Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

***Science Letters:***

# How to realize a negative refractive index material at the atomic level in an optical frequency range?*

SHEN Jian-qi (沈建其)†1, RUAN Zhi-chao (阮智超)[1], HE Sai-ling (何赛灵)†1,2

(*[1]Centre for Optical and Electromagnetic Research, Joint Research Centre of Photonics of the Royal Institute of Technology (Sweden) and Zhejiang University, Zhejiang University, Hangzhou 310027, China*)

(*[2]Laboratory of Photonics and Microwave Engineering, Department of Microelectronics and Information Technology, Royal Institute of Technology, Electrum 229, SE-164 40 Kista, Sweden*)

†E-mail: jqshen@coer.zju.edu.cn; sailing@kth.se

Received Sept. 10, 2004; revision accepted Sept. 20, 2004

**Abstract:** The theoretical mechanism for realizing a negative refractive index material in an optical frequency range with an atomic gas system of electromagnetically induced transparency (EIT) is studied. It is shown that under certain conditions such a dense gas can exhibit simultaneously negative permittivity and negative permeability, and negligibly small loss.

**Key words:** Negative refractive index, Left-handed medium, Electromagnetically induced transparency, Atomic level, Optical frequency

**doi:**10.1631/jzus.2004.1322 **Document code:** A **CLC number:** O413

## INTRODUCTION

Recently, a new type of artificial composite materials (so-called left-handed media or negative refractive index media) whose electric permittivity and magnetic permeability are simultaneously negative (corresponding to a negative refractive index) over a certain frequency band has attracted considerable attention due to its extraordinary electromagnetic properties (Pendry, 2000; Shelby *et al*., 2001; Grbic and Eleftheriades, 2004; Chen *et al*., 2004). The negative refractive index media that have been fabricated successfully so far (Shelby *et al*., 2001; Grbic and Eleftheriades, 2004) are metallic periodic structures (like arrays of split ring resonators and wires) at microwave frequencies, and actually anisotropic in nature. Thus, the impact would be enormous if an isotropic and homogeneous (preferably at the atomic level) negative refractive index material can be realized in an optical frequency range. In this Letter we show that under certain conditions an atomic gas of three-level electromagnetically induced transparency (EIT) system (Harris, 1997; Li and Xiao, 1995; Schmidt and Imamoğlu, 1996; Davanco *et al*., 2003) can exhibit simultaneously negative permittivity and negative permeability in an optical frequency range.

## PROBLEM FORMULATION AND EXPRESSIONS FOR THE PERMITTIVITY AND PERMEABILITY

Consider a Λ-type atomic gas system of three-levels with one upper level $|a\rangle$ and two lower

*Project supported by the National Natural Science Foundation (NSF) of China (Nos. 90101024 and 60378037), and the National Basic Research Program (973) of China (No. 2004CB719805)

levels $|b\rangle$ and $|c\rangle$. The system interacts with two optical fields of the coupling and probe lasers, which couple the level pairs $|a\rangle$-$|c\rangle$ and $|a\rangle$-$|b\rangle$, respectively (Fig.1). The coupling laser is assumed to be in resonance with the $|a\rangle$-$|c\rangle$ transition, while the probe laser has a frequency detuning $\Delta=\omega_{ab}-\omega$, where $\omega_{xy}$ and $\omega$ denote the $|x\rangle$-$|y\rangle$ transition frequency (here $x$,$y$=$a$,$b$ or $c$) and the probe beam frequency, respectively. Here we show that the electric-dipole transition ($|a\rangle$-$|b\rangle$) and the magnetic-dipole transition ($|c\rangle$-$|a\rangle$) will produce the electric polarizability and the magnetic susceptibility for the probe beam, respectively. In general, the dimensionless ratio of the magnetic dipole matrix element to the electric dipole matrix element, i.e. $m_{cb}/\wp_{ab}c$ ($c$ is the speed of light in vacuum), has the order of $10^{-2}$ for an ordinary atomic system. Thus the magnetic-dipole transition is usually neglected for the wave propagation in a conventional material. However, in a three-level EIT medium where the intensity of the coupling laser is much larger than that of the probe light, the population in level $|c\rangle$ is much greater than that in the upper level $|a\rangle$. Thus, the magnitude of the density matrix element $\rho_{cb}$ may be much larger than that of $\rho_{ab}$. This means the magnetic dipole moment ($2m_{cb}^*\rho_{cb}$) may possibly be of the same order as the electric dipole moment ($2c\wp_{ab}^*\rho_{ab}$). Here $\wp_{ab}$ and $m_{cb}$ are the electric and magnetic dipole matrix elements, respectively.

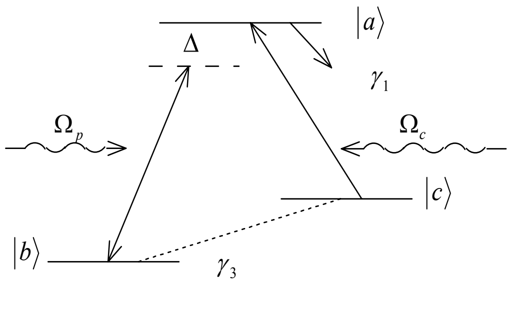


**Fig.1 The schematic diagram of a Λ-type three-level EIT atomic system**

First we consider the steady density matrix elements of this EIT system. The density matrix elements $\rho_{ab}$ and $\rho_{cb}$ of such a three-level system can be rewritten as $\rho_{ab}=\tilde{\rho}_{ab}\exp(-i\omega t)$ and $\rho_{cb}=\tilde{\rho}_{cb}\exp[-i(\omega+\omega_{ca})t]$. The intensity of the probe laser is assumed to be sufficiently weak, and therefore nearly all the atoms remain in the ground state, i.e. the atomic population in level $|b\rangle$ is unity. Under this assumption, $\tilde{\rho}_{ab}$ and $\tilde{\rho}_{cb}$ satisfy the following matrix equation (Scully and Zubairy, 1997)

$$\frac{\partial}{\partial t}\begin{pmatrix}\tilde{\rho}_{ab}\\ \tilde{\rho}_{cb}\end{pmatrix}=\begin{pmatrix}-(\gamma_1+i\Delta) & \frac{i}{2}\Omega_c\\ \frac{i}{2}\Omega_c^* & -(\gamma_3+i\Delta)\end{pmatrix}\begin{pmatrix}\tilde{\rho}_{ab}\\ \tilde{\rho}_{cb}\end{pmatrix}+\begin{pmatrix}\frac{i\wp_{ab}E_p}{2\hbar}\\ 0\end{pmatrix}, \tag{1}$$

where $\gamma_1$ and $\gamma_3$ represent the spontaneous decay rate of level $|a\rangle$ and the dephasing rate (nonradiative decay rate) of level $|c\rangle$, respectively. $E_p$ and $\Omega_c$ denote the probe field envelope and the Rabi frequency of the coupling laser ($\Omega_c=\wp_{ac}E_c/\hbar$ with $E_c$ being the electric field envelope of the coupling laser). It can be readily verified that the steady solution of Eq.(1) is

$$\begin{cases}\tilde{\rho}_{ab}=\dfrac{i\wp_{ab}E_p(\gamma_3+i\Delta)}{2\hbar\left[(\gamma_1+i\Delta)(\gamma_3+i\Delta)+\dfrac{\Omega_c^*\Omega_c}{4}\right]},\\ \tilde{\rho}_{cb}=-\dfrac{\wp_{ab}E_p\Omega_c^*}{4\hbar\left[(\gamma_1+i\Delta)(\gamma_3+i\Delta)+\dfrac{\Omega_c^*\Omega_c}{4}\right]}\end{cases} \tag{2}$$

The electric (or magnetic) susceptibility $\chi_e$ (or $\chi_m$) can be obtained from the simple relation $\chi_{e,m}=N\alpha_{e,m}$ (instead of the Clausius-Mossotti-Lorentz relation), where $N$ and $\alpha_{e,m}$ denote the atomic density (total number of atoms per volume) and the polarizability, respectively (Jackson, 2001). Therefore, in such a three-level atomic system, the electric susceptibility $\chi_e$ and the magnetic susceptibility $\chi_m$ are $\chi_e = 2N\wp_{ab}^*\tilde{\rho}_{ab}/\varepsilon_0 E_p$ and $\chi_m = 2Nm_{cb}^*\tilde{\rho}_{cb}/H_p$, where $H_p$ denotes the magnetic field envelope of the probe field. Thus, with the help of the steady Solution (2), one can obtain the relative permittivity $\varepsilon_r = 1+\chi_e$ and the relative permeability $\mu_r = 1+\chi_m$ of the above atomic system. By using the relation $H_p = \sqrt{\varepsilon_r\varepsilon_0/\mu_r\mu_0}E_p$ between the envelopes of magnetic and electric fields, we obtain the following expression and relation for the electric and magnetic susceptibilities,

$$\begin{cases} \chi_e = \dfrac{iN\left|\wp_{ab}\right|^2(\gamma_3+i\Delta)}{\varepsilon_0\hbar\left[(\gamma_1+i\Delta)(\gamma_3+i\Delta)+\dfrac{\Omega_c^*\Omega_c}{4}\right]}, \\ \chi_m = \dfrac{m_{cb}^*}{\wp_{ab}^* c}\sqrt{\dfrac{1+\chi_m}{1+\chi_e}}\left(\dfrac{i}{2}\cdot\dfrac{\Omega_c^*}{\gamma_3+i\Delta}\right)\chi_e \end{cases} \quad (3)$$

Since the dephasing rate $\gamma_3$ is small compared with the spontaneous decay rate $\gamma_1$ ($\gamma_3$ is in general two or three orders smaller than $\gamma_1$) (Li and Xiao, 1995), we will ignore $\gamma_3$ hereafter. It thus follows from Eq.(3) that

$$\begin{cases} \varepsilon_r = 1+\chi_e = 1-\dfrac{N\left|\wp_{ab}\right|^2\Delta}{\varepsilon_0\hbar\left[i\Delta(\gamma_1+i\Delta)+\dfrac{\Omega_c^*\Omega_c}{4}\right]}, \\ \mu_r^{\pm} = 1+\dfrac{\varsigma^2\pm\sqrt{\varsigma^4+4\varsigma^2}}{2} \end{cases} \quad (4)$$

with $\varsigma = (m_{cb}^*/\wp_{ab}^* c)(\Omega_c^*/2\Delta)\left(\chi_e/\sqrt{1+\chi_e}\right)$. In a negatively refractive EIT medium, if $\chi_e$ has a very small imaginary part, parameter $\varsigma$ is dominated by the imaginary part. The sign of the roots should be determined by inserting the values of $\mu_r^{\pm}$ into the second relation of Eq.(3), and only one of the two roots ($\mu_r^{\pm}$) satisfies this relation for any frequency detuning $\Delta$.

EXISTENCE OF NEGATIVE REFRACTIVE INDEX

We first analyze the real and imaginary parts of the electric susceptibility $\chi_e$. Let $\chi_e = \chi_e' + i\chi_e''$, then it follows from Eq.(3) that

$$\begin{cases} \chi_e' = \dfrac{N\left|\wp_{ab}\right|^2\Delta\left(\Delta^2-\dfrac{\Omega_c^*\Omega_c}{4}\right)}{\varepsilon_0\hbar\left[\left(\Delta^2-\dfrac{\Omega_c^*\Omega_c}{4}\right)^2+\Delta^2\gamma_1^2\right]} \\ \chi_e'' = \dfrac{N\left|\wp_{ab}\right|^2\Delta^2\gamma_1}{\varepsilon_0\hbar\left[\left(\Delta^2-\dfrac{\Omega_c^*\Omega_c}{4}\right)^2+\Delta^2\gamma_1^2\right]} \end{cases} \quad (5)$$

To make the loss of the EIT medium small, the imaginary part $\chi_e''$ should be negligibly small (i.e. $\chi_e'' \ll \left|1+\chi_e'\right|$). In an ordinary EIT experiment, the Rabi frequency $\Omega_c$ of the coupling laser often has the same order of magnitude of $\gamma_1$, e.g. $10^7$~$10^8$ $s^{-1}$ (Li and Xiao, 1995; Schmidt and Imamoğlu, 1996). Thus, it follows from Eq.(5) that, for a low-loss EIT medium, the frequency detuning of the probe field should be much less than the Rabi frequency of the coupling laser, i.e. $\Delta << \Omega_c$. A negative electric permittivity requires $\chi_e' < -1$, which is equivalent to [cf. Eq.(5)]

$$\zeta\Delta\left(\Delta^2-\frac{\Omega_c^*\Omega_c}{4}\right)+\left(\Delta^2-\frac{\Omega_c^*\Omega_c}{4}\right)^2+\Delta^2\gamma_1^2 < 0 \quad (6)$$

with $\zeta = N\left|\wp_{ab}\right|^2/\varepsilon_0\hbar$. Thus, the requirement for the realization of negative permittivity is $\Omega_c \gg \Delta > \Omega_c^*\Omega_c/4\zeta$. The condition $\Omega_c \gg \Omega_c^*\Omega_c/4\zeta$ requires $N \gg \varepsilon_0\hbar\left|\Omega_c^*\right|/4\left|\wp_{ab}\right|^2$. Therefore, in order to realize a low-loss negative permittivity, one should choose a dense-gas EIT medium.

With Eq.(4) one can show that if $\mathrm{Re}\{\varsigma^2\} < -4$

the magnetic permeability $\mu_r$ will have a negative real part and a nearly vanishing imaginary part. For simplicity we consider the case of large negative permittivity with $\chi_e' < -1$ (which is true in the region $\Delta > 5\times10^{-3}\gamma_1$ of Fig.2). Then one can show that the restriction $\mathrm{Re}\{\varsigma^2\} < -4$ can be rewritten as $\Delta \ll (m_{cb}^*/\wp_{ab}^* c)^2 \zeta/4$. Note that in an atomic system, $m_{cb}^*/\wp_{ab}^* c \sim 10^{-2}$. From the above discussions, it can be verified that, for a typical EIT experiment in which $\gamma_1 \sim \Omega_c = 10^8$ s$^{-1}$ and $\wp_{ab} = 1.2\times10^{-31}$ Debye (hyperfine-split Na D lines) (Cowan, 1981), a low-loss negative refractive index can exist when the three-level atomic system is a dense gas with the atomic density $N$ larger than $10^{27}$ m$^{-3}$ and the frequency detuning $\Delta$ is in the range of $\min\{\Omega_c, (m_{cb}^*/\wp_{ab}^* c)^2 \zeta/4\} \gg \Delta > \Omega_c^*\Omega_c/4\zeta$. Fig.2 shows the behaviors of the permittivity and the permeability for the probe beam in such an EIT gas system with $N=10^{27}$ m$^{-3}$, $\gamma_1=\Omega_c=10^8$ s$^{-1}$ $\wp_{ab} = 1.2\times10^{-31}$ Debye and $m_{cb}^*/\wp_{ab}^* c = 10^{-2}$. From this figure one sees that the permittivity has a negative real part and its imaginary part is very small if the frequency detuning $\Delta$ is greater than $2.3\times10^{-3}\gamma_1$. When $\Delta$ is in the range of $[2.3\times10^{-3}\gamma_1, 7.7\times10^{-3}\gamma_1]$, the dense EIT atomic gas will exhibit a negative permeability. When $\Delta$ is in the range of $[2.3\times10^{-3}\gamma_1, 4.6\times10^{-3}\gamma_1]$, the imaginary part of the permeability is negligibly small. The corresponding absorption coefficient $\alpha = -2\pi\,\mathrm{Im}\{\sqrt{\varepsilon_r\mu_r}\}$ (indicating the loss of the medium per unit wavelength) when $\Delta$ is in the range of $[2.3\times10^{-3}\gamma_1, 7.7\times10^{-3}\gamma_1]$ (where both the permittivity and permeability have negative real parts) is shown in Fig.3. This figure shows that the loss is small when $\Delta < 4.6\times10^{-3}\gamma_1$, but becomes large when $\Delta > 4.6\times10^{-3}\gamma_1$ (since the permeability has a large imaginary part).

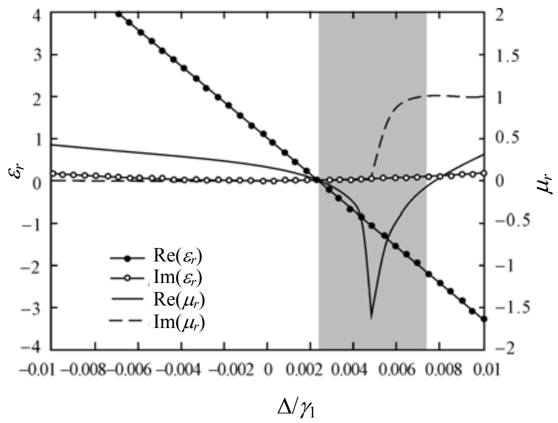


**Fig.2 The real and imaginary parts of the permittivity (circles connected with the solid and dashed lines, respectively) and the permeability (the solid and dashed lines, respectively) of an EIT gas system as the frequency detuning Δ varies**

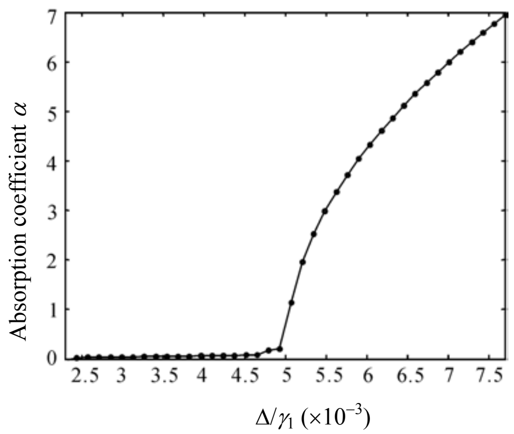


**Fig.3 The corresponding absorption coefficient when the frequency detuning Δ is in the grey range of Fig.2 (where both the permittivity and permeability have negative real parts)**

## CONCLUSION

We have studied how to realize a negative refractive index material with an atomic EIT gas system in an optical frequency range. Explicit formulas for the permittivity and permeability have been given. It is shown that under certain conditions such a dense gas can exhibit simultaneously negative permittivity and negative permeability, and thus become a negative refractive index material. The loss can also be very small. Compared with the other existing schemes for the realization of negati-

After we have finished the present Letter we found that Oktel *et al.* had also considered a similar problem [M.Ö. Oktel and Ö.E. Müstecaphoğlu, arXiv: physics/0406309 (2004)], however, gave wrong formulae and wrong results.

ve refractive index, the present scheme has three main advantages: (i) it can be used to fabricate *isotropic* and homogeneous negative refractive index material; (ii) it can be used to fabricate negative refractive index material in the visible or infrared frequency band; (iii) it can be used to tune the negative refractive index by an external field (the coupling laser). For these reasons, we believe that the present scheme for realizing a negative refractive index medium deserves further investigation both theoretically and experimentally.

ACKNOWLEGEMENTS

The realization of a negative refractive index material with EIT is one of the main subjects in our 973 program (China's national keystone basic research program) application submitted in April, 2004, in which Prof. Jian-Ying Zhou (Sun Yat-sen University) was also involved.

## References

Chen, L., He, S., Shen, L., 2004. Finite-size effects of a left-handed material slab on the image quality. *Phys. Rev. Lett.*, **92**:107404-(1-4).

Cowan, R.D., 1981. The Theory of Atomic Structure and Spectra. Univ. of California Press, Berkeley, Calif., Appendix D, p.640.

Davanco, M., Holmström, P., Blumenthal, D.J., Thylén, L., 2003. Directional coupler wavelength filters based on waveguides exhibiting electromagnetically induced transparency. *IEEE J. Quantum Electron*, **39**: 608-613.

Grbic, A., Eleftheriades, G.V., 2004. Overcoming the diffraction limit with a planar left-handed transmission-line lens. *Phys. Rev. Lett.*, **92**:117403-(1-4).

Harris, S.E., 1997. Electromagnetically induced transparency. *Phys. Today*, **50**(7):36-42.

Jackson, J.D., 2001. Classical Electrodynamics, 3rd Ed., Wiley, New York, p.158-162.

Li, Y.Q., Xiao, M., 1995. Transient properties of an electromagnetically induced transparency in three-level atoms. *Opt. Lett.*, **20**:1489-1491.

Pendry, J.B., 2000. Negative refraction makes a perfect lens. *Phys. Rev. Lett.*, **85**:3966-3969.

Schmidt, H., Imamoğlu, A., 1996. Giant Kerr nonlinearities obtained by electromagnetically induced transparency. *Opt. Lett.*, **21**:1936-1938.

Scully, M.O., Zubairy, M.S., 1997. Quantum Optics. Cambridge Univ. Press, Cambridge, Sec.7.

Shelby, R.A., Smith, D.R., Schultz, S., 2001. Experimental verification of a negative index of refraction. *Science*, **292**:77-79.