t-h_i(t))+\boldsymbol{E}_{30}f_3(\sigma(t))+\sum_{i=1}^{k}\boldsymbol{E}_{3i}f_{3i}(\sigma(t-h_i(t))) \\ &\quad +(\boldsymbol{D}_{30}+\Delta\boldsymbol{D}_{30}(x,t))\boldsymbol{w}(t)+\sum_{i=1}^{k}(\boldsymbol{D}_{3i}+\Delta\boldsymbol{D}_{3i}(x,t))\cdot \\ &\quad \boldsymbol{w}(t-h_i(t))\end{aligned} \tag{3}$$

$$\sigma(t)=\boldsymbol{C}\boldsymbol{x}(t),\ \boldsymbol{x}(t)=0,\ \boldsymbol{w}(t)=0,\ \boldsymbol{u}(t)=0,$$
$$t\in[-\max((h_j(t),g_j(t)),0]$$

The system Eqs.(1), (2) and (3) is denoted as Σ_Δ, where $\boldsymbol{x}(t)\in\mathrm{R}^n$ is the state vector, $\boldsymbol{u}(t)\in\mathrm{R}^m$ is control input vector, $\boldsymbol{w}(t)\in\mathrm{R}^p$ is the disturbance input vector from $L_2[0,\infty)$, $\boldsymbol{y}(t)\in\mathrm{R}^r$ is the measurement vector, $\boldsymbol{z}(t)\in\mathrm{R}^q$ is controlled output vector. $\boldsymbol{C}$, $\boldsymbol{A}_i$, $\boldsymbol{B}_{1i}$, $\boldsymbol{B}_{2i}$, $\boldsymbol{C}_{1i}$, $\boldsymbol{C}_{2i}$, $\boldsymbol{D}_{1i}$, $\boldsymbol{D}_{2i}$, $\boldsymbol{D}_{3i}$, $\boldsymbol{E}_{1i}$, $\boldsymbol{E}_{2i}$ and $\boldsymbol{E}_{3i}$ (i=0,1,2,…k) are known real constant matrices with appropriate dimensions. $\Delta\boldsymbol{A}_i(\cdot)$, $\Delta\boldsymbol{B}_{1i}(\cdot)$, $\Delta\boldsymbol{B}_{2i}(\cdot)$, $\Delta\boldsymbol{C}_{1i}(\cdot)$, $\Delta\boldsymbol{C}_{2i}(\cdot)$, $\Delta\boldsymbol{D}_{1i}(\cdot)$, $\Delta\boldsymbol{D}_{2i}(\cdot)$, and $\Delta\boldsymbol{D}_{3i}(\cdot)$ (i=0,1,2,…k) are time-variant matrices representing norm-bounded parameter uncertainties, and are assumed to be of the following form,

$$\begin{bmatrix} \Delta\boldsymbol{A}_0 & \Delta\boldsymbol{A}_1 & \Delta\boldsymbol{B}_{10} & \Delta\boldsymbol{B}_{11} & \Delta\boldsymbol{B}_{20} & \Delta\boldsymbol{B}_{21} \\ \Delta\boldsymbol{C}_{10} & \Delta\boldsymbol{C}_{11} & \Delta\boldsymbol{D}_{10} & \Delta\boldsymbol{D}_{11} & \Delta\boldsymbol{D}_{20} & \Delta\boldsymbol{D}_{21} \\ \Delta\boldsymbol{C}_{20} & \Delta\boldsymbol{C}_{21} & \Delta\boldsymbol{D}_{30} & \Delta\boldsymbol{D}_{31} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_{11} \\ \boldsymbol{G}_{21} \\ \boldsymbol{G}_{31} \end{bmatrix} \boldsymbol{F}(x,t) \begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} & \boldsymbol{H}_{13} & \boldsymbol{H}_{14} & \boldsymbol{H}_{15} & \boldsymbol{H}_{16} \end{bmatrix} \tag{4}$$

where $\boldsymbol{G}_{11}$, $\boldsymbol{G}_{21}$, $\boldsymbol{G}_{31}$, $\boldsymbol{H}_{11}$, $\boldsymbol{H}_{12}$, $\boldsymbol{H}_{13}$, $\boldsymbol{H}_{14}$, $\boldsymbol{H}_{15}$, and $\boldsymbol{H}_{16}$ are known real constant matrices with appropriate dimensions. The time-variant matrix $\boldsymbol{F}(x,t)$ with Lebesgue measurable elements satisfies

$$\boldsymbol{F}^{\mathrm{T}}(x,t)\boldsymbol{F}(x,t)\le I,\ \forall t. \tag{5}$$

$h_i(t)$ and $g_i(t)$ are unknown scalars denoting the delays in the state and control, respectively, and it is assumed that there exist positive numbers h, g, h_i, g_i and τ such that

$$\begin{aligned} &0\le h_i(t)\le h\le\tau;\quad \dot{h}_i(t)\le h_i<1; \\ &0\le g_i(t)\le g\le\tau;\quad \dot{g}_i(t)\le g_i<1 \end{aligned} \tag{6}$$

hold for all t, i=1,…,k. $\phi(t)$ is smooth vector-valued continuous initial function defined in the Banach space $\boldsymbol{C}_\tau$. In this paper, nonlinear terms are assumed to be of the following form

$$\boldsymbol{f}_j(\cdot)=\{\boldsymbol{f}_j(\sigma)\,|\,\boldsymbol{f}_j(0)=0,\ 0<\sigma\boldsymbol{f}_j(\sigma)\le\boldsymbol{K}_j\sigma^2(\sigma\ne0)\}\quad j=1,2,3$$

$$\boldsymbol{f}_{ji}(\cdot)=\{\boldsymbol{f}_{ji}(\sigma)\,|\,\boldsymbol{f}_{ji}(0)=0,\ 0<\sigma\boldsymbol{f}_{ji}(\sigma)\le\boldsymbol{K}_{ji}\sigma^2(\sigma\ne0)\}\quad i=1,2,\dots k \tag{7}$$

where $\boldsymbol{K}_j$ and $\boldsymbol{K}_{ji}$ are diagonal matrices composed of the elements of positive scalars.

Throughout this paper, we shall use the following concepts and introduce the following useful lemmas.

Definition 1 (The problem of robust H_∞ output feedback control) The uncertain Lur'e time-delay systems (Σ_Δ) is said to be robust H_∞ output feedback controllable if there exists a linear output feedback control law

$$\sum\nolimits_{spc}:\begin{cases} \dot{\boldsymbol{x}}_c=\boldsymbol{A}_c\boldsymbol{x}_c+\boldsymbol{B}_c\boldsymbol{y} \\ \boldsymbol{u}=\boldsymbol{C}_c\boldsymbol{x}_c \end{cases} \tag{8}$$

such that the resultant closed-loop system is not only robustly stable but also satisfies the following condition,

$$\sup_{0\ne\boldsymbol{w}(t),\boldsymbol{w}(t-h_1(t))\in L_2[0,\infty)}\left(\frac{\|\boldsymbol{z}(t)\|_2}{\|\boldsymbol{w}(t)\|_2+\|\boldsymbol{w}(t-h_1(t))\|_2}\right)<\gamma \tag{9}$$

for a given scalar $\gamma>0$, for all non-zero $\boldsymbol{w}(t)$, $\boldsymbol{w}(t-h_1(t))\in L_2[0,\infty)$ and for all admissible parameter uncertainties. In this case, Σ_{spc} is said to be a robust H_∞ output feedback control law for system (Σ_Δ).

Lemma 1 (Boyd *et al.*, 1994) Given vectors $\boldsymbol{x}$, $\boldsymbol{y}$, a positive definite symmetric matrix $\boldsymbol{R}$ with appropriate dimensions, we have

$$\pm2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}\le\boldsymbol{x}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x}+\boldsymbol{y}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{y}$$

Lemma 2 (Boyd *et al.*, 1994) Given matrices Θ, Γ and Ξ with appropriate dimensions and Θ is symmetric, then

$$\Theta+\Gamma\boldsymbol{F}(\delta)\Xi+(\Gamma\boldsymbol{F}(\delta)\Xi)^{\mathrm{T}}<0$$

for all $\boldsymbol{F}(\delta)$ satisfying $\boldsymbol{F}^{\mathrm{T}}(\delta)\boldsymbol{F}(\delta)\le I$, if and only if there exists a scalar $\varepsilon>0$ such that

$$\Theta+\varepsilon\Gamma\Gamma^{\mathrm{T}}+\varepsilon^{-1}\Xi^{\mathrm{T}}\Xi<0.$$

ROBUST H_∞ OUTPUT FEEDBACK CONTROL

In this section, the problem of robust H_∞ output feedback control for system Eqs.(1)–(3) is discussed. First, the sufficient condition for the existence of robust H_∞ output feedback control without parameter uncertainties is derived.

For simplicity and without loss of generality, we assume k=1. Define $\bar{\boldsymbol{x}}^{\mathrm{T}}=[\boldsymbol{x}^{\mathrm{T}}\ \boldsymbol{x}_c^{\mathrm{T}}]$, $\bar{\boldsymbol{f}}_1^{\mathrm{T}}=[\boldsymbol{f}_1^{\mathrm{T}}\ \boldsymbol{f}_2^{\mathrm{T}}]$, and $\bar{\boldsymbol{f}}_{11}^{\mathrm{T}}=[\boldsymbol{f}_{11}^{\mathrm{T}}(\sigma(t-h_1(t))\ \boldsymbol{f}_{31}^{\mathrm{T}}(\sigma(t-h_1(t))]$; the following augmented model ($\bar{\Sigma}$) can be derived from Eqs.(1), (2), (3) and (8),

$$\bar{\Sigma}:\begin{cases}\dot{\bar{\boldsymbol{x}}}=\bar{\boldsymbol{A}}_0\bar{\boldsymbol{x}}+\bar{\boldsymbol{A}}_1\bar{\boldsymbol{x}}(t-h_1(t))+\bar{\boldsymbol{A}}_2\bar{\boldsymbol{x}}(t-g_1(t))\\ \quad+\bar{\boldsymbol{B}}_0\boldsymbol{w}+\bar{\boldsymbol{B}}_1\boldsymbol{w}(t-h_1(t))+\bar{\boldsymbol{E}}_1\bar{\boldsymbol{f}}_1+\bar{\boldsymbol{E}}_{11}\bar{\boldsymbol{f}}_{11}\\ \boldsymbol{z}=\bar{\boldsymbol{C}}_0\bar{\boldsymbol{x}}+\bar{\boldsymbol{C}}_1\bar{\boldsymbol{x}}(t-h_1(t))+\bar{\boldsymbol{C}}_2\bar{\boldsymbol{x}}(t-g_1(t))\\ \quad+\boldsymbol{D}_{10}\boldsymbol{w}+\boldsymbol{D}_{11}\boldsymbol{w}(t-h_1(t))+\boldsymbol{E}_{20}\boldsymbol{f}_2+\boldsymbol{E}_{21}\boldsymbol{f}_{21}\end{cases}\tag{10}$$

where

$$\bar{\boldsymbol{A}}_0=\begin{bmatrix}\boldsymbol{A}_0 & \boldsymbol{B}_{20}\boldsymbol{C}_c\\ \boldsymbol{B}_c\boldsymbol{C}_{20} & \boldsymbol{A}_c\end{bmatrix},\ \bar{\boldsymbol{A}}_1=\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\\ \boldsymbol{B}_c\boldsymbol{C}_{21} & \boldsymbol{0}\end{bmatrix},$$

$$\bar{\boldsymbol{A}}_2=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{B}_{21}\boldsymbol{C}_c\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},\ \bar{\boldsymbol{B}}_0=\begin{bmatrix}\boldsymbol{B}_{10}\\ \boldsymbol{B}_c\boldsymbol{D}_{30}\end{bmatrix},\ \bar{\boldsymbol{B}}_1=\begin{bmatrix}\boldsymbol{B}_{11}\\ \boldsymbol{B}_c\boldsymbol{D}_{31}\end{bmatrix},$$

$$\bar{\boldsymbol{C}}_0=[\boldsymbol{C}_{10}\quad \boldsymbol{D}_{20}\boldsymbol{C}_c],\ \bar{\boldsymbol{C}}_1=[\boldsymbol{C}_{11}\quad \boldsymbol{0}],\ \bar{\boldsymbol{C}}_2=[\boldsymbol{0}\quad \boldsymbol{D}_{21}\boldsymbol{C}_c],$$

$$\bar{\boldsymbol{E}}_1=\begin{bmatrix}\boldsymbol{E}_{10} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}_c\boldsymbol{E}_{30}\end{bmatrix},\ \bar{\boldsymbol{E}}_{11}=\begin{bmatrix}\boldsymbol{E}_{11} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}_c\boldsymbol{E}_{31}\end{bmatrix}$$

An equivalent descriptor form representation of Eq.(10) is given by Fridman (2001),

$$\begin{aligned}&\dot{\bar{\boldsymbol{x}}}(t)=\xi(t)\\ &0=-\xi(t)+\sum_{i=0}^{2}\bar{\boldsymbol{A}}_i\bar{\boldsymbol{x}}(t)-\bar{\boldsymbol{A}}_1\int_{t-h_1(t)}^{t}\xi(s)\mathrm{d}s\\ &\quad-\bar{\boldsymbol{A}}_2\int_{t-g_1(t)}^{t}\xi(s)\mathrm{d}s+\bar{\boldsymbol{B}}_0\boldsymbol{w}+\bar{\boldsymbol{B}}_1\boldsymbol{w}(t-h_1(t))\\ &\quad+\bar{\boldsymbol{E}}_1\bar{\boldsymbol{f}}_1+\bar{\boldsymbol{E}}_{11}\bar{\boldsymbol{f}}_{11}\end{aligned}\tag{11}$$

Setting

$$\bar{\boldsymbol{B}}_H=[\lambda_1\bar{\boldsymbol{E}}_1\quad \lambda_{11}\bar{\boldsymbol{E}}_{11}\quad \boldsymbol{0}\quad \boldsymbol{0}],\bar{\boldsymbol{D}}_H=[\boldsymbol{0}\quad \boldsymbol{0}\quad \lambda_2\boldsymbol{E}_{20}\quad \lambda_{21}\boldsymbol{E}_{21}]$$

$$\bar{\boldsymbol{C}}_H^{\mathrm{T}}=[\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_1/\lambda_1\quad \boldsymbol{0}\quad \boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_2/\lambda_2\quad \boldsymbol{0}],$$

$$\bar{\boldsymbol{C}}_{1H}^{\mathrm{T}}=[\boldsymbol{0}\quad \boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_{11}/\lambda_{11}\quad \boldsymbol{0}\quad \boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_{21}/\lambda_{21}],$$

$$\begin{aligned}\boldsymbol{\alpha}^{\mathrm{T}}=[&\bar{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_1^{\mathrm{T}}/\lambda_1\quad \bar{\boldsymbol{x}}^{\mathrm{T}}(t-h_1(t))\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_{11}^{\mathrm{T}}/\lambda_{11}\\ &\bar{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_2^{\mathrm{T}}/\lambda_2\quad \bar{\boldsymbol{x}}^{\mathrm{T}}(t-h_1(t))\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}_{21}^{\mathrm{T}}/\lambda_{21}],\end{aligned}$$

$$\begin{aligned}\boldsymbol{\beta}^{\mathrm{T}}=[&\bar{\boldsymbol{f}}_1^{\mathrm{T}}(\sigma(t))/\lambda_1\quad \bar{\boldsymbol{f}}_{11}^{\mathrm{T}}(\sigma(t-h_1(t)))/\lambda_{11}\\ &\boldsymbol{f}_2^{\mathrm{T}}(\sigma(t))/\lambda_2\quad \boldsymbol{f}_{21}^{\mathrm{T}}(\sigma(t-h_1(t)))/\lambda_{21}].\end{aligned}$$

where λ_1, λ_2, λ_{11}, and λ_{21}, are positive scalars and

$$\bar{\boldsymbol{K}}_1=\begin{bmatrix}\boldsymbol{K}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{K}_3\end{bmatrix},\ \bar{\boldsymbol{K}}_2=\begin{bmatrix}\boldsymbol{K}_2 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},$$

$$\bar{\boldsymbol{K}}_{11}=\begin{bmatrix}\boldsymbol{K}_{11} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{K}_{31}\end{bmatrix},\ \bar{\boldsymbol{K}}_{21}=\begin{bmatrix}\boldsymbol{K}_{21} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}$$

The nonlinear terms described in Eq.(7) are equivalent to the following expression,

$$\boldsymbol{f}_j(\sigma)(\boldsymbol{f}_j(\sigma)-\boldsymbol{K}_j\boldsymbol{C}\boldsymbol{x}(t))\le 0$$

and

$$\boldsymbol{f}_{ji}(\sigma)(\boldsymbol{f}_{ji}(\sigma)-\boldsymbol{K}_{ji}\boldsymbol{C}\boldsymbol{x}(t))\le 0,\ j=1,2,3,\ i=1.\ \ldots,k$$

which implies that

$$\|\boldsymbol{f}_j(\sigma)\|^2\le\|\boldsymbol{K}_j\boldsymbol{C}\boldsymbol{x}(t)\|^2$$

and

$$\|\boldsymbol{f}_{ji}(\sigma)\|^2\le\|\boldsymbol{K}_{ji}\boldsymbol{C}\boldsymbol{x}(t)\|^2.\tag{12}$$

From inequality (12) and the above description, we can get

$$\|\boldsymbol{\alpha}(t)\|^2-\|\boldsymbol{\beta}(t)\|^2\ge 0\tag{13}$$

Introduce the following Lyapunov-Krasovskii functional for the system Eq.(11),

$$\begin{aligned}V(t)=&[\bar{\boldsymbol{x}}^{\mathrm{T}}(t)\ \ \xi^{\mathrm{T}}(t)]\boldsymbol{E}\boldsymbol{P}\begin{bmatrix}\bar{\boldsymbol{x}}(t)\\ \xi(t)\end{bmatrix}\\ &+\int_{-h_1(t)}^{0}\int_{t+\theta}^{t}\xi^{\mathrm{T}}(s)\,\boldsymbol{R}_1\xi(s)\mathrm{d}s\mathrm{d}\theta\\ &+\int_{-g_1(t)}^{0}\int_{t+\theta}^{t}\xi^{\mathrm{T}}(s)\boldsymbol{R}_2\xi(s)\mathrm{d}s\mathrm{d}\theta\\ &+\int_{t-h_1(t)}^{t}\bar{\boldsymbol{x}}^{\mathrm{T}}(s)\bar{\boldsymbol{C}}_1^{\mathrm{T}}\sum_{i=1}^{4}\boldsymbol{Q}_{1i}\bar{\boldsymbol{C}}_1\bar{\boldsymbol{x}}(s)\mathrm{d}s\\ &+\int_{t-g_1(t)}^{t}\bar{\boldsymbol{x}}^{\mathrm{T}}(s)\bar{\boldsymbol{C}}_2^{\mathrm{T}}\sum_{i=1}^{4}\boldsymbol{Q}_{2i}\bar{\boldsymbol{C}}_2\bar{\boldsymbol{x}}(s)\mathrm{d}s\\ &+\int_{0}^{t}(\|\boldsymbol{\alpha}(s)\|^2-\|\boldsymbol{\beta}(s)\|^2)\mathrm{d}s\end{aligned}\tag{14}$$

where

$$\boldsymbol{E}=\begin{bmatrix}\boldsymbol{I}_n & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},\quad \boldsymbol{P}=\begin{bmatrix}\boldsymbol{P}_1 & \boldsymbol{0}\\ \boldsymbol{P}_2 & \boldsymbol{P}_3\end{bmatrix},$$

$$\boldsymbol{P}_1>0,\quad \boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_3\in \mathrm{R}^{2n\times 2n},\quad \boldsymbol{R}_1,\boldsymbol{R}_2>0$$

Then, the following Lemma can be obtained.

Lemma 3 Consider the system Eqs.(1), (2) and (3), for a given scalar $\gamma>0$, the inequality Eq.(9) is satisfied for all nonzero $\boldsymbol{w}(t)$, $\boldsymbol{w}(t-h_1(t))\in L_2[0,\infty)$, if there exist matrices $\boldsymbol{P}_1>0$, $\boldsymbol{P}_2$, $\boldsymbol{P}_3$, positive definite symmetric matrices $\overline{\boldsymbol{R}}_1=\boldsymbol{R}_1^{-1}$, $\overline{\boldsymbol{R}}_2=\boldsymbol{R}_2^{-1}$, $\overline{\boldsymbol{X}}_{1j}=\boldsymbol{X}_{1j}^{-1}$, $\boldsymbol{X}_{2j}^{-1}=\overline{\boldsymbol{X}}_{2j}$, j=1,…,4, and positive scalars λ_1, λ_2, λ_{11}, λ_{21} such that satisfy the following linear matrix inequality (LMI), as shown in Eq.(15),

$$\boldsymbol{M}=\begin{bmatrix}
\Psi & \boldsymbol{P}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0}\\ \overline{\boldsymbol{B}}_0\end{bmatrix} & \boldsymbol{P}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0}\\ \overline{\boldsymbol{B}}_1\end{bmatrix} & \boldsymbol{P}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0}\\ \overline{\boldsymbol{B}}_H\end{bmatrix} & \begin{bmatrix}\overline{\boldsymbol{C}}_0^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix} & h_1\boldsymbol{P}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0}\\ \overline{\boldsymbol{A}}_1\end{bmatrix} & g_1\boldsymbol{P}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0}\\ \overline{\boldsymbol{A}}_2\end{bmatrix} & \Phi_1\\
\begin{bmatrix}\boldsymbol{0} & \overline{\boldsymbol{B}}_0^{\mathrm{T}}\end{bmatrix}\boldsymbol{P} & -\gamma^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_{10}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \Phi_2\\
\begin{bmatrix}\boldsymbol{0} & \overline{\boldsymbol{B}}_1^{\mathrm{T}}\end{bmatrix}\boldsymbol{P} & \boldsymbol{0} & -\gamma^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{D}_{11}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \Phi_3\\
\begin{bmatrix}\boldsymbol{0} & \overline{\boldsymbol{B}}_H^{\mathrm{T}}\end{bmatrix}\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I} & \overline{\boldsymbol{D}}_H^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \Phi_4\\
\begin{bmatrix}\overline{\boldsymbol{C}}_0 & \boldsymbol{0}\end{bmatrix} & \boldsymbol{D}_{10} & \boldsymbol{D}_{11} & \overline{\boldsymbol{D}}_H & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
h_1\begin{bmatrix}\boldsymbol{0} & \overline{\boldsymbol{A}}_1^{\mathrm{T}}\end{bmatrix}\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -h_1\boldsymbol{R}_1 & \boldsymbol{0} & \boldsymbol{0}\\
g_1\begin{bmatrix}\boldsymbol{0} & \overline{\boldsymbol{A}}_2^{\mathrm{T}}\end{bmatrix}\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -g_1\boldsymbol{R}_2 & \boldsymbol{0}\\
\Phi_1^{\mathrm{T}} & \Phi_2^{\mathrm{T}} & \Phi_3^{\mathrm{T}} & \Phi_4^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \Phi_5
\end{bmatrix} \quad (15)$$

where

$$\Psi=\boldsymbol{P}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ \sum_{i=0}^{2}\overline{\boldsymbol{A}}_i & -\boldsymbol{I}\end{bmatrix}+\begin{bmatrix}\boldsymbol{0} & \sum_{i=0}^{2}\overline{\boldsymbol{A}}_i^{\mathrm{T}}\\ \boldsymbol{I} & -\boldsymbol{I}\end{bmatrix}\boldsymbol{P}+\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{U}_1\end{bmatrix},$$

$$\Phi_1=\begin{bmatrix}\overline{\boldsymbol{C}}_0^{\mathrm{T}} & \overline{\boldsymbol{C}}_0^{\mathrm{T}} & \overline{\boldsymbol{C}}_0^{\mathrm{T}} & \overline{\boldsymbol{C}}_1^{\mathrm{T}} & \cdots & \overline{\boldsymbol{C}}_1^{\mathrm{T}} & \overline{\boldsymbol{C}}_2^{\mathrm{T}} & \cdots & \overline{\boldsymbol{C}}_2^{\mathrm{T}} & \overline{\boldsymbol{C}}_H^{\mathrm{T}} & \overline{\boldsymbol{C}}_{1H}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & h_1\boldsymbol{I} & g_1\boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0}\end{bmatrix}$$

$$\Phi_2=[\underbrace{\boldsymbol{0}\ \boldsymbol{0}\ \cdots\cdots\boldsymbol{0}}_{17}\ \ \boldsymbol{D}_{10}^{\mathrm{T}}\ \ \boldsymbol{D}_{10}^{\mathrm{T}}\ \ \underbrace{\boldsymbol{0}\ \cdots\cdots\ \boldsymbol{0}}_{4}],\ \Phi_3=[\underbrace{\boldsymbol{0}\ \boldsymbol{0}\ \cdots\cdots\boldsymbol{0}}_{19}\ \ \boldsymbol{D}_{11}^{\mathrm{T}}\ \ \boldsymbol{D}_{11}^{\mathrm{T}}\ \ \boldsymbol{0}\ \ \boldsymbol{0}],\ \Phi_4=[\underbrace{\boldsymbol{0}\ \boldsymbol{0}\ \cdots\cdots\boldsymbol{0}}_{21}\ \ \overline{\boldsymbol{D}}_H^{\mathrm{T}}\ \ \overline{\boldsymbol{D}}_H^{\mathrm{T}}],$$

$$\Phi_5=\mathrm{diag}\{-(1-h_1)\boldsymbol{X}_{11},-(1-g_1)\boldsymbol{X}_{21},-\boldsymbol{I},-\boldsymbol{I},\overline{\boldsymbol{X}}_{11},\overline{\boldsymbol{X}}_{12},\overline{\boldsymbol{X}}_{13},\overline{\boldsymbol{X}}_{14},-\boldsymbol{I},\overline{\boldsymbol{X}}_{21},\overline{\boldsymbol{X}}_{22},\overline{\boldsymbol{X}}_{23},\overline{\boldsymbol{X}}_{24},-\boldsymbol{I},-\boldsymbol{I},-h_1\overline{\boldsymbol{R}}_1,$$
$$-g_1\overline{\boldsymbol{R}}_2,-(1-h_1)\boldsymbol{X}_{12},-(1-g_1)\boldsymbol{X}_{22},-(1-h_1)\boldsymbol{X}_{13},-(1-g_1)\boldsymbol{X}_{23},-(1-h_1)\boldsymbol{X}_{14},-(1-g_1)\boldsymbol{X}_{24}\}$$

Proof Note that if inequality Eq.(15) holds, from the reported results (Yu and Chen, 1997; Yu and Chu, 1999), we can easily obtain that the system ($\overline{\Sigma}$) is asymptotically stable.

To prove Eq.(9), we have

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t}+\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(t-h_1(t))\cdot$$
$$\boldsymbol{w}(t-h_1(t))$$

$$=\begin{bmatrix}\overline{\boldsymbol{x}}^{\mathrm{T}}(t) & \xi^{\mathrm{T}}(t)\end{bmatrix}\left(\boldsymbol{P}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ \sum_{i=0}^{2}\overline{\boldsymbol{A}}_i & -\boldsymbol{I}\end{bmatrix}+\begin{bmatrix}\boldsymbol{0} & \sum_{i=0}^{2}\overline{\boldsymbol{A}}_i^{\mathrm{T}}\\ \boldsymbol{I} & -\boldsymbol{I}\end{bmatrix}\boldsymbol{P}\right.$$
$$\left.+\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{U}_1\end{bmatrix}\right)\begin{bmatrix}\overline{\boldsymbol{x}}(t)\\ \xi(t)\end{bmatrix}+2\xi^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}[\overline{\boldsymbol{F}}\xi(t-d_1(t))$$
$$-\overline{\boldsymbol{A}}_1\int_{t-h_1(t)}^{t}\xi(s)\mathrm{d}s-\overline{\boldsymbol{A}}_2\int_{t-g_1(t)}^{t}\xi(s)\mathrm{d}s+\overline{\boldsymbol{B}}_0\boldsymbol{w}$$
$$+\overline{\boldsymbol{B}}_1\boldsymbol{w}(t-h_1(t))+\overline{\boldsymbol{B}}_H\boldsymbol{\beta}]+h_1\xi^{\mathrm{T}}(t)\boldsymbol{R}_1\xi(t)+g_1\xi^{\mathrm{T}}(t)\cdot$$

$$\boldsymbol{R}_2\xi(t)-\int_{t-h_1(t)}^{t}\xi^{\mathrm{T}}(s)\boldsymbol{R}_1\xi(s)\mathrm{d}s-\int_{t-g_1(t)}^{t}\xi^{\mathrm{T}}(s)\boldsymbol{R}_2\cdot$$
$$\xi(s)\mathrm{d}s+\boldsymbol{\bar{x}}^{\mathrm{T}}(t)\boldsymbol{\bar{C}}_H^{\mathrm{T}}\boldsymbol{\bar{C}}_H\boldsymbol{\bar{x}}(t)+\boldsymbol{\bar{x}}^{\mathrm{T}}(t-h_1(t))\boldsymbol{\bar{C}}_{1H}^{\mathrm{T}}\boldsymbol{\bar{C}}_{1H}\cdot$$
$$\boldsymbol{\bar{x}}(t-h_1(t))-\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{\beta}+\boldsymbol{\bar{x}}^{\mathrm{T}}(t)\boldsymbol{\bar{C}}_1^{\mathrm{T}}\sum_{i=1}^{4}\boldsymbol{X}_{1i}\boldsymbol{\bar{C}}_1\boldsymbol{\bar{x}}(t)$$
$$-(1-h_1)\boldsymbol{\bar{x}}^{\mathrm{T}}(t-h_1(t))\boldsymbol{\bar{C}}_1^{\mathrm{T}}\sum_{i=1}^{4}\boldsymbol{X}_{1i}\boldsymbol{\bar{C}}_1\boldsymbol{\bar{x}}(t-h_1(t))$$
$$+\boldsymbol{\bar{x}}^{\mathrm{T}}(t)\boldsymbol{\bar{C}}_2^{\mathrm{T}}\sum_{i=1}^{4}\boldsymbol{X}_{2i}\boldsymbol{\bar{C}}_2\boldsymbol{\bar{x}}(t)-(1-g_1)\boldsymbol{\bar{x}}^{\mathrm{T}}(t-g_1(t))\cdot$$
$$\boldsymbol{\bar{C}}_2^{\mathrm{T}}\sum_{i=1}^{4}\boldsymbol{X}_{2i}\boldsymbol{\bar{C}}_2\boldsymbol{\bar{x}}(t-g_1(t))+\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)$$
$$-\gamma^2\boldsymbol{w}^{\mathrm{T}}(t-h_1(t))\boldsymbol{w}(t-h_1(t)) \quad (16)$$

Due to the asymptotic stability of $\boldsymbol{x}(t)$, and $\boldsymbol{w}(t)$ is square integrable on $[0,\infty)$, it follows that $\xi(t)\in L_2[0,\infty)$ from Eq.(11). Similar to the prove of Theorem 2.1 (Fridman, 2001), some inequalities can be obtained, substitute the obtained inequalities into Eq.(16) and integrate the resulting inequality in t from 0 to ∞. At the same time, consider the following equation,

$$\int_0^{\infty}[\boldsymbol{\bar{x}}^{\mathrm{T}}(t-h_1(t))(\boldsymbol{\bar{C}}_1^{\mathrm{T}}\boldsymbol{\bar{C}}_1+\boldsymbol{\bar{C}}_{1H}^{\mathrm{T}}\boldsymbol{\bar{C}}_{1H})\boldsymbol{\bar{x}}(t-h_1(t))$$
$$+\boldsymbol{\bar{x}}^{\mathrm{T}}(t-g_1(t))\boldsymbol{\bar{C}}_2^{T}\boldsymbol{\bar{C}}_2\bar{x}(t-g_1(t))]\mathrm{d}t$$
$$=\int_0^{\infty}[\boldsymbol{\bar{x}}^{\mathrm{T}}(t)(\boldsymbol{\bar{C}}_1^{\mathrm{T}}\boldsymbol{\bar{C}}_1+\boldsymbol{\bar{C}}_{1H}^{\mathrm{T}}\boldsymbol{\bar{C}}_{1H}+\boldsymbol{\bar{C}}_2^{\mathrm{T}}\boldsymbol{\bar{C}}_2)\boldsymbol{\bar{x}}(t)\mathrm{d}t \quad (17)$$

we finally can obtain that Eq.(9) is satisfied if the LMI Eq.(15) holds, by Schur complements. This completes the proof.

From Eq.(15), we can observe that $\Psi<0$. By expansion of the block matrices, we have $-(\boldsymbol{P}_3+\boldsymbol{P}_3^{\mathrm{T}})<0$, it implies that $\boldsymbol{P}$ is a nonsingular matrix. Defining

$$\boldsymbol{P}^{-1}=\boldsymbol{Q}=\begin{bmatrix}\boldsymbol{Q}_1 & \boldsymbol{0}\\ \boldsymbol{Q}_2 & \boldsymbol{Q}_3\end{bmatrix} \quad (18)$$

then multiply Eq.(15) by diag$\{\boldsymbol{Q}^{\mathrm{T}},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$ and diag$\{\boldsymbol{Q},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$ on the left side and right side, respectively, and denotes the result as $\boldsymbol{M}'$, i.e.

$$\boldsymbol{M}'=\mathrm{diag}\{\boldsymbol{Q}^{\mathrm{T}},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}\boldsymbol{M}\mathrm{diag}\{\boldsymbol{Q},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\} \quad (19)$$

In order to linearize the resulting optimization problem, we look for $\boldsymbol{Q}_1$ that has the following block diagonal structure,

$$\boldsymbol{Q}_1=\begin{bmatrix}\boldsymbol{I} & \\ & \boldsymbol{Q}_{12}\end{bmatrix} \quad (20)$$

where $\boldsymbol{Q}_{12}$ is a positive definite matrix. This restriction is adopted to clear up the bilinear terms appeared in Eq.(15), and will introduce an additional conservation to the solution proposed, but compared with the reported result (Guo, 2002) on H_∞ output feedback control, its conservation is still lessened. In particular, if we choose

$$\boldsymbol{R}_2=\boldsymbol{C}_c^{\mathrm{T}}\boldsymbol{R}_2'\boldsymbol{C}_c,\ \boldsymbol{R}_3=(\boldsymbol{C}_c^{\mathrm{T}}\boldsymbol{R}_2'\boldsymbol{C}_c)^{-1}, \quad (21)$$

by expansion of the block matrices, we have

$$\boldsymbol{M}'=\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c^{\mathrm{T}}\boldsymbol{I}\}\boldsymbol{M}''\cdot$$
$$\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c\boldsymbol{I}\} \quad (22)$$

where $\boldsymbol{M}''<0$ is shown in Eq.(23),

$$\boldsymbol{M}''=\begin{bmatrix}
\Theta_{11} & \Theta_{12} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \Theta_{17} & \Theta_{18} & \boldsymbol{0} & \boldsymbol{0} & \Theta_{111}\\
* & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} & \Theta_{26} & \Theta_{27} & \boldsymbol{0} & \Theta_{29} & \Theta_{210} & \Theta_{211}\\
* & * & -\gamma^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_{10}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & -\gamma^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_{11}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\bar{D}}_H^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & -\boldsymbol{U}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & -\boldsymbol{\bar{U}}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & -\mathbf{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & -h_1\boldsymbol{R}_1 & \boldsymbol{0} & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & * & -g_1\boldsymbol{R}_2' & \boldsymbol{0}\\
* & * & * & * & * & * & * & * & * & * & \Phi_5'
\end{bmatrix}<0 \quad (23)$$

and Φ_5' is Φ_5 in Eq.(15) that $\bar{\boldsymbol{R}}_2$ is replaced by $\boldsymbol{R}_3$ in Eq.(21),

$$\Theta_{11}=\begin{bmatrix}\boldsymbol{Q}_{21}+\boldsymbol{Q}_{21}^{\mathrm{T}} & \boldsymbol{Q}_{22}+\boldsymbol{Q}_{23}^{\mathrm{T}}\\ \boldsymbol{Q}_{22}^{\mathrm{T}}+\boldsymbol{Q}_{23} & \boldsymbol{Q}_{24}^{\mathrm{T}}+\boldsymbol{Q}_{24}\end{bmatrix},$$

$$\Theta_{12}=\begin{bmatrix}\boldsymbol{Q}_{31}-\boldsymbol{Q}_{21}^{\mathrm{T}}+\boldsymbol{A}_0+\boldsymbol{A}_1 & \boldsymbol{Q}_{32}-\boldsymbol{Q}_{23}^{\mathrm{T}}+(\boldsymbol{C}_{20}+\boldsymbol{C}_{21})^{\mathrm{T}}\boldsymbol{B}_c^{\mathrm{T}}\\ \boldsymbol{Q}_{33}-\boldsymbol{Q}_{22}^{\mathrm{T}}+\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{B}_{20}+\boldsymbol{B}_{21})^{\mathrm{T}} & \boldsymbol{Q}_{34}-\boldsymbol{Q}_{24}^{\mathrm{T}}+\boldsymbol{H}^{\mathrm{T}}\end{bmatrix}$$

$$\Theta_{22}=\begin{bmatrix}-\boldsymbol{Q}_{31}-\boldsymbol{Q}_{31}^{\mathrm{T}} & -\boldsymbol{Q}_{32}-\boldsymbol{Q}_{33}^{\mathrm{T}}\\ -\boldsymbol{Q}_{32}^{\mathrm{T}}-\boldsymbol{Q}_{33} & -\boldsymbol{Q}_{34}^{\mathrm{T}}-\boldsymbol{Q}_{34}\end{bmatrix},$$

$$\Theta_{23}=\begin{bmatrix}\boldsymbol{B}_{10}\\ \boldsymbol{B}_c\boldsymbol{D}_{30}\end{bmatrix},\ \Theta_{24}=\begin{bmatrix}\boldsymbol{B}_{11}\\ \boldsymbol{B}_c\boldsymbol{D}_{31}\end{bmatrix},$$

$$\Theta_{25}=\left[\begin{bmatrix}\lambda_1\boldsymbol{E}_{10} & \\ & \lambda_1\boldsymbol{B}_c\boldsymbol{E}_{30}\end{bmatrix}\ \begin{bmatrix}\lambda_{11}\boldsymbol{E}_{11} & \\ & \lambda_{11}\boldsymbol{B}_c\boldsymbol{E}_{31}\end{bmatrix}\ \boldsymbol{0}\ \ \boldsymbol{0}\right],$$

$$\Theta_{17}=\begin{bmatrix}\boldsymbol{Q}_{21}^{\mathrm{T}} & \boldsymbol{Q}_{23}^{\mathrm{T}}\\ \boldsymbol{Q}_{22}^{\mathrm{T}} & \boldsymbol{Q}_{24}^{\mathrm{T}}\end{bmatrix},\ \Theta_{26}=\begin{bmatrix}\boldsymbol{F}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},\ \Theta_{27}=\begin{bmatrix}\boldsymbol{Q}_{31}^{\mathrm{T}} & \boldsymbol{Q}_{33}^{\mathrm{T}}\\ \boldsymbol{Q}_{32}^{\mathrm{T}} & \boldsymbol{Q}_{34}^{\mathrm{T}}\end{bmatrix},$$

$$\Theta_{18}=\begin{bmatrix}\boldsymbol{C}_{10}^{\mathrm{T}}\\ \boldsymbol{G}^{\mathrm{T}}\boldsymbol{D}_{20}^{\mathrm{T}}\end{bmatrix},\ \Theta_{29}=\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\\ \boldsymbol{B}_c\boldsymbol{C}_{21} & \boldsymbol{0}\end{bmatrix},\ \Theta_{210}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{B}_{21}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},$$

$$\Theta_{211}=\left[\boldsymbol{0}\ \cdots\ \boldsymbol{0}\ h_1\begin{bmatrix}\boldsymbol{Q}_{31}^{\mathrm{T}} & \boldsymbol{Q}_{33}^{\mathrm{T}}\\ \boldsymbol{Q}_{32}^{\mathrm{T}} & \boldsymbol{Q}_{34}^{\mathrm{T}}\end{bmatrix}\ g_1\begin{bmatrix}\boldsymbol{Q}_{31}^{\mathrm{T}} & \boldsymbol{Q}_{33}^{\mathrm{T}}\\ \boldsymbol{Q}_{32}^{\mathrm{T}} & \boldsymbol{Q}_{34}^{\mathrm{T}}\end{bmatrix}\ \boldsymbol{0}\ \cdots\ \boldsymbol{0}\right]$$

$$\Theta_{111}=\left[\Theta_{191}\ \ \Theta_{191}\ \ \Theta_{191}\ \ \Theta_{192}\ \cdots\ \Theta_{192}\ \ \Theta_{193}\ \cdots\ \Theta_{193}\ \ \Theta_{194}\ \ \Theta_{195}\ \ \boldsymbol{0}\ \cdots\ \boldsymbol{0}\right],$$

where

$$\Theta_{191}=\begin{bmatrix}\boldsymbol{C}_{10}^{\mathrm{T}}\\ \boldsymbol{G}^{\mathrm{T}}\boldsymbol{D}_{20}^{\mathrm{T}}\end{bmatrix},\ \Theta_{192}=\begin{bmatrix}\boldsymbol{C}_{11}^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix},\ \Theta_{193}=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{G}^{\mathrm{T}}\boldsymbol{D}_{21}^{\mathrm{T}}\end{bmatrix},$$

$$\Theta_{194}=\left[\begin{bmatrix}\boldsymbol{C}^{\mathrm{T}}K_1/\lambda_1 & \\ & \boldsymbol{Q}_{12}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}K_3/\lambda_1\end{bmatrix}\ \boldsymbol{0}\ \begin{bmatrix}\boldsymbol{C}^{\mathrm{T}}K_2/\lambda_1 & \\ & \boldsymbol{0}\end{bmatrix}\ \boldsymbol{0}\right]$$

$$\Theta_{195}=\left[\boldsymbol{0}\ \begin{bmatrix}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{K}_{11}/\lambda_{11} & \\ & \boldsymbol{Q}_{12}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{K}_{31}/\lambda_{11}\end{bmatrix}\ \boldsymbol{0}\ \begin{bmatrix}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{K}_{21}/\lambda_{11} & \\ & \boldsymbol{0}\end{bmatrix}\right]$$

Now, we are in a position to give the design method for a robust H_∞ output feedback controller.

Theorem 1 Consider the system of Eq.(10), for a given scalar $\gamma>0$, Eq.(9) is satisfied for all nonzero $\boldsymbol{w}(t)$, $\boldsymbol{w}(t-h_1(t))\in L_2[0,\infty)$, if there exist matrices $\boldsymbol{Q}_{12}>0$, $\boldsymbol{Q}_{21}$, $\boldsymbol{Q}_{22}$, $\boldsymbol{Q}_{23}$, $\boldsymbol{Q}_{24}$, $\boldsymbol{Q}_{31}$, $\boldsymbol{Q}_{32}$, $\boldsymbol{Q}_{33}$, $\boldsymbol{Q}_{34}$, $\boldsymbol{H}$, $\boldsymbol{G}$, $\boldsymbol{B}_c$, positive definite symmetric matrices $\bar{\boldsymbol{U}}_1=\boldsymbol{U}_1^{-1}$, $\bar{\boldsymbol{R}}_1=\boldsymbol{R}_1^{-1}$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\bar{\boldsymbol{X}}_{ij}=\boldsymbol{X}_{ij}^{-1}$, j=1,…,4, i=1,2, and positive scalars $\lambda_1,\lambda_2,\lambda_{11},\lambda_{21}$ such that linear matrix inequality (LMI) Eq.(23) holds. In this case, the parameters of the controller can be solved and

$$\boldsymbol{A}_c=\boldsymbol{H}\boldsymbol{Q}_{12}^{-1},\quad \boldsymbol{C}_c=\boldsymbol{G}\boldsymbol{Q}_{12}^{-1},\quad \boldsymbol{B}_c=\boldsymbol{B}_c.$$

Remark 1 For the scalar γ, we can obtain the mini mum value by the following optimal algorithm:

$$\min\ \gamma,$$
subject to Eq.(23).

The upper bound of delays h_1, g_1 can be obtained by stepwise iteration, the steps are stated as follows: First, the LMI (23) is solved in terms of the given values h_1, $g_1>0$, if there exists a feasible solution, then the values of h_1, g_1 are increased step by step; otherwise the values are reduced to half values step by step. Repeat the above procedure, finally, the upper bound of delays h_1, g_1 can be obtained according to any precision.

Assuming k=1, the augmented model is described as follows,

$$\bar{\Sigma}_{\Delta L}:\begin{cases}\dot{\bar{\boldsymbol{x}}}=\bar{\boldsymbol{A}}_{0\Delta}\bar{\boldsymbol{x}}(t)+\bar{\boldsymbol{A}}_{1\Delta}\bar{\boldsymbol{x}}(t-h_1(t))\\ \quad+\bar{\boldsymbol{A}}_{2\Delta}\bar{\boldsymbol{x}}(t-g_1(t))+\bar{\boldsymbol{B}}_{\Delta}\boldsymbol{w}\\ \quad+\bar{\boldsymbol{B}}_{1\Delta}\boldsymbol{w}(t-h_1(t))+\bar{\boldsymbol{E}}_1\bar{\boldsymbol{f}}_1+\bar{\boldsymbol{E}}_{11}\bar{\boldsymbol{f}}_{11}\\ \boldsymbol{z}=\bar{\boldsymbol{C}}_{0\Delta}\bar{\boldsymbol{x}}(t)+\bar{\boldsymbol{C}}_{1\Delta}\bar{\boldsymbol{x}}(t-h_1(t))\\ \quad+\bar{\boldsymbol{C}}_{2\Delta}\bar{\boldsymbol{x}}(t-g_1(t))+\boldsymbol{D}_{10\Delta}\boldsymbol{w}\\ \quad+\boldsymbol{D}_{11\Delta}\boldsymbol{w}(t-h_1(t))+\boldsymbol{E}_{20}\boldsymbol{f}_2+\boldsymbol{E}_{21}\boldsymbol{f}_2\end{cases}\tag{24}$$

where $(\cdot)_\Delta=(\cdot)+\Delta(\cdot)$, the parameter uncertainties in the system ($\bar{\Sigma}_{\Delta L}$) can be rewritten as

$$\begin{bmatrix}\Delta\bar{\boldsymbol{A}}_0 & \Delta\bar{\boldsymbol{A}}_1 & \Delta\bar{\boldsymbol{A}}_2 & \Delta\bar{\boldsymbol{B}}_0 & \Delta\bar{\boldsymbol{B}}_1\\ \Delta\bar{\boldsymbol{C}}_0 & \Delta\bar{\boldsymbol{C}}_1 & \Delta\bar{\boldsymbol{C}}_2 & \Delta\boldsymbol{D}_{10} & \Delta\boldsymbol{D}_{11}\end{bmatrix}$$

$$=\begin{bmatrix} \boldsymbol{G}_1 \\ \boldsymbol{G}_2 \end{bmatrix} \boldsymbol{F}(x,t) \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_2 & \boldsymbol{H}_3 & \boldsymbol{H}_4 & \boldsymbol{H}_5 \end{bmatrix} \tag{25}$$

where

$$\boldsymbol{G}_1 = \begin{bmatrix} \boldsymbol{G}_{11} & \boldsymbol{G}_{11} \\ \boldsymbol{B}_c \boldsymbol{G}_{31} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{G}_2 = \begin{bmatrix} \boldsymbol{G}_{21} & \boldsymbol{G}_{21} \end{bmatrix},$$

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{H}_{11} & \\ & \boldsymbol{H}_{15}\boldsymbol{C}_c \end{bmatrix},\ \boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{H}_{12} & \\ & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{H}_3 = \begin{bmatrix} \boldsymbol{0} & \\ & \boldsymbol{H}_{15}\boldsymbol{C}_c \end{bmatrix},\ \boldsymbol{H}_4 = \begin{bmatrix} \boldsymbol{H}_{13} \\ \boldsymbol{0} \end{bmatrix},\ \boldsymbol{H}_5 = \begin{bmatrix} \boldsymbol{H}_{14} \\ \boldsymbol{0} \end{bmatrix}.$$

Then the problem of robust H_∞ output feedback control can be presented as the following Theorem 2.

Theorem 2 Consider the uncertain Lur'e delay system Eq.(24), for a given scalar $\gamma>0$ Eq.(9) is satisfied for all nonzero $\boldsymbol{w}(t)$, $\boldsymbol{w}(t-h_1(t))\in L_2[0,\infty)$, if there exist matrices $\boldsymbol{Q}_{12}>0$, $\boldsymbol{Q}_{21}$, $\boldsymbol{Q}_{22}$, $\boldsymbol{Q}_{23}$, $\boldsymbol{Q}_{24}$, $\boldsymbol{Q}_{31}$, $\boldsymbol{Q}_{32}$, $\boldsymbol{Q}_{33}$, $\boldsymbol{Q}_{34}$, $\boldsymbol{H}$, $\boldsymbol{G}$, $\boldsymbol{B}_c$, positive definite symmetric matrix $\boldsymbol{U}_1$, $\bar{\boldsymbol{U}}_1=\boldsymbol{U}_1^{-1}$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\bar{\boldsymbol{R}}_1=\boldsymbol{R}_1^{-1}$, $\bar{\boldsymbol{X}}_{ij}=\boldsymbol{X}_{ij}^{-1}$, j=1,…4, i=1, 2, and positive scalars λ_1, λ_2, λ_{11}, λ_{21}, α such that linear matrix inequality (LMI) Eq.(26) holds.

$$\boldsymbol{M}_N = \begin{bmatrix} \boldsymbol{M}'' & \Omega_{11} & \Omega_{12}^{\mathrm{T}} \\ \Omega_{11}^{\mathrm{T}} & -\alpha^{-1}\boldsymbol{I} & \boldsymbol{0} \\ \Omega_{12} & \boldsymbol{0} & -\alpha\boldsymbol{I} \end{bmatrix} < 0 \tag{26}$$

where

$$\Omega_{11} = \begin{bmatrix} \Gamma_{11} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \Gamma_{12} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \Gamma_{13} \end{bmatrix},\quad \Omega_{12} = \begin{bmatrix} \Gamma_{21} & \Gamma_{22} & \Gamma_{23} \end{bmatrix},$$

$$\Gamma_{11} = \begin{bmatrix} \Xi_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix},\quad \Gamma_{13} = \begin{bmatrix} \Xi_2 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \ddots & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \boldsymbol{0} & \Xi_2 & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix},$$

$$\Gamma_{21}^{\mathrm{T}} = \begin{bmatrix} \Pi_1^{\mathrm{T}} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \Pi_6^{\mathrm{T}} & \boldsymbol{0} & \Pi_6^{\mathrm{T}} & \cdots & \Pi_7^{\mathrm{T}} & \cdots & \Pi_8^{\mathrm{T}} & \cdots & \Pi_8^{\mathrm{T}} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix},$$

$$\Gamma_{12} = \begin{bmatrix} \Xi_2 & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \Xi_2 & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \Xi_2 \end{bmatrix},$$

$$\Gamma_{22} = \begin{bmatrix} \Pi_2 & \Pi_3 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \Pi_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \Pi_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \Pi_3 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \Pi_3 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},\ \Gamma_{23} = \begin{bmatrix} \Pi_4 & \Pi_5 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix}$$

$$\Xi_1 = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \begin{bmatrix} \boldsymbol{G}_{11} & \boldsymbol{G}_{11} \\ \boldsymbol{B}_c\boldsymbol{G}_{31} & \boldsymbol{0} \end{bmatrix} & \boldsymbol{0} \end{bmatrix},\quad \Xi_2 = \begin{bmatrix} \begin{bmatrix} \boldsymbol{G}_{21} & \boldsymbol{G}_{21} \end{bmatrix} & \boldsymbol{0} \end{bmatrix},$$

$$\Pi_1 = \begin{bmatrix} \begin{bmatrix} \boldsymbol{H}_{11}+\boldsymbol{H}_{12} & \boldsymbol{0} \\ \boldsymbol{0} & (\boldsymbol{H}_{15}+\boldsymbol{H}_{16})\boldsymbol{G} \end{bmatrix} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},\ \Pi_2 = \begin{bmatrix} \begin{bmatrix} \boldsymbol{H}_{13} \\ \boldsymbol{0} \end{bmatrix} \\ \boldsymbol{0} \end{bmatrix},$$

$$\Pi_3 = \begin{bmatrix} \begin{bmatrix} \boldsymbol{H}_{14} \\ \boldsymbol{0} \end{bmatrix} \\ \boldsymbol{0} \end{bmatrix},\ \Pi_4 = \begin{bmatrix} h_1 \begin{bmatrix} \boldsymbol{H}_{12} & \\ & \boldsymbol{0} \end{bmatrix} \\ \boldsymbol{0} \end{bmatrix},\ \Pi_5 = \begin{bmatrix} g_1 \begin{bmatrix} \boldsymbol{0} & \\ & \boldsymbol{H}_{16} \end{bmatrix} \\ \boldsymbol{0} \end{bmatrix},$$

$$\Pi_6 = \begin{bmatrix} \begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_{15}\boldsymbol{G} \end{bmatrix} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},\ \Pi_7 = \begin{bmatrix} \begin{bmatrix} \boldsymbol{H}_{12} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\Pi_8 = \begin{bmatrix} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_{16}\boldsymbol{G} \end{bmatrix} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}.$$

Proof If LMI Eq.(26) holds, by Schur complement, it follows that

$$\boldsymbol{M}'' + \alpha\Omega_{11}\Omega_{11}^{\mathrm{T}} + \alpha^{-1}\Omega_{12}^{\mathrm{T}}\Omega_{12} < 0 \tag{27}$$

Therefore, we can deduce

$$\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c^{\mathrm{T}},\boldsymbol{I}\}(\boldsymbol{M}''+\alpha\Omega_{11}\Omega_{11}^{\mathrm{T}} + \alpha^{-1}\Omega_{12}^{\mathrm{T}}\Omega_{12})\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c,\boldsymbol{I}\} < 0$$

Eq.(22) implies that

$$\boldsymbol{M}'+\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c^{\mathrm{T}},\boldsymbol{I}\}(\alpha\boldsymbol{\Omega}_{11}\boldsymbol{\Omega}_{11}^{\mathrm{T}} + \alpha^{-1}\boldsymbol{\Omega}_{12}^{\mathrm{T}}\boldsymbol{\Omega}_{12})\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c,\boldsymbol{I}\}<0$$

Defining

$$\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c^{\mathrm{T}},\boldsymbol{I}\}\boldsymbol{\Omega}_{11} = \mathrm{diag}\{\boldsymbol{Q}^{\mathrm{T}},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}\boldsymbol{L}_1,$$

$$\boldsymbol{\Omega}_{12}\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{C}_c,\boldsymbol{I}\} = \boldsymbol{L}_2\mathrm{diag}\{\boldsymbol{Q},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}.$$

Then we can deduce from Eq.(19) that the following inequality holds,

$$\boldsymbol{M}+\alpha\boldsymbol{L}_1\boldsymbol{L}_1^{\mathrm{T}}+\alpha^{-1}\boldsymbol{L}_2^{\mathrm{T}}\boldsymbol{L}_2<0 \qquad (28)$$

On the other hand, substituting Eq.(25) into the LMI Eq.(15), considering the description Eq.(4) of uncertainties, and transforming the linear matrix $\boldsymbol{M}$ in Eq.(15) into $\boldsymbol{M}_\Delta$, we have

$$\boldsymbol{M}_\Delta=\boldsymbol{M}+\boldsymbol{L}_1\boldsymbol{F}(x(t),t)\boldsymbol{L}_2+(\boldsymbol{L}_1\boldsymbol{F}(x(t),t)\boldsymbol{L}_2)^{\mathrm{T}} \qquad (29)$$

By Lemma 2 and Eq.(28), we can obtain

$$\boldsymbol{M}_\Delta<0 \qquad (30)$$

Thus, from Lemma 3 and Eq.(30), this completes the proof of Theorem 2.

NUMERAL EXAMPLES

Example 1 Consider the system $\overline{\Sigma}$, whose system matrices (Guo, 2002) are given by

$$\boldsymbol{F}=0, \boldsymbol{A}_0=\begin{bmatrix}-1.1 & -0.5\\ 0 & 0.2\end{bmatrix}, \boldsymbol{A}_1=\begin{bmatrix}0.2 & 0.25\\ -0.3 & 0.1\end{bmatrix},$$

$$\boldsymbol{B}_{10}=\begin{bmatrix}0.1\\0.1\end{bmatrix}, \boldsymbol{B}_{11}=\begin{bmatrix}0\\0\end{bmatrix}, \boldsymbol{B}_{20}=\begin{bmatrix}-1.5\\2.0\end{bmatrix}, \boldsymbol{B}_{21}=\begin{bmatrix}0.8\\-0.5\end{bmatrix},$$

$$\boldsymbol{C}_{10}=\begin{bmatrix}0.1 & 0.1\end{bmatrix}, \boldsymbol{C}_{11}=\begin{bmatrix}0.05 & 0.01\end{bmatrix}, \boldsymbol{C}_{20}=\begin{bmatrix}1 & 0.5\\ 0 & 0.5\end{bmatrix},$$

$$\boldsymbol{D}_{10}=0.1, \boldsymbol{D}_{11}=0, \boldsymbol{D}_{20}=0.5, \boldsymbol{D}_{21}=0.1,$$

$$\boldsymbol{C}_{21}=\begin{bmatrix}0.1 & 0.1\\ 0 & 0.1\end{bmatrix}, \boldsymbol{D}_{30}=\begin{bmatrix}0.1\\0\end{bmatrix}, \boldsymbol{D}_{31}=\begin{bmatrix}0\\0\end{bmatrix},$$

$$\boldsymbol{E}_{10}=\begin{bmatrix}0.3 & 0\\ 0 & -0.1\end{bmatrix}, \boldsymbol{E}_{11}=\begin{bmatrix}0.05 & 0\\ 0 & -0.1\end{bmatrix},$$

$$\boldsymbol{E}_{30}=\begin{bmatrix}0.5 & 0\\ 0 & -0.1\end{bmatrix}, \boldsymbol{C}=\begin{bmatrix}1.0 & 0\\ 0 & 1..0\end{bmatrix}, \boldsymbol{K}_1=\begin{bmatrix}0.2 & 0\\ 0 & 0.1\end{bmatrix},$$

$$\boldsymbol{K}_2=\begin{bmatrix}-0.15 & 0\\ 0 & 0.1\end{bmatrix}, \boldsymbol{K}_{11}=\begin{bmatrix}-0.2 & 0\\ 0 & -0.1\end{bmatrix}.$$

Choosing $\lambda_1=\lambda_2=\lambda_{11}=\lambda_{21}=1$, based on Theorem 1 and by using the LMI-toolbox in Matlab, we obtain the minimum values of γ, as a function of the bound h_1 and g_1, they are described in Table 1. A minimum value of γ=0.7335 is obtained, compared to the achievable value (Guo, 2002) of γ=1.0000.

Table 1 Relation of the bound of time-delays with performance index

h_1	g_1	$\gamma_{\min}$
0	0	0.7335
0.3	0.2	0.9215
0.4	0.3	0.9865
0.5	0.5	1.1743
0.6	0.53	1.4987
0.7	0.6	2.3816
0.745	0.64	4.3659

The upper bound of delays h_1 and g_1 are $0\le h_1\le 0.7456$ and $0\le g_1\le 0.6402$, respectively, compared with the bound $0\le h_1=g_1\le 0.5$ (Guo, 2002). When $h_1=g_1=0.5$, the corresponding matrices of the robust H_∞ output feedback controller are

$$\boldsymbol{A}_c=\begin{bmatrix}-1.9842 & 1.2866\\ 1.2977 & -2.8531\end{bmatrix},$$

$$\boldsymbol{B}_c=\begin{bmatrix}-1.5266 & 2.5252\\ 0.8444 & -3.0860\end{bmatrix},$$

$$\boldsymbol{C}_c=\begin{bmatrix}-0.4071 & 0.6216\end{bmatrix}.$$

Example 2 Consider the system $\overline{\Sigma}_{\Delta L}$ with the same matrices as $\overline{\Sigma}$ in example 1 and with parametric uncertainties (Guo, 2002) described by

$$\boldsymbol{G}_{11}=\begin{bmatrix}-0.1\\0.2\end{bmatrix},\ \boldsymbol{G}_{21}=0,\ \boldsymbol{G}_{31}=\begin{bmatrix}0.1\\-0.1\end{bmatrix},\ \boldsymbol{H}_{11}=[0.1\quad 0.1],$$
$$\boldsymbol{H}_{12}=[0.01\quad 0],\ \boldsymbol{H}_{13}=0.1,\ \boldsymbol{H}_{14}=-0.2,\ \boldsymbol{H}_{15}=\boldsymbol{H}_{16}=0.$$

Based on Theorem 2, a minimum value of γ= 0.8869 is obtained, compared with the achievable value (Guo, 2002) of γ=1.2000; it is found that this system is robustly stable and has H_∞ performance for any time-delay $0\le h_1\le 0.8974$, $0\le g_1\le 0.7179$, compared with the bound (Guo, 2002) $0\le h_1=g_1\le 0.5$. When $h_1=g_1=0.5$, the corresponding matrices of the robust H_∞ output feedback controller are

$$\boldsymbol{A}_c=\begin{bmatrix}-3.9796 & 2.8539\\ 2.6516 & -3.4823\end{bmatrix},\boldsymbol{B}_c=\begin{bmatrix}-4.0545 & -4.0543\\ -4.4230 & -4.4232\end{bmatrix},$$
$$\boldsymbol{C}_c=\begin{bmatrix}0.2649 & -0.2428\end{bmatrix}.$$

CONCLUSION

In this paper, a design method of robust H_∞ output feedback controller has been presented for a class of Lur'e systems with time-varying multi-delays in the states, input and measurement outputs, and with both nonlinear and parametric uncertainty appeared in all system matrices. Feasible design procedures are provided based on the LMI-based convex optimization approach. The sufficient conditions are presented which guarantee that the Lur'e systems have robust H_∞ performance, moreover, the results obtained are less conservative than the reported results due to the efficient BRL that was derived for uncertain Lur'e time-delay systems based on an equivalent descriptor representation of the system and due to the Park's efficient overbounding method. The numerical examples show that the presented results have the less conservation.

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