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Passive control of Permanent Magnet Synchronous Motor chaotic systems*

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Abstract: Permanent Magnet Synchronous Motor model can exhibit a variety of chaotic phenomena under some choices of system parameters and external input. Based on the property of passive system, the essential conditions were studied, by which Permanent Magnet Synchronous Motor chaotic system could be equivalent to passive system. Using Lyapunov stability theory, the convergence condition deciding the system's characters was discussed. In the convergence condition area, the equivalent passive system could be globally asymptotically stabilized by smooth state feedback.

Key words: Permanent Magnet Synchronous Motor, Passive system, Convergence condition
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INTRODUCTION

Chaos plays an important role in dynamical systems and is applied in many fields such as electrical engineering, chemistry, economics and so on (Zhao and Qi, 2001; Pecora and Carroll, 1991). The characters of chaos have been proved to be useful in describing and diagnosing nonlinear dynamical systems. However, it is harmful to many dynamical systems, especially for Permanent Magnet Synchronous Motor system (Zhang *et al*., 2002). People seek for ways to avoid and eliminate it (Chen and Chen, 1999; Qi and Zhao, 2003). In general, through modifying parameters or controlling chaotic system, we can find some ways to affect the exiting conditions of chaos in Permanent Magnet Synchronous Motor system so that chaos can be avoided.

Many people had paid attention to passive network theory (Byrnes and Isidori, 1991; Wen, 1999). The passive system is a network theory concept, and has dissipative network characteristics. A system's dynamical characteristic, such as stabilization, etc., can be analyzed by using passive network theory. In this paper, a new chaos control method is proposed for avoiding chaos in Permanent Magnet Synchronous Motor system. The remainder of the paper is organized as follows. In Section I, through giving some definitions and reaching some conclusions by reasoning, the properties of passive system were studied. In Section II, a new chaos control method is proposed. Based on the properties of passive system, we studied the essential conditions, by which PMSM chaotic system could be equivalent to passive system. In Section III, the convergence condition is discussed in which the system can be globally asymptotically stabilized. Section IV gives the conclusion, which is that weakly minimum phase and minimum phase nonlinear system transformed by Permanent Magnet Synchronous Motor chaotic system could be globally asymptotically stabilized by smooth state feedback.

PROPERTIES OF PASSIVE SYSTEM

Consider a continuous chaotic system Σ de-

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scribed by differential equation as follows

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(x) \\ \boldsymbol{y} &= \boldsymbol{h}(x) \end{aligned} \tag{1}$$

where, $\dot{\boldsymbol{x}} \in \mathbb{R}^n$ is state variable, $\boldsymbol{y} \in \mathbb{R}^p$ is the output, $\boldsymbol{f}$ are smooth vector fields, function $\boldsymbol{h}$ is smooth mapping.

In order to analyzing the control method with which the chaotic dynamical system could be stable, we can find a smooth function $\boldsymbol{g}$ and input $\boldsymbol{u} \in \mathbb{R}^m$ to construct a new system:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(x) + \boldsymbol{g}(x) \cdot \boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{h}(x) \end{aligned} \tag{2}$$

where, $\dot{\boldsymbol{x}} \in \mathbb{R}^n$ is state variable, $\boldsymbol{u} \in \mathbb{R}^m$ is the input, $\boldsymbol{y} \in \mathbb{R}^p$ is the output, $\boldsymbol{f}$ and $\boldsymbol{g}$ are smooth vector fields, function $\boldsymbol{h}$ is smooth mapping. We suppose that the vector field $\boldsymbol{f}$ has at least one equilibrium point, without loss of generality, we can assume that the equilibrium point is $x=0$. If the equilibrium point is not at $x=0$, through coordinate shift we can transfer the equilibrium points to $x=0$.

Definition 1 A system Σ of the form Eq.(2) is said to be passive if functions $\boldsymbol{f}(x)$ and $\boldsymbol{g}(x)$ exist and there is a real-valued constant β, such that for any $\forall t \geq 0$

$$\int_0^t \boldsymbol{u}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau \geq \beta \tag{3}$$

or if there is a constant $\rho>0$ and real-valued constant β, such that for any $\forall t \geq 0$

$$\int_0^t \boldsymbol{u}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho \cdot \boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau \tag{4}$$

A system Σ of the form Eq.(2) is said to be passive if there is a nonnegative function $V(x)$: $X \to \mathbb{R}$, $V(0)=0$, named storage function, such that for any $\forall x \in X$, $\forall t \geq 0$

$$V(x) - V(x_0) \leq \int_0^t \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{u}(s)\mathrm{d}s \tag{5}$$

A passive system with storage function $V(x)$ is said to be strictly passive if there exists a positive definite function $S(x)$, such that for any $\forall x \in X$, $\forall t \geq 0$

$$V(x) - V(x_0) = \int_0^t \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{u}(s)\mathrm{d}s - \int_0^t S(x(s))\mathrm{d}s \tag{6}$$

Theorem 1 (Byrnes and Isidori, 1991) A system Σ of the form Eq.(2) which has the KYP property is passive, with storage function $V(x)$. Conversely, a passive system having a continuous and smooth storage function $V(x)$ has the KYP property.

Theorem 2 (Byrnes and Isidori, 1991) Suppose system Σ is passive with storage function $V(x)$ which is positive definite and suppose system Σ is locally zero-state detectable. Let φ be any smooth function such that $\boldsymbol{y}^{\mathrm{T}}\varphi(\boldsymbol{y})>0$ and $\varphi(0)=0$ for each nonzero $\boldsymbol{y}$. The control law

$$\boldsymbol{u}(t) = -\varphi(\boldsymbol{y}) \tag{7}$$

asymptotically stabilizes the equilibrium $x=0$.

Theorem 3 Suppose $x=0$ is an equilibrium point of chaotic dynamical system (2). If system (2) has relative degree {1, 1, …, 1} at $x=0$ and the system is weakly minimum phase, then system (2) can be locally feedback equivalent to a passive system with proper storage function *V(x)*.

Proof Let $\boldsymbol{y}=\boldsymbol{h}(x)$ be a new state variable and choose $\eta=\varphi(x)$. The system (2) is represented by equations of the form

$$\begin{cases} \dot{\boldsymbol{\eta}} = \boldsymbol{c}(\boldsymbol{\eta}, \boldsymbol{y}) + \boldsymbol{d}(\boldsymbol{\eta}, \boldsymbol{y})\boldsymbol{u} \\ \dot{\boldsymbol{y}} = \boldsymbol{b}(\boldsymbol{\eta}, \boldsymbol{y}) + \boldsymbol{a}(\boldsymbol{\eta}, \boldsymbol{y})\boldsymbol{u} \end{cases} \tag{8}$$

where $\boldsymbol{a}(\boldsymbol{\eta}, \boldsymbol{y})$ is nonsingular for all $(\boldsymbol{\eta}, \boldsymbol{y})$ at (0, 0). Choose the new feedback law

$$\boldsymbol{u} = \mathbf{a}(\boldsymbol{\eta}, \boldsymbol{y})^{-1}[-\boldsymbol{b}(\boldsymbol{\eta}, \boldsymbol{u}) + \boldsymbol{v}] \tag{9}$$

we have

$$\begin{cases} \dot{\boldsymbol{\eta}} = \boldsymbol{\theta}(\boldsymbol{\eta}, \boldsymbol{y}) + \gamma(\boldsymbol{\eta}, \boldsymbol{y})\boldsymbol{v} \\ \dot{\boldsymbol{y}} = \boldsymbol{v} \end{cases} \tag{10}$$

Let $\boldsymbol{z} = \boldsymbol{\eta} - \gamma(\boldsymbol{\eta}, 0)\boldsymbol{y}$, then

$$\begin{cases} \dot{\boldsymbol{z}} = \boldsymbol{f}^*(\boldsymbol{z}) + \boldsymbol{p}(\boldsymbol{z}, \boldsymbol{y})\boldsymbol{y} + (\sum_{i=1}^{m} \boldsymbol{q}_i(\boldsymbol{z}, \boldsymbol{y})\boldsymbol{y}_i) \cdot \boldsymbol{v} \\ \dot{\boldsymbol{y}} = \boldsymbol{v} \end{cases} \tag{11}$$

where $\boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})$, $\boldsymbol{q}_i(\boldsymbol{z},\boldsymbol{y})$ are proper matrices.

If system (2) is weakly minimum phase, there exists a positive definite function $W^*(\boldsymbol{z})$, with $W^*(0)=0$, such that $\boldsymbol{L}_{f^*}V(\boldsymbol{z})\le 0$ for each $z\neq 0$. Define

$$\boldsymbol{M}(\boldsymbol{z},\boldsymbol{y})=\begin{bmatrix}\boldsymbol{L}_{\boldsymbol{q}_1(\boldsymbol{z},\boldsymbol{y})}W^*(\boldsymbol{z})\\ \vdots \\ \boldsymbol{L}_{\boldsymbol{q}_m(\boldsymbol{z},\boldsymbol{y})}W^*(\boldsymbol{z})\end{bmatrix} \tag{12}$$

where $\boldsymbol{M}(0,\boldsymbol{y})=0$. Feedback control law

$$\boldsymbol{v}=[\boldsymbol{I}+\boldsymbol{M}(\boldsymbol{z},\boldsymbol{y})]^{-1}[-(\boldsymbol{L}_{\boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})}W^*(\boldsymbol{z}))^{\mathrm{T}}+\boldsymbol{w}] \tag{13}$$

is well defined in a neighborhood of (0, 0). Therefore, we have

$$\begin{bmatrix}\dot{\boldsymbol{z}}\\ \dot{\boldsymbol{y}}\end{bmatrix}=\bar{\boldsymbol{f}}(\boldsymbol{z},\boldsymbol{y})+\bar{\boldsymbol{g}}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{w} \tag{14}$$

Together with the positive function

$$V(\boldsymbol{z},\boldsymbol{y})=W^*(\boldsymbol{z})+\frac{1}{2}\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y}\,.$$

There is

$$\boldsymbol{L}_{\bar{f}}V(\boldsymbol{z},\boldsymbol{y})+\boldsymbol{L}_{\bar{g}}V(\boldsymbol{z},\boldsymbol{y})\boldsymbol{w}=\boldsymbol{L}_{f^*}W^*(\boldsymbol{z})+\boldsymbol{y}^{\mathrm{T}}\boldsymbol{w} \tag{15}$$

Therefore

$$\boldsymbol{L}_{\bar{f}}V(\boldsymbol{z},\boldsymbol{y})=\boldsymbol{L}_{f^*}W^*(\boldsymbol{z})\le 0$$

$$(\boldsymbol{L}_{\bar{g}}V(\boldsymbol{z},\boldsymbol{y}))^{\mathrm{T}}=\boldsymbol{y}$$

and system (2) is passive system.

PASSIVE CONTROL OF PMSM

Given the chaotic dynamical system of PMSM (Zhang *et al*., 2002):

$$\begin{cases}\mathrm{d}\tilde{i}_d/\mathrm{d}t=-\tilde{i}_d+\tilde{\omega}\tilde{i}_q+\tilde{u}_d\\ \mathrm{d}\tilde{i}_q/\mathrm{d}t=-\tilde{i}_q-\tilde{\omega}\tilde{i}_d+\tilde{u}_q+\gamma\tilde{\omega}\\ \mathrm{d}\tilde{\omega}/\mathrm{d}t=\sigma(\tilde{i}_q-\tilde{\omega})-\tilde{T}_L\end{cases} \tag{16}$$

where $\tilde{u}_d$ and $\tilde{u}_q$ is the voltage of d axes and q axes respectively, $\tilde{T}_L$ is external input torque, γ and σ are parameters.

Considering a kind of class condition, we suppose $\tilde{u}_d=\tilde{u}_q=\tilde{T}_L=0$, γ=20, σ=5.46 and $x_1=\tilde{i}_d$, $x_2=\tilde{\omega}$, $x_3=\tilde{i}_q$, Eq.(16) can be expressed by differential equation, that is

$$\begin{cases}\dot{x}_1=-x_1+x_2x_3\\ \dot{x}_2=5.46\cdot(x_3-x_2)\\ \dot{x}_3=-x_3-x_1x_2+20x_2\end{cases} \tag{17}$$

whereas "·" represent differentiation.

Design the controller

Add the controller u into the third equation of Eq.(17). Suppose that state vector x_3 is the output of PMSM system and suppose $z_1=x_1$, $z_2=x_2$, $y=x_3$, then the system can be expressed by normal form:

$$\begin{cases}\dot{z}_1=-z_1+z_2y\\ \dot{z}_2=5.46\cdot(y-z_2)\\ \dot{y}=-y-z_1z_2+20z_2+u\end{cases} \tag{18}$$

We have

$$\begin{cases}\dot{z}=f_0(z)+p(z,y)y\\ \dot{y}=b(z,y)+a(z,y)u\end{cases} \tag{19}$$

where

$f_0(z)=[-z_1,\ -5.46z_2]^{\mathrm{T}}$, $p(z,y)=[z_2,\ 5.46]^{\mathrm{T}}$, $a(z,y)=1$, $b(z,y)=-y-z_1z_2+20z_2$.

Choose storage function

$$V(z,y)=W(z)+\frac{1}{2}y^2 \tag{20}$$

where $W(z)$ is the Lyapunov function of $f_0(z)$, and $W(0)=0$.

$$W(z)=\frac{1}{2}z_1^2+\frac{1}{2}z_2^2 \tag{21}$$

From Eq.(21), we have that $W(z)$ is Lyapunov function of $f_0(z)$ because $\mathrm{d}W/\mathrm{d}t\le 0$.

According to Theorem 4, we know that $f_0(z)$ should be globally stable if Eq.(17) is equivalent to passive system, where the eigenvalues of $f_0(z)$ are $\lambda_1=-1$, $\lambda_2=-5.46$, so $f_0(z)$ is globally stable at (0, 0). The PMSM chaotic dynamical system can be equivalent to passive system.

According to the above analysis, the zero dynamical character of PMSM system is Lyapunov stable, so the PMSM chaotic system is minimum phase system. From Eq.(20)

$$\begin{aligned}\dot{V} &= \frac{\partial W}{\partial z}\dot{z} + y\dot{y} \\ &= \frac{\partial}{\partial z}W(z)\cdot f_0(z) + \frac{\partial}{\partial z}W(z)\cdot p(z,y)\cdot y \\ &\quad + [b(z,y) + a(z,y)u]y \end{aligned} \quad (22)$$

Because the equivalent system is minimum phase and have KYP property, we have

$$\frac{\partial}{\partial}W(z)\cdot f_0(z) \le 0$$

From Eq.(22)

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le \frac{\partial}{\partial z}W(z)\cdot p(z,y)\cdot y + [b(z,y) + a(z,y)u]y \quad (23)$$

Let

$$u = a^{-1}(z,y)[-b^{\mathrm{T}}(z,y) - \frac{\partial}{\partial z}W(z)p(z,y) - \alpha y + v]$$

where α is positive real constant, v is external input signal. We have

$$u = (1-\alpha)y + v - 25.46z_2 \quad (24)$$

According to Eq.(17) and Eq.(24), we have

$$\begin{cases} \dot{x}_1 = -x_1 + x_2x_3 \\ \dot{x}_2 = 5.46\cdot(x_3 - x_2) \\ \dot{x}_3 = -\alpha x_3 - x_1x_2 - 5.46x_2 + v \end{cases} \quad (25)$$

Convergence condition

In order to discuss the stabilization of the system at fixed point x_e=0, nonlinear function can be evolved as Taylor progression near fixed point x_e=0.

$$\dot{x} = J(x - x_e) + R(x)$$

where $R(x)$ is the high rank derivative item of the evolving expression, $\boldsymbol{J}(x) = f'(x) = \frac{\partial f(x)}{\partial x}$ is the Jacobin matrix.

According to nonlinear control theory, the nonlinear system can be stabilized if all of the eigenvalues of Jacobin matrix $\boldsymbol{J}$ have negative real parts.

To stabilize the chaotic synchronization system, we solve the eigenvalues of Jacobin matrix $\boldsymbol{J}$ at fixed point x_e=0, so

$$\lambda_i = \lambda_i(\alpha, x, v) \;\; i = 1, 2, \ldots, n \quad (26)$$

where λ_i are the eigenvalues of $\boldsymbol{J}$.

Let all of the real parts of eigenvalues less than zero be

$$\mathrm{Re}\{\lambda_i(\alpha, x, v)\} < 0 \;\; i = 1, 2, \ldots, n \quad (27)$$

Therefore, we can get the convergence condition.
Let $\mathrm{Re}\{\lambda_i(\alpha, x, v)\} < 0, i = 1, 2, 3,$ we have

$$\alpha > -5.46\,.$$

The parameter α>0 was defined when we designed the controller, therefore the convergence condition of the PMSM chaotic system is α>0.

Simulation

Suppose the initial point of PMSM system is (1, −1, 0.6) and suppose α=2, v=0. Fig.1 is the output of PMSM system when α=2, v=0. It is about 13 seconds

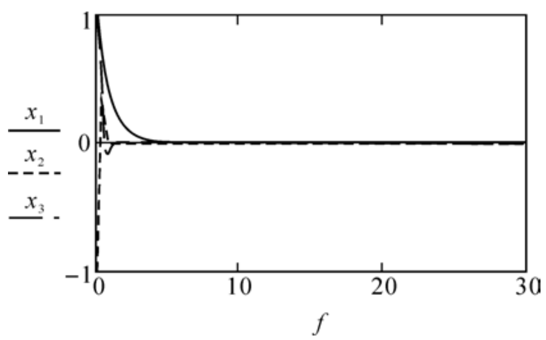

Fig.1 Outputs of PMSM when α=2, v=0

that the system is stabilized at the initial point under the control of $u(t)$.

Fig.2 is the output of the controlled PMSM system when additive noise n=0.01. The system also can be stabilized at the desired point although the additive noise influences the controlled system. The controlled PMSM chaotic dynamical system is robust.

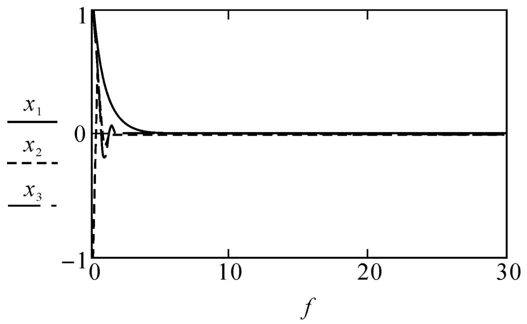

Fig.2 Output of PMSM when additive noise *n*=0.01

CONCLUSIONS

The character of passive system is a network theory concept, and has properties of dissipative network. Based on the property of passive system, we studied the essential conditions, by which PMSM chaotic system could be equivalent to passive system. Theoretical proof revealed that use of state feedback could make the passive system transformed by PMSM chaotic dynamical system globally asymptotically stabilized.

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