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Waves scattering induced by an interface crack in a coated material*

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Abstract: This paper deals with the two-dimensional problem of elastic wave scattering from a finite crack at the interface between a coated material layer and its substrate. By adopting the Fourier transform method and introducing the crack opening displacement function, the boundary value problem is simplified for numerically solving a system of Cauchy-type singular integral equations by means of Jacobi polynomial expansion. The stress intensity factors and the crack opening displacements are defined in terms of the integral equations solutions. The influence of the dimensionless wave number and the ratio of crack length to layer thickness on the stress intensity factors and crack opening displacements are discussed.

Key words: Wave scattering, Interfacial crack, Coated materials
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INTRODUCTION

Cracks are likely to occur on the interfaces of coated materials widely applied in engineering. It is important to detect the interface cracks by non-destructive means. Detecting the scattered waves induced by interfacial cracks by using ultrasonic technique can be considered as one of the most feasible methods. This paper focuses on the theoretical basis for the study of wave scattering induced by interfacial cracks.

In the last two decades, there has been a large number of published works bearing on the problem of the interaction of elastic waves with interface cracks. Yang and Bogy (1985) used Green's theorem to derive the integral equations of the scattered field induced by an interface crack in a layered half space, and numerically obtained the stress intensity factors for various material combinations. Qu (1994; 1995) studied the two-dimensional problem of a finite interface crack between two dissimilar solids, which are isotropic, and loaded by a plane wave using Fourier transform technique. The same method was adopted by Wang (1997) and Feng (1999) to study the scattering of elastic waves by interface cylindrical cracks. In the case of wave scattering from the interfacial cracks in piezoelectric media, Shen *et al.*(2000) and Wang (2001) researched the dynamic electromechanical behavior under anti-plane mechanical loading. Wu (2004) investigated the diffraction of a plane stress wave by a semi-infinite crack in anisotropic elastic materials by employing Stroh-like formalism to obtain the solution directly in the time domain.

This work is aimed at analyzing plane wave scattering induced by an interface crack between the coating layer and its substrate. Fourier transform technique and numerical integral method were used. In the numerical analysis, two fracture mechanics parameters, stress intensity factors and crack opening displacement, are numerically computed for the incident longitudinal wave from the free surface.

PROBLEM STATEMENT AND FORMULATION

Consider the steady time-harmonic plane-strain problem of the cracked layered elastic half space

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shown in Fig.1. The layered half space is composed of an isotropic layer bonded at its lower surface to the upper surface of an isotropic substrate of a different material. A crack with length of $2a$ is located at the interface between the coating layer and the substrate, and the crack faces are assumed to be traction free.

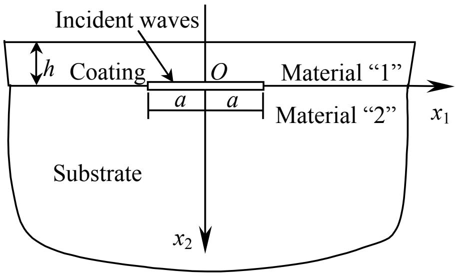

Fig.1 The layered half space with interface crack

In the case of the plane strain problem, with the time factor $e^{-i\omega t}$ and the body force omitted, the wave motion equations for the layer and the substrate can be expressed in terms of two scalar displacement potentials.

$$\nabla^2\varphi^{(n)}+(k_L^{(n)})^2\varphi^{(n)}=0,\quad \nabla^2\psi^{(n)}+(k_T^{(n)})^2\psi^{(n)}=0 \tag{1}$$

in which

$$k_L^{(n)}=\omega\sqrt{\rho^{(n)}/(\lambda^{(n)}+2\mu^{(n)})},\quad k_T^{(n)}=\omega\sqrt{\rho^{(n)}/\mu^{(n)}} \tag{2}$$

and where n=1,2 denotes the layer and the substrate, respectively; ω is the circular frequency of the incident wave; $\rho^{(n)}$, $\lambda^{(n)}$, $\mu^{(n)}$ are the densities and Láme constants.

From the linear superposition, the total wave field caused by the incident wave can be written as

$$[\varphi_t^{(n)},\psi_t^{(n)}]^{\mathrm{T}}=[\varphi_P^{(n)},\psi_P^{(n)}]^{\mathrm{T}}+[\varphi^{(n)},\psi^{(n)}]^{\mathrm{T}} \tag{3}$$

where $[\varphi_t^{(n)},\psi_t^{(n)}]^{\mathrm{T}}$ are the total wave fields; $[\varphi_P^{(n)},\psi_P^{(n)}]^{\mathrm{T}}$ the primary wave fields without the influence of interfacial crack; $[\varphi^{(n)},\psi^{(n)}]^{\mathrm{T}}$ the scattered fields induced by the interfacial crack. This paper focuses on the scattered fields. The displacement and stress components can be expressed as

$$u_\alpha^{(n)}=\varphi_{,\alpha}^{(n)}+e_{\alpha\beta}\psi_{,\beta}^{(n)} \tag{4}$$

$$\sigma_{\alpha\beta}^{(n)}=\lambda^{(n)}\nabla^2\varphi^{(n)}\delta_{\alpha\beta}+2\mu^{(n)}\varphi_{,\alpha\beta}^{(n)}+\mu^{(n)}(e_{\alpha\gamma}\psi_{,\beta\gamma}^{(n)}+e_{\beta\gamma}\psi_{,\alpha\gamma}^{(n)}) \tag{5}$$

here, α,β,γ=1,2 and $e_{11}=e_{22}=0$, $e_{12}=-e_{21}=1$.

Defining $\boldsymbol{\sigma}^{(n)}=[\sigma_{12}^{(n)},\sigma_{22}^{(n)}]^{\mathrm{T}}$, $\boldsymbol{u}^{(n)}=[u_1^{(n)},u_2^{(n)}]^{\mathrm{T}}$, the scattered field solution satisfies the following free boundary conditions and interfacial conditions

$$\boldsymbol{\sigma}^{(1)}=\mathbf{0},\ (x_2=-h) \tag{6a}$$
$$\boldsymbol{\sigma}^{(1)}=\boldsymbol{\sigma}^{(2)}=-\boldsymbol{\sigma}_0;\ (x_2=0,\ |x_1|<a) \tag{6b}$$
$$\boldsymbol{\sigma}^{(1)}=\boldsymbol{\sigma}^{(2)},\quad \boldsymbol{u}^{(1)}=\boldsymbol{u}^{(2)};\ (x_2=0,\ |x_1|>a) \tag{6c}$$

where $\boldsymbol{\sigma}_0$ is the primary traction at the interface.

Eqs.(6a)~(6c) provide enough boundary conditions to uniquely determine $\boldsymbol{u}^{(1)}$ and $\boldsymbol{u}^{(2)}$ if the following radiation conditions are enforced.

$$\boldsymbol{u}^{(2)}=0;\ (x_2\rightarrow+\infty) \tag{7}$$

The Fourier transform method is employed to solve the scattered field. For clarity, the overbar symbol " ¯ " is used to denote the transformed function, and ξ is the transform variable. Eq.(1) can be transformed to

$$\bar{\varphi}_{,22}^{(n)}+(q_L^{(n)})^2\bar{\varphi}^{(n)}=0,\quad \bar{\psi}_{,22}^{(n)}+(q_T^{(n)})^2\bar{\psi}^{(n)}=0 \tag{8}$$

where $(q_L^{(n)})^2=(k_L^{(n)})^2-\xi^2$, $(q_T^{(n)})^2=(k_T^{(n)})^2-\xi^2$.

By using inverse Fourier transform, and substituting the solutions of Eq.(8) into Eqs.(4) and (5), we can obtain the scattered displacement and stress fields. Let

$$\boldsymbol{u}^{(n)}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\Big[\boldsymbol{P}_1^{(n)}(\xi)\boldsymbol{E}_1^{(n)}(x_2)\boldsymbol{\eta}_1^{(n)}(\xi)+\boldsymbol{P}_2^{(n)}(\xi)\boldsymbol{E}_2^{(n)}(x_2)\boldsymbol{\eta}_2^{(n)}(\xi)\Big]e^{-i\xi x_1}d\xi \tag{9}$$

$$\boldsymbol{\sigma}^{(n)}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\Big[\boldsymbol{Q}_1^{(n)}(\xi)\boldsymbol{E}_1^{(n)}(x_2)\boldsymbol{\eta}_1^{(n)}(\xi)+\boldsymbol{Q}_2^{(n)}(\xi)\boldsymbol{E}_2^{(n)}(x_2)\boldsymbol{\eta}_2^{(n)}(\xi)\Big]e^{-i\xi x_1}d\xi \tag{10}$$

where

$$\boldsymbol{P}_m^{(n)}(\xi)=\begin{bmatrix}-i\xi & (-1)^m iq_T^{(n)}\\ (-1)^m iq_L^{(n)} & i\xi\end{bmatrix},$$

$$\boldsymbol{Q}_m^{(n)}(\xi)=\mu^{(n)}\begin{bmatrix}2(-1)^m\xi q_L^{(n)} & \left[\xi^2-(q_T^{(n)})^2\right]\\ \left[\xi^2-(q_T^{(n)})^2\right] & -2(-1)^m\xi q_T^{(n)}\end{bmatrix},$$

$$\boldsymbol{E}_m^{(n)}(x_2)=\begin{bmatrix}\mathrm{e}^{(-1)^m\mathrm{i}q_L^{(n)}x_2} & 0\\ 0 & \mathrm{e}^{(-1)^m\mathrm{i}q_T^{(n)}x_2}\end{bmatrix},$$

$$\boldsymbol{\eta}_m^{(n)}(\xi)=\begin{bmatrix}A_m^{(n)}(\xi)\\ B_m^{(n)}(\xi)\end{bmatrix},\ m=1,2.$$

here, $\boldsymbol{\eta}_1^{(n)}$ and $\boldsymbol{\eta}_2^{(n)}$ are unknown vectors. Considering the radiation condition of scattered waves in the substrate, we get $\boldsymbol{\eta}_1^{(2)}(\xi)=\boldsymbol{0}$. To determine other unknown vectors, boundary conditions Eqs.(6a)~(6c) must be used.

First, the free boundary condition Eq.(6a) implies

$$\boldsymbol{K}_1(\xi)\boldsymbol{\eta}_1^{(1)}(\xi)+\boldsymbol{K}_2(\xi)\boldsymbol{\eta}_2^{(1)}(\xi)=0 \tag{11}$$

in which $\boldsymbol{K}_m(\xi)=\boldsymbol{Q}_m^{(1)}(\xi)\boldsymbol{E}_m^{(1)}(-h)$, m=1,2.

From the stress continuity condition $\boldsymbol{\sigma}^{(1)}=\boldsymbol{\sigma}^{(2)}$ at the interface between the coating layer and the substrate, the following equation can be derived.

$$\boldsymbol{Q}_1^{(1)}\boldsymbol{\eta}_1^{(1)}+\boldsymbol{Q}_2^{(1)}\boldsymbol{\eta}_2^{(1)}=\boldsymbol{Q}_2^{(2)}\boldsymbol{\eta}_2^{(2)} \tag{12}$$

Furthermore, define the crack opening displacement and the dislocation density, respectively, as

$$\Delta\boldsymbol{u}(x_1)=\boldsymbol{u}^{(2)}(x_1,0)-\boldsymbol{u}^{(1)}(x_1,0) \tag{13a}$$

$$\boldsymbol{f}(x_1)=\mathrm{d}[\Delta\boldsymbol{u}(x_1)]/\mathrm{d}x_1,\ \int_{-a}^{a}\boldsymbol{f}(x_1)\mathrm{d}x_1=0 \tag{13b}$$

Taking the Fourier transform of Eq.(13a) yields

$$\Delta\bar{\boldsymbol{u}}(\xi)=\mathrm{i}\xi^{-1}\int_{-a}^{a}\boldsymbol{f}(\varsigma)\mathrm{e}^{\mathrm{i}\varsigma\xi}\mathrm{d}\varsigma \tag{14}$$

Combining Eqs.(9), (11) and (12), yields Eq.(14) which can be written as

$$\boldsymbol{\eta}_2^{(2)}=\mathrm{i}\xi^{-1}\boldsymbol{G}(\xi)\int_{-a}^{a}\boldsymbol{f}(\varsigma)\mathrm{e}^{\mathrm{i}\varsigma\xi}\mathrm{d}\varsigma \tag{15}$$

in which $\boldsymbol{G}(\xi)=[\boldsymbol{P}_2^{(n)}-\boldsymbol{M}_P\boldsymbol{M}_Q^{-1}\boldsymbol{Q}_2^{(2)}]^{-1}$, $\boldsymbol{M}_P(\xi)=\boldsymbol{P}_1^{(1)}-\boldsymbol{P}_2^{(1)}\boldsymbol{K}_2^{-1}\boldsymbol{K}_1$, $\boldsymbol{M}_Q(\xi)=\boldsymbol{Q}_1^{(1)}-\boldsymbol{Q}_2^{(1)}\boldsymbol{K}_2^{-1}\boldsymbol{K}_1$. Thus, the stress free condition Eq.(6b) on the crack faces becomes

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\boldsymbol{Q}_2^{(2)}\boldsymbol{\eta}_2^{(2)}\mathrm{e}^{-\mathrm{i}\xi x_1}\mathrm{d}\xi=-\boldsymbol{\sigma}_0(x_1) \tag{16}$$

Substitution of Eq.(15) into Eq.(16) results in an integral equation of the dislocation density

$$\frac{\mathrm{i}}{2\pi}\int_{-\infty}^{+\infty}\int_{-a}^{a}\xi^{-1}\boldsymbol{Q}_2^{(2)}\boldsymbol{G}\boldsymbol{f}(\varsigma)\mathrm{e}^{\mathrm{i}\xi(\varsigma-x_1)}\mathrm{d}x\mathrm{d}\xi=-\boldsymbol{\sigma}_0(x_1) \tag{17}$$

By changing the integral order, Eq.(17) can be expressed as

$$\int_{-a}^{a}\boldsymbol{H}(\xi,\varsigma,x_1)\boldsymbol{f}(x)\mathrm{d}x=-\boldsymbol{\sigma}_0(x_1) \tag{18}$$

where $\boldsymbol{H}(\varsigma,x_1)=\frac{\mathrm{i}}{2\pi}\int_{-\infty}^{+\infty}\xi^{-1}\boldsymbol{Q}_2^{(2)}\boldsymbol{G}\mathrm{e}^{\mathrm{i}(\varsigma-x_1)\xi}\mathrm{d}\xi$.

The integral Eq.(18) will be reduced to a standard Cauchy singular integral equation of the second kind. First, let us analyze the limit of $\xi^{-1}\boldsymbol{Q}_2^{(2)}\boldsymbol{G}$ when $\xi\rightarrow\pm\infty$. After some asymptotic analysis, one gets

$$\lim_{\xi\to\infty}(\xi^{-1}\boldsymbol{Q}_2^{(2)}\boldsymbol{G})=L_1\xi^{-1}\left|\xi\right|\boldsymbol{I}+\mathrm{i}L_2\boldsymbol{I}_0 \tag{19}$$

in which $\boldsymbol{I}$ is the identity matrix, and $L_1=S_1/S$, $L_2=S_2/S$. Here,

$$S=\left[\left(k_L^{(1)}\right)^2(\mu^{(1)}-\mu^{(2)})-\left(k_T^{(1)}\right)^2(\mu^{(1)}+\mu^{(2)})\right]\times\left[\left(k_L^{(2)}\right)^2(\mu^{(1)}-\mu^{(2)})+\left(k_T^{(2)}\right)^2(\mu^{(1)}+\mu^{(2)})\right],$$

$$S_1=2\mu^{(1)}\mu^{(2)}\left[\left(k_T^{(1)}k_T^{(2)}\right)^2(\mu^{(1)}+\mu^{(2)})-\left(k_L^{(1)}k_T^{(2)}\right)^2\mu^{(1)}-\left(k_T^{(1)}k_L^{(2)}\right)^2\mu^{(2)}\right],$$

$$S_2=2\mu^{(1)}\mu^{(2)}\left[-\left(k_L^{(1)}k_L^{(2)}\right)^2(\mu^{(1)}-\mu^{(2)})+\left(k_T^{(1)}k_L^{(2)}\right)^2\mu^{(1)}-\left(k_L^{(1)}k_T^{(2)}\right)^2\mu^{(2)}\right],$$

$$\boldsymbol{I}_0=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}.$$

Let $r=L_2/L_1$, $\boldsymbol{H}_0(\varsigma,x_1)=\frac{\mathrm{i}}{2\pi L_1}\int_{-\infty}^{+\infty}(\xi^{-1}\boldsymbol{Q}_2^{(2)}\boldsymbol{G}$

$-L_1\xi^{-1}|\xi|\boldsymbol{I}-\mathrm{i}L_2\boldsymbol{I}_0)\mathrm{e}^{\mathrm{i}(\varsigma-x_1)\xi}\mathrm{d}\xi$, substituting Eq.(19) into Eq.(18), yields the singular integral equation in the following form

$$r\boldsymbol{I}_0\boldsymbol{f}(x_1)+\frac{1}{\pi}\int_{-a}^{a}\frac{\boldsymbol{f}(\varsigma)}{(\varsigma-x_1)}\mathrm{d}\varsigma-\int_{-a}^{a}\boldsymbol{H}_0\boldsymbol{f}(\varsigma)\mathrm{d}\varsigma=\boldsymbol{\sigma}_0(x_1)/L_1 \quad (20)$$

This is the governing equation for an interface crack in a coated medium. Once the dislocation density $\boldsymbol{f}(x_1)$ is solved from it, the stress and displacement field of the scattered waves can be obtained.

NUMERICAL METHOD FOR SOLVING THE INTEGRAL EQUATION

In this section, a numerical method is introduced for solving the integral Eq.(20). First, define

$$\boldsymbol{R}=\begin{bmatrix}\mathrm{i} & 1\\ 1 & \mathrm{i}\end{bmatrix},\ \boldsymbol{\varLambda}=\begin{bmatrix}\mathrm{i} & 0\\ 0 & -\mathrm{i}\end{bmatrix} \quad (21)$$

After introducing $x=x_1/a$, $\zeta=\varsigma/a$, $\boldsymbol{f}(x_1)=\boldsymbol{R}\boldsymbol{g}(x)$, $\boldsymbol{W}(\zeta,x)=-a\boldsymbol{R}^{-1}\boldsymbol{I}_0^{-1}\boldsymbol{H}_0\boldsymbol{R}/L_1$, $\boldsymbol{t}(x)=\boldsymbol{R}^{-1}\boldsymbol{I}_0^{-1}\boldsymbol{\sigma}_0(x_1)/L_1$, Eq.(20) can be written as

$$r\boldsymbol{g}(x)+\frac{1}{\pi}\int_{-1}^{1}\frac{\boldsymbol{\varLambda}\boldsymbol{g}(\zeta)}{(\zeta-x)}\mathrm{d}\zeta+\int_{-1}^{1}\boldsymbol{W}(\zeta-x)\boldsymbol{g}(\zeta)\mathrm{d}\zeta=\boldsymbol{t}(x),\ \ (|x|<1) \quad (22)$$

Eq.(22)'s solution can be approximated by a Jacobi polynomial

$$\boldsymbol{g}(x)=\boldsymbol{Q}(x)\sum_{n=1}^{N}\boldsymbol{P}_n(x)\boldsymbol{c}_n \quad (23)$$

in which

$$\boldsymbol{Q}(x)=\begin{bmatrix}(1-x)^{\gamma_1}(1+x)^{\gamma_2} & 0\\ 0 & (1-x)^{\gamma_2}(1+x)^{\gamma_1}\end{bmatrix},$$

$$\boldsymbol{P}_n(x)=\begin{bmatrix}P_n^{(\gamma_1,\gamma_2)}(x) & 0\\ 0 & P_n^{(\gamma_2,\gamma_1)}(x)\end{bmatrix},$$

where $\gamma_1=-\frac{1}{2}-\mathrm{i}\varepsilon$, $\gamma_2=-\frac{1}{2}+\mathrm{i}\varepsilon$, $\varepsilon=\frac{1}{2\pi}\ln\frac{(1+r)}{(1-r)}$, $\boldsymbol{c}_n$ is a constant vector to be determined.

Since there exists the following integral relationship

$$\int_{-1}^{1}\frac{\boldsymbol{Q}(\zeta)\boldsymbol{P}_n(\zeta)\mathrm{d}\zeta}{\pi(\zeta-x)}=r\boldsymbol{\varLambda}\boldsymbol{Q}(x)\boldsymbol{P}_n(x)+\frac{\sqrt{1-r^2}\boldsymbol{N}_{n-1}(x)}{2}\quad (|x|<1) \quad (24)$$

where $\boldsymbol{N}_n(x)=\begin{bmatrix}P_n^{(-\gamma_1,-\gamma_2)}(x) & 0\\ 0 & P_n^{(-\gamma_2,-\gamma_1)}(x)\end{bmatrix}$.

Substituting Eq.(23) into Eq.(22) yields

$$\sum_{n=1}^{N}\left[\frac{\sqrt{1-r^2}\boldsymbol{\varLambda}\boldsymbol{N}_{n-1}(x)}{2}+\int_{-1}^{1}\boldsymbol{W}(\zeta-x)\boldsymbol{Q}(\zeta)\boldsymbol{P}_n(\zeta)\mathrm{d}\zeta\right]\times\boldsymbol{c}_n=\boldsymbol{t}(x) \quad (25)$$

Next, pre-multiplying Eq.(25) by $\boldsymbol{N}_{m-1}(x)\boldsymbol{Q}^{-1}(x)$ and integrating it from –1 to 1 with respect to x yields the following system of equations for $\boldsymbol{c}_n$. In the deriving process, the following orthogonality property of the Jacobi polynomials was used.

$$\sum_{n=1}^{N}[\boldsymbol{V}_{mn}+y_m\delta_{mn}\boldsymbol{\varLambda}]\boldsymbol{c}_n=\boldsymbol{s}_m,\ m=1,2,\ldots,N \quad (26)$$

where

$$y_m=\sqrt{1-r^2}\Gamma(m-\gamma_1)\Gamma(m-\gamma_2)/[m!]^2 \quad (27)$$

$$\boldsymbol{V}_{mn}=\int_{-1}^{1}\int_{-1}^{1}\boldsymbol{N}_{m-1}(x_1)\boldsymbol{Q}^{-1}(x_1)\boldsymbol{W}(x-x_1)\boldsymbol{Q}(x)\boldsymbol{P}_n(x)\boldsymbol{c}_n\mathrm{d}x]\mathrm{d}x_1=v_{mn}\int_{-\infty}^{\infty}(\mathrm{i}a\xi)^{m+n-1}\boldsymbol{Y}_m(a\xi)\tilde{\boldsymbol{H}}_0(\xi)\boldsymbol{F}_n(a\xi)\mathrm{d}\xi \quad (28)$$

$$\boldsymbol{s}_m=\int_{-1}^{1}\boldsymbol{N}_{m-1}(x)\boldsymbol{Q}^{-1}(x)\boldsymbol{t}(x)\mathrm{d}x \quad (29)$$

In Eq.(28),

$$v_{mn}=\frac{(-1)^{m+1}2^{m+n+1}}{(m-1)!n!}B(m-\gamma_1,n-\gamma_2)\times B(n+1+\gamma_1,n+1+\gamma_2),$$

$$\tilde{\boldsymbol{H}}_0(\xi)=\frac{a\mathrm{i}}{2\pi}\boldsymbol{R}^{-1}\boldsymbol{I}_0^{-1}(\xi^{-1}|\xi|\boldsymbol{I}+\mathrm{i}r\boldsymbol{I}_0-\xi^{-1}\boldsymbol{Q}_2^{(2)}\boldsymbol{G}/L_1)\boldsymbol{R},$$

$$\boldsymbol{Y}_m(x)=\begin{bmatrix}Z_1 & 0\\ 0 & Z_2\end{bmatrix},\ \boldsymbol{F}_n(x)=\begin{bmatrix}Z_3 & 0\\ 0 & Z_4\end{bmatrix},$$

in which $Z_1={}_1F_1(m-\gamma_2,2m-\gamma_1-\gamma_2,-2\mathrm{i}x)$,
$Z_2={}_1F_1(m-\gamma_1,2m-\gamma_1-\gamma_2,-2\mathrm{i}x)$,
$Z_3={}_1F_1(n+1+\gamma_2,2n+2+\gamma_1+\gamma_2,2\mathrm{i}x)$,
$Z_4={}_1F_1(n+1+\gamma_1,2n+2+\gamma_1+\gamma_2,2\mathrm{i}x)$.

Here, $B(x,y)$ is the Beta function and ${}_1F_1(x,y,z)$ is the confluent hyper geometric function.

To obtain the unknown vector $\boldsymbol{c}_n$ from Eq.(26), the matrix $\boldsymbol{V}_{mn}$ must be first computed numerically by truncating the infinite integral. After these equations are solved, an approximate solution to $\boldsymbol{g}(x)$ can be obtained.

NUMERICAL RESULTS

In this section, two important fracture mechanics parameters, namely, stress intensity factors and crack opening displacement, are derived in terms of the dislocation density. According to the conventional definition of the stress intensity factors for interface crack, at the interface the stress can be expressed as

$$\sigma_{22}(ax,0)+\mathrm{i}\sigma_{12}(ax,0)=\frac{K_1+\mathrm{i}K_2}{\sqrt{2\pi a(x-1)}}\left(\frac{x-1}{2}\right)^{\mathrm{i}\varepsilon} \quad (30)$$

After introducing the matrix $\tilde{\boldsymbol{Q}}(x)$, $\boldsymbol{k}$ can be expressed by Jacobi polynomials.

$$\tilde{\boldsymbol{Q}}(x)=\begin{bmatrix}[(x-1)/(x+1)]^{\mathrm{i}\varepsilon} & 0\\ 0 & [(x+1)/(x-1)]^{\mathrm{i}\varepsilon}\end{bmatrix} \quad (31)$$

$$\boldsymbol{k}=[K_2,K_1]^{\mathrm{T}}=\lim_{x\to 1^+}[\sqrt{2\pi a(x-1)}\boldsymbol{R}\tilde{\boldsymbol{Q}}\boldsymbol{R}^{-1}\boldsymbol{\sigma}(ax,0)]$$
$$=L_1\sqrt{\pi a(1-r^2)}\boldsymbol{R}\sum_{n=1}^{N}\boldsymbol{P}_n(1)\boldsymbol{c}_n \quad (32)$$

The stress intensity vector normalized by the static mode I stress intensity factor $K_1^0=\sigma_0\sqrt{\pi a}$ can be defined as

$$\tilde{\boldsymbol{k}}=[\tilde{K}_2,\tilde{K}_1]^{\mathrm{T}}=\boldsymbol{k}/K_1^0 \quad (33)$$

Next, according to Eqs.(13) and (23), we can define the crack opening displacement vector $\Delta\boldsymbol{u}(ax)$. Moreover, the displacement vector can be normalized by $\Delta u_2^0(0)$, which indicates the static crack opening displacement along the direction of x_2.

$$\Delta\boldsymbol{u}(ax)=\boldsymbol{R}\left[\sum_{n=1}^{N}\int_{-1}^{x}\boldsymbol{Q}(\varsigma)\boldsymbol{P}_n(\varsigma)\mathrm{d}\varsigma\right]\boldsymbol{c}_n \quad (34)$$

$$\Delta\tilde{\boldsymbol{u}}=[\Delta\tilde{u}_1,\Delta\tilde{u}_2]^{\mathrm{T}}=\Delta\boldsymbol{u}/\Delta u_2^0(0) \quad (35)$$

According to the above analysis, the numerical results of $\tilde{\boldsymbol{k}}$ and $\Delta\tilde{\boldsymbol{u}}$ can be obtained for various plane waves propagating in the coated medium. As an example, the incident wave is taken to be a longitudinal plane wave normal to the interface from the upper face of the coating layer. The materials of the coating layer and the substrate are nickel and iron, respectively. Their properties are listed in Table 1.

The normalized crack opening displacements are presented in Fig.2 for the case when the ratio of the crack length to the layer thickness equals to 1. It is observed that the frequency of the incident wave has great influence on the crack opening displacements.

Fig.3 shows the normalized stress intensity factors $\tilde{K}_1$ and $\tilde{K}_2$ as functions of dimensionless wave number for various ratios of crack length to layer thickness. It is worth noting that even for the case of incident longitudinal wave being normal to the interface, the mode II stress intensity factor is not zero. Moreover, there exist resonances of the stress intensity factors at specific values of $k_L^{(1)}a$. Clearly, the peak values of the resonances depend strongly on the ratio of the crack length to layer thickness.

CONCLUSION

The problem of elastic wave scattering induced by an interface crack in a coated material was investigated in this work. Uses of Fourier transform and numerical integral method yield the stress intensity

Table 1 Material properties

Material	Poisson's ratio	Shear modulus μ (GPa)	Density ρ (kg/m^3)	Longitudinal wave velocity c_{L} (m/s)	Transverse wave velocity c_{T} (m/s)
Nickel (Ni)	0.31	66.5	8800	5240	2750
Iron (Fe)	0.28	77.0	7700	5720	3160

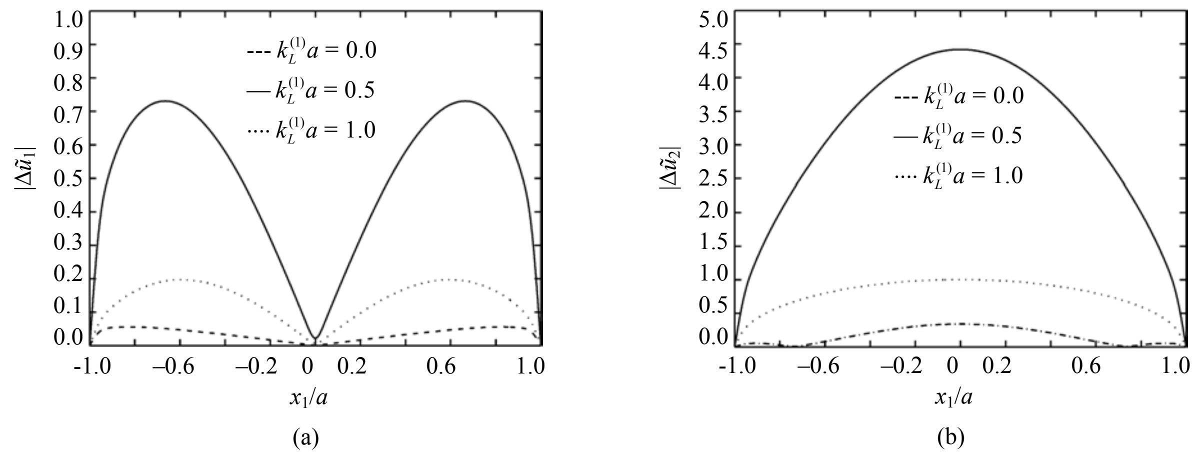

Fig.2 Normalized crack opening displacements (*a*/*h*=1)
(a) The displacement in the x_1 direction; (b) The displacement in the x_2 direction

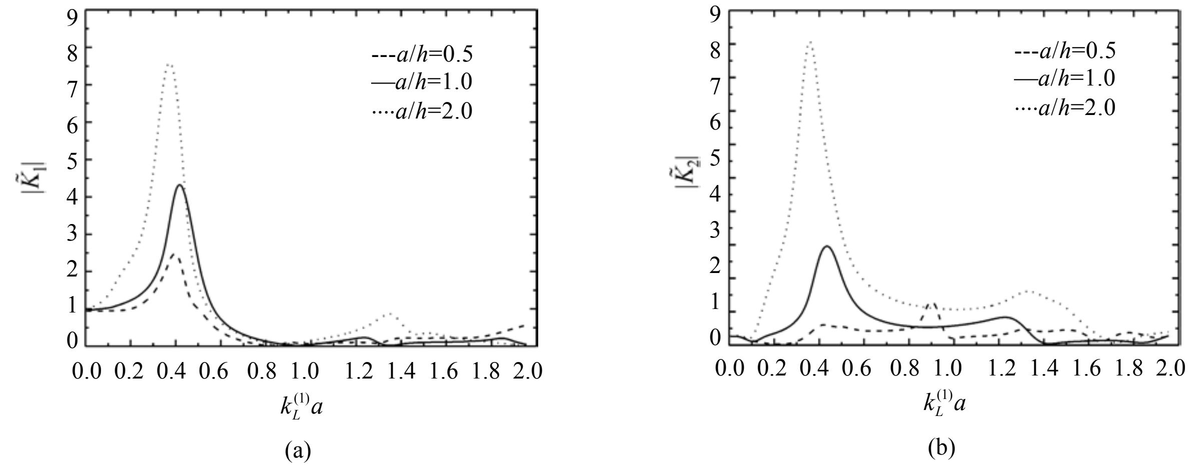

Fig.3 Normalized stress intensity factors as a function of dimensionless wave number
(a) The stress intensity factor $\tilde{K}_1$; (b) The stress intensity factor $\tilde{K}_2$

factors and the crack opening displacements numerically. For a special dimensionless wave number $k_L^{(1)}a$, the stress intensity factors show resonances, the peak values of which depend strongly on ratio of the crack length to the layer thickness.

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