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Active control of structural vibration by piezoelectric stack actuators*

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Abstract: This paper presents a general analytical model of flexible isolation system for application to the installation of high-speed machines and lightweight structures. Piezoelectric stack actuators are employed in the model to achieve vibration control of flexible structures, and dynamic characteristics are also investigated. Mobility technique is used to derive the governing equations of the system. The power flow transmitted into the foundation is solved and considered as a cost function to achieve optimal control of vibration isolation. Some numerical simulations revealed that the analytical model is effective as piezoelectric stack actuators can achieve substantial vibration attenuation by selecting proper value of the input voltage.

Key words: Piezoelectric actuator, Power flow, Active control
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INTRODUCTION

Passive methods have played an important role for a long time (Snowdon, 1973; Nelson, 1982; Pan *et al*., 1992) in vibration and noise control. However, with inrush of some novel engineering structures and machines, such as super high-speed machine tools, long-span bridges, thin shell structures, micro electromechanical systems and so on, traditional passive control approaches are not efficient enough for controlling structure noise and vibration from these novel systems and machines, because these passive methods cannot adjust the parameters of the systems automatically. So, many researchers have focused their energies to research new theories and techniques for vibration and noise reduction of structures, and have obtained some exciting results (Pan *et al*., 1993; Gardonio *et al*., 1997; Sciulli and Inman, 1998). Recently, techniques of active vibration and noise control are drawing much attention because active control methods are becoming cost efficient due to the rapid development of electronic technologies.

In active systems, the actuators are the key parts. Piezoelectric materials can be easily bonded on or imbedded into conventional structures, are lightweight and have relatively high actuating force and relatively low power consumption characteristics, and so, are used most widely to control vibration and noise (Crawley and Deluis, 1987; Dimitriadis, 1991; Han *et al*., 1997; Niu *et al*., 2004). However, most researches on piezoelectric actuators focus on distributed bending actuators which are bonded or imbedded into the host beams, plates, shells, or other flexible structures, and few focus on piezoelectric stack actuators which are usually used in the active isolation system.

In order to evaluate vibration and noise level of flexible and lightweight structures efficiently, power flow, as an ideal combined performance index, is

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becoming increasingly popular and drawing much attention (Goyder and White, 1980; Pan *et al*., 1992; Gardonio *et al*., 1997; Niu *et al*., 2004) because power flow is an energy index, which includes the magnitude and phase of general force and corresponding velocity, simultaneously. Therefore, it is very convenient and reasonable to consider power flow as a cost function used in the optimal control strategy of vibration isolation.

Aiming at active control of flexible system, piezoelectric stack actuators are employed to achieve vibration control of flexible structures. Mobility technique is used to derive the governing equations of the system. The power flow transmitted into the foundation is considered as cost function used for optimal vibration control. Simulations showed that the analytical model is effective.

MODEL AND ANYLYSIS OF SYSTEM

In order to simulate the isolation system in practical engineering, the foundation machines installed are modeled as rectangular flexible thin plate, simply supported at the four edges, as shown in Fig.1. The machines are rigid. The mounts are composed of passive isolators and inside piezoelectric actuators, as shown in Fig.2. The configuration of piezoelectric material we consider is the stack arrangement, working against an applied external force. The stack is defined to be a single or multi-layered piezoelectric element, which is relatively long in the *z* (or 3) direction. This configuration is intended to induce motion in the *z* direction by applying voltage on electrodes at the top and bottom of the element. The ring-like rubber isolators outside are in parallel with the piezoelectric stacks.

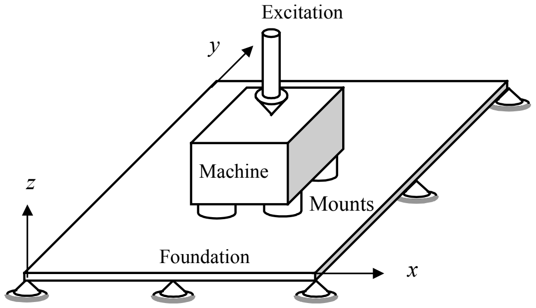

Fig.1 Model of flexible active isolation system

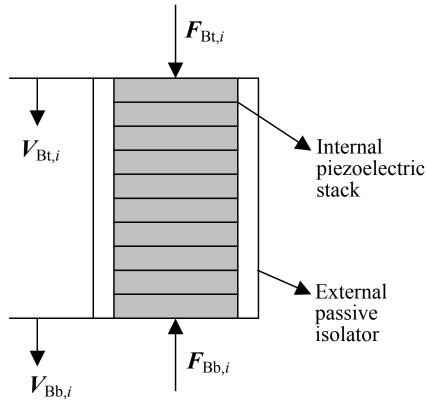

Fig.2 Configuration of *i*th combined mount

Mobility of the machine

The machine dynamic characteristics were studied using mobility technique, so the governing equation of the machine is given by

$$\begin{Bmatrix} \boldsymbol{V}_{\mathrm{At}} \\ \boldsymbol{V}_{\mathrm{Ab}} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{F}_{\mathrm{At}} \\ \boldsymbol{F}_{\mathrm{Ab}} \end{Bmatrix} \tag{1}$$

where $\boldsymbol{F}_{\mathrm{At}}$, $\boldsymbol{F}_{\mathrm{Ab}}$, $\boldsymbol{V}_{\mathrm{At}}$, $\boldsymbol{V}_{\mathrm{Ab}}$ are, respectively, the upper and the lower forces and their corresponding velocities of the subsystem A (machine), and subscripts b, and t denote the bottom and top output of the corresponding subsystem, respectively. The elements of the equation above $\boldsymbol{A}_{ij}$ (i,j=1,2) are the mobility of the machine, given by

$$\boldsymbol{A}_{11} = 1/\bar{M} \tag{2}$$

$$\boldsymbol{A}_{12} = -\boldsymbol{A}_{21}^{\mathrm{T}} = [-1/\bar{M} \quad -1/\bar{M}] \tag{3}$$

$$\boldsymbol{A}_{22} = -\begin{bmatrix} \dfrac{1}{\bar{M}} + \dfrac{x_{o1}^2}{\bar{J}} & \dfrac{1}{\bar{M}} + \dfrac{x_{o2}x_{o1}}{\bar{J}} \\ \dfrac{1}{\bar{M}} + \dfrac{x_{o2}x_{o1}}{\bar{J}} & \dfrac{1}{\bar{M}} + \dfrac{x_{o2}^2}{\bar{J}} \end{bmatrix} \tag{4}$$

where $\bar{M}$=jωm, $\bar{J}$=jωJ, and m, J respectively are the mass and the inertia moment of the machine, x_{oi} (i=1,2) is the local coordinate of the junction of the ith mount attached to the machine, as shown in Fig.3, and j=$\sqrt{-1}$.

Mobility analysis of mounts

Since the mounts consist of passive isolators and

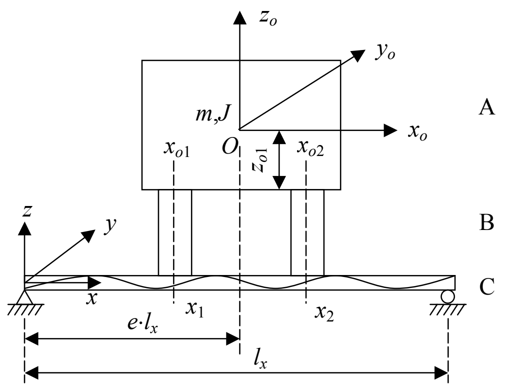

Fig.3 Installation size of machine and mounts

actuators, which are in parallel with each other, they should be considered separately.

When no external constrained force is present, the unconstrained strained displacement of the piezoelectric actuator in the z (or 3) direction is given by

$$\boldsymbol{\varepsilon}_{\rm p}=d_{33}\boldsymbol{u}/L_{\rm p} \tag{5}$$

where, $\boldsymbol{u}$ is the applied voltage, d_{33} is the piezoelectric strain constant, and $L_{\rm p}$ is the thickness of a single piezoelectric element in the z direction. Thus the unconstrained displacement of the actuator is obtained by

$$\boldsymbol{z}_{\rm p}=d_{33}\boldsymbol{u} \tag{6}$$

If a passive isolator is installed in parallel with the piezoelectric stack actuator as a combined mount shown in Fig.2, the stiffness of the passive isolator resists the motion of the actuator to $\boldsymbol{z}$, then the internal force $\boldsymbol{F}_{\rm p}$ that the actuator exerts in the positive z direction is related to the constrained motion of the actuator by

$$\boldsymbol{F}_{\rm p}=A_{\rm p}E_{\rm p}(\boldsymbol{z}_{\rm p}-\boldsymbol{z})/L_{\rm p} \tag{7}$$

where $A_{\rm p}$ and $E_{\rm p}$ are the area and the Young's modulus of the piezoelectric element, respectively. Applying a pair of balance force $\boldsymbol{F}_{{\rm Bt},i}$ and $\boldsymbol{F}_{{\rm Bb},i}$ at both ends of the ith combined mount, one can obtain the expression as

$$\boldsymbol{F}_{{\rm Bt},i}=k\boldsymbol{z}-A_{\rm p}E_{\rm p}(\boldsymbol{z}_{\rm p}-\boldsymbol{z})/L_{\rm p} \tag{8}$$

where k is the constant of the external passive isolator. Substituting Eq.(6) into Eq.(8) yields

$$\boldsymbol{F}_{{\rm Bt},i}=(k+k_{\rm p})\boldsymbol{z}-k_{\rm p}d_{33}\boldsymbol{u}_i \tag{9}$$

where $k_{\rm p}=A_{\rm p}E_{\rm p}/L_{\rm p}$ is the equivalent stiffness of the piezoelectric stack actuator. Neglecting the weights of mounts and considering the structural damping of the passive isolators, the complex stiffness matrixes of the mount systems can be described as

$$\boldsymbol{K}=\mathrm{diag}(k_1^*,k_2^*) \tag{10}$$

where $k_i^*=k_i(1+\mathrm{j}\eta_i)$ and η_i are the complex stiffness and the damping loss of the ith isolator, respectively. Thus the governing equation of the subsystem B (mounts) can be concisely given by

$$\boldsymbol{F}_{\rm Bt}=\bar{\boldsymbol{K}}\boldsymbol{V}-k_{\rm p}d_{33}\boldsymbol{U} \tag{11}$$

where $\bar{\boldsymbol{K}}=\boldsymbol{K}/\mathrm{j}\omega$ is the complex stiffness matrixes of the mounts.

Mobility analysis of foundation

For the plate-like foundation, which is excited by multiple distributions and simply supported at the four edges, the governing equation describing mobility is given by

$$\begin{Bmatrix}\boldsymbol{V}_{\rm C1}\\ \boldsymbol{V}_{\rm C2}\end{Bmatrix}=\begin{bmatrix}\boldsymbol{C}_{11} & \boldsymbol{C}_{12}\\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22}\end{bmatrix}\begin{Bmatrix}\boldsymbol{F}_{\rm C1}\\ \boldsymbol{F}_{\rm C2}\end{Bmatrix} \tag{12}$$

or more concisely

$$\boldsymbol{V}_{\rm C}=\boldsymbol{C}\boldsymbol{F}_{\rm C} \tag{13}$$

The transfer mobility from the point $\sigma_i(x_i,y_i)$ to the point $\sigma_j(x_j,y_j)$ can be determined by

$$\boldsymbol{C}_{ij}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mathrm{j}\omega\psi_{mn}(\sigma_p)\psi_{mn}(\sigma_q)}{\Lambda[\omega_{mn}^2(1+\mathrm{j}\delta)-\omega^2]} \tag{14}$$

where $\Lambda=pabh/4$ and δ, respectively, are the modal mass and the loss factor of the plate, ω_{mn} and φ_{mn} are the (m,n)th natural frequency and eigenfunction of the rectangular thin plate, respectively.

Power flow transmitted into the foundation

The relationships of the transmitted forces and the corresponding velocities on the interfaces of the subsystems can be easily known.

Through the dynamic transitive relationship of subsystems, as shown in Fig.4, the force transmitted onto the foundation is given by

$$\boldsymbol{F}_{\mathrm{C}}=[\boldsymbol{I}+\bar{\boldsymbol{K}}(\boldsymbol{C}-\boldsymbol{A}_{22})]^{-1}(\bar{\boldsymbol{K}}\boldsymbol{A}_{21}\boldsymbol{F}_0-k_{\mathrm{a}}d_{33}\boldsymbol{U}) \quad (15)$$

setting $\boldsymbol{T}=[\boldsymbol{I}+\bar{\boldsymbol{K}}(\boldsymbol{C}-\boldsymbol{A}_{22})]^{-1}$, $\boldsymbol{T}_1=-k_{\mathrm{a}}d_{33}\boldsymbol{T}$, and $\boldsymbol{T}_2=\boldsymbol{T}\bar{\boldsymbol{K}}\boldsymbol{A}_{21}$, the equation above can be concisely rewritten as

$$\boldsymbol{F}_{\mathrm{C}}=\boldsymbol{T}_1\boldsymbol{U}+\boldsymbol{T}_2\boldsymbol{F}_0 \quad (16)$$

and then the corresponding velocity of the foundation is given by

$$\boldsymbol{V}_{\mathrm{C}}=\boldsymbol{T}_3\boldsymbol{U}+\boldsymbol{T}_4\boldsymbol{F}_0 \quad (17)$$

where $\boldsymbol{T}_3=\boldsymbol{C}\boldsymbol{T}_1$, and $\boldsymbol{T}_4=\boldsymbol{C}\boldsymbol{T}_2$. The equations above are based on the general vibration system. If the active terms in Eqs.(16) and (17) are set to zeros, the active isolation system will degenerate into a passive one, i.e., if $\boldsymbol{U}$=0, then

$$\boldsymbol{F}=\boldsymbol{T}_2d \quad (18)$$
$$\boldsymbol{V}=\boldsymbol{T}_4d \quad (19)$$

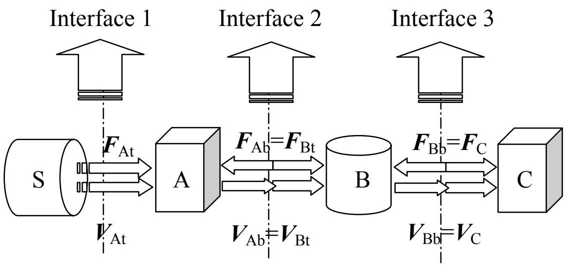

Fig.4 Dynamic relationship of the system

CONTROL STRATEGIES

In controlling the vibration of coupled flexible systems, both forces and velocities transmitted through mounts into the foundation should be considered. Power flow is an ideal index, because it includes the magnitude and phase of force and velocity, simultaneously. Therefore, considering power flow as an index is convenient for achieving optimal control of the vibration transmission into the foundation.

In a vibrating system, according to the definition of power flow (Goyder and White, 1980), the power flow transmitted into the foundation is given by

$$P=\frac{1}{2}\mathrm{Re}\left\{\boldsymbol{F}_{\mathrm{C}}^{\mathrm{H}}\cdot\boldsymbol{V}_{\mathrm{C}}\right\}=\frac{1}{4}\left(\boldsymbol{F}_{\mathrm{C}}^{\mathrm{H}}\cdot\boldsymbol{V}_{\mathrm{C}}+\boldsymbol{V}_{\mathrm{C}}^{\mathrm{H}}\cdot\boldsymbol{F}_{\mathrm{C}}\right) \quad (20)$$

where the superscript H indicates the complex conjugate and transpose of matrices or vectors. Considering practical engineering, to find the optimal control vector, power flow is utilized as an updated cost function expressed by

$$J=P+\boldsymbol{U}^{\mathrm{H}}\boldsymbol{Q}\boldsymbol{U} \quad (21)$$

where $\boldsymbol{Q}=qd_{33}\mathrm{diag}(1,1)$ is the weight of the input voltage.

Substituting Eqs.(17), (18) and (20) into Eq.(21) yields

$$J=\boldsymbol{U}^{\mathrm{H}}\boldsymbol{A}\boldsymbol{U}+\boldsymbol{U}^{\mathrm{H}}\boldsymbol{b}+\boldsymbol{b}^{\mathrm{H}}\boldsymbol{U}+c \quad (22)$$

where

$$\boldsymbol{A}=\frac{1}{4}(\boldsymbol{T}_1^{\mathrm{H}}\boldsymbol{T}_3+\boldsymbol{T}_3^{\mathrm{H}}\boldsymbol{T}_1)+\boldsymbol{Q}$$

and

$$b=\frac{1}{4}(\boldsymbol{T}_1^{\mathrm{H}}\boldsymbol{T}_4+\boldsymbol{T}_3^{\mathrm{H}}\boldsymbol{T}_2)d$$

Eq.(22) is of Hermitian quadratic format. Its unique minimum is assured provided that the matrix $\boldsymbol{A}$ is positive-definite. The corresponding optimal voltage vector yields (Fuller *et al*., 1996)

$$\boldsymbol{U}_{\mathrm{opt}}=-\boldsymbol{A}^{-1}b \quad (23)$$

Substituting the optimal vector into Eq.(20), the total power flow transmitted into the foundation can be computed, easily.

NUMERICAL EXAMPLES

To demonstrate the effectiveness of the presented model, some numerical examples are presented here. In the examples, the system is symmetrical about *xOy* plane, and two combined mounts are asymmetrically installed on the foundation, as shown in Fig.1 and Fig.2.

Fig.5a shows the power flow transmitted into the foundation when the weight $\boldsymbol{Q}$ is a zero matrix. The solid line indicates the uncontrolled power flow and the dash line shows the controlled power flow in case of q=0. Obviously, the active piezoelectric actuators reduce the power flow transmitted into the foundation to almost zero dramatically, but the value of the input voltage is so high that the piezoelectric actuators may not stand it at low frequencies as shown in Fig.5c. Therefore, a suitable weight of the control force should be adopted in practice.

Considering the output capacity of the piezoelectric actuators, a suitable weight of the voltage vector is selected to simulate the power flow transmitted into the foundation. It is obvious that the power flow is reduced significantly, as shown in Fig.5b, especially at high frequencies. It is easily understood that a smaller weight of voltage vector (denoting larger active force) induces more cancellation of the power flow, as shown in Fig.5d. In this case, one also finds that the maximum input voltage not exceeding 5 kN is an acceptable and reasonable value, in comparison with Fig.5c.

In the presented model, the passive isolator and the piezoelectric stack actuators are constructed in parallel. The piezoelectric stacks are encompassed by the ring-like rubber isolators, which can protect the inside piezoelectric materials. However, the stiffness of the isolators will affect the efficiency of the piezoelectric actuators.

Fig.6 indicates the variety of the power flow transmitted into the foundation in case of different stiffness of the isolators. It is concluded that larger

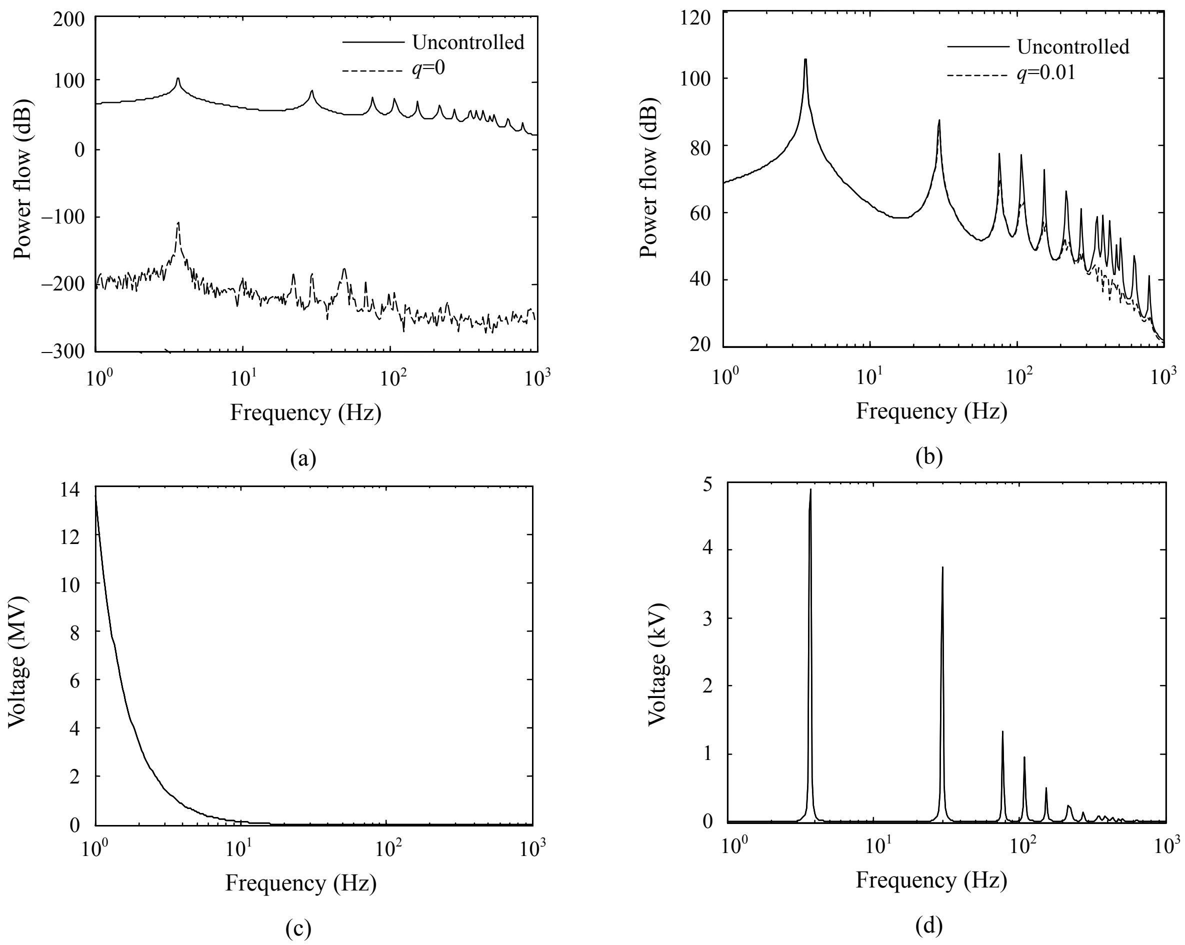

Fig.5 Power flow transmitted into the foundation in case of q=0 (a) and q=0.01 (b), and the input voltage of the piezoelectric stack actuators in case of q=0 (c), and q=0.01 (d)

stiffness of isolators provide less vibration cancellation, on the contrary smaller stiffness of the passive isolator will achieve more vibration reduction. Therefore, a passive stiffness should match the equivalent stiffness of the piezoelectric stack actuators to achieve good vibration cancellation and ensure that external forces do not damage the piezoelectric actuators.

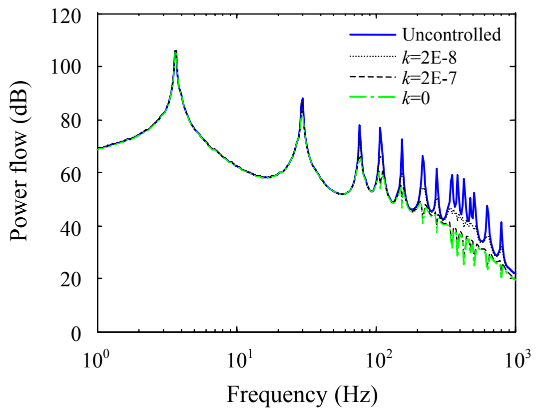

Fig.6 Power flow transmitted into the foundation in case of different isolator stiffness

CONCLUSION

An analytical model of the flexible isolation system is presented. In the model, piezoelectric material actuators are employed to achieve vibration control of the flexible structures. Mobility technique is used to derive the governing equations of the system. The power flow transmitted into the foundation is solved and considered as cost function for optimal control of vibration isolation.

From some numerical simulations, it is clear that the analytical model is effective as piezoelectric stack actuators can achieve vibration attenuation substantially through selecting proper weight of the input voltage. It is also concluded that a smaller passive stiffness can achieve more vibration reduction of the flexible system.

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