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SAW Reflection and Scattering by Electrodes*

WANG Wei-biao (王为标)[1], HAN Tao (韩 韬)[†1,2], ZHANG Xiao-dong (张晓东)[1],
WU Hao-dong (吴浩东)[1], SHUI Yong-an (水永安)[†1,2]
([1]State Key Laboratory of Modern Acoustics, Nanjing University, Nanjing 310093, China)
([2]Department of Instrumentation Engineering, Shanghai Jiao Tong University, Shanghai 200030, China)
[†]E-mail: than@sjtu.edu.cn; yashui@nju.edu.cn
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Abstract: A rigorous analysis of surface acoustic wave (SAW) reflection and scattering by electrodes is of paramount importance in the design of SAW identification tags and sensors. In this paper, a new method based on Green's function concept is used to study reflection and scattering coefficients. By this method the reflection coefficient with its phase angle, transmission coefficient, and bulk wave scattering coefficient, can be obtained rapidly and accurately. To get precise result, the influence of static charge must be taken into account. In the work, we successfully cancelled out the effect of static charge and the validity of the results was checked. As an example, the reflection, transmission and scattering coefficients of a single grounded electrode on 128° YX $LiNbO_3$ is shown.

Key words: SAW reflection, SAW scattering, Green's function
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INTRODUCTION

Electrodes or grooves in the path of surface acoustic wave (SAW) propagation give rise to SAW reflection and cause SAW scattering into bulk acoustic wave. This is a fundamental SAW problem on which had been conducted many studies. It is especially important for designing SAW radio frequency identification tags (RFID). Global SAW Tag (GST) (Hartmann, 2004) has many advantages over traditional SAW RFID, but requires accurate knowledge of the reflectivity and reflective phase of every reflector in the design. Moreover, the energy loss due to SAW scattering causes decrease of SAW pulse energy after a series of reflectors, so in SAW RFID design serious attention should be directed to the scattering loss reduction, so that quantitative calculation of SAW reflection and scattering is important.

Lehtonen *et al*.(2004a; 2004b; 2004c; 2003) calculated frequency domain signals by Fourier transformation to get the impulse response, and separated the incident impulse from the reflected echo by using time gating. To achieve good time resolution and large time scope, they did calculations on a great deal of data with wide frequency range and fine frequency separation, which requires long CPU time and limits accuracy. The reflective phase is especially sensitive to error, and is important in the design of SAW ID tag. They calculated the bulk wave scattering coefficient s from reflectivity r and transmission coefficient t according to the relationship

$$|s|^2=1-|r|^2-|t|^2 \quad (1)$$

As $|r|^2$ and $|s|^2$ are of the order of 10^{-3}, it means that they must have very high accuracy in their calculation of t.

We use a new method to simplify the calculation greatly. The reflective coefficient (including the amplitude and phase) and scattering coefficient are obtained directly at each frequency, so the calculation is

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very fast and precise. In Section II, the theory of the method is described. In Section III, some results are given as examples. Section IV discusses various aspects of SAW reflection and scattering.

THEORY

Assume a 2D problem is under discussion. The aperture is large enough so that the distribution along the aperture can be considered as uniform. FEM/BEM (Vantura *et al*., 1995) is used to transform the 2D problem into a 1-dimensional one by relating all along depth quantities (including fields within electrodes and in the substrate) to the field on the surface or interface.

Green's function formula

We start our theory from 1-dimensional Green's function.

$$\boldsymbol{F}(x)=\int \boldsymbol{G}(x-x')\boldsymbol{S}(x')\mathrm{d}x' \tag{2}$$

x is the coordinate along the wave propagation direction, $\boldsymbol{G}$ is Green's function, a matrix, $\boldsymbol{F}$ and $\boldsymbol{S}$ are the field and source vector of the wave. Because of the substrate piezoelectricity, $\boldsymbol{F}$ and $\boldsymbol{S}$ include both mechanical and electrical components.

$$\boldsymbol{F}=\begin{bmatrix}\boldsymbol{u}\\ \varphi\end{bmatrix},\ \boldsymbol{S}=\begin{bmatrix}\boldsymbol{T}_\mathrm{n}\\ \sigma\end{bmatrix} \tag{3}$$

$\boldsymbol{u}$ consists of vibration components in 3 space coordinates (in case of decoupling, $\boldsymbol{u}$ is 2D vector), $\boldsymbol{T}_\mathrm{n}$ is comprised of normal stress components, φ and σ represent electric potential and electric charge, respectively. Fourier transformation can be used to transform Eqs.(2) and (3) into k-space

$$\overline{\boldsymbol{F}}(k)=\overline{\boldsymbol{G}}(k)\overline{\boldsymbol{S}}(k) \tag{4}$$

$$\overline{\boldsymbol{F}}=\begin{bmatrix}\overline{\boldsymbol{u}}\\ \overline{\varphi}\end{bmatrix},\ \overline{\boldsymbol{S}}=\begin{bmatrix}\overline{\boldsymbol{T}}_\mathrm{n}\\ \overline{\sigma}\end{bmatrix} \tag{5}$$

$\boldsymbol{G}$ can be obtained according to references (Vantura *et al*., 1995; Milsom *et al*., 1977). Under certain frequency, wave number k can be replaced by slowness.

Green's function can be divided into three parts: static electric part $\boldsymbol{G}^0$, Rayleigh wave eigenmode part $\boldsymbol{G}^r$ and bulk wave contribution $\boldsymbol{G}^b$.

$$\boldsymbol{G}=\boldsymbol{G}^0+\boldsymbol{G}^r+\boldsymbol{G}^b \tag{6}$$

The Rayleigh wave field $\boldsymbol{F}$ excited by certain source $\boldsymbol{S}$ is

$$\overline{\boldsymbol{F}}=\overline{\boldsymbol{G}}^r\overline{\boldsymbol{S}} \tag{7}$$

Structure under discussion, reflective coefficient and transmission coefficient

The structure for our calculation is shown in Fig.1. The device is composed of a transducer and a reflector. Field point a is located between transducer and reflector. Field point b is located behind reflector. Similar to (Vantura *et al*., 1995), we can solve Eq.(4) for this structure, obtain source $\boldsymbol{S}^\mathrm{T}$ on the transducer and source $\boldsymbol{S}^\mathrm{R}$ on the reflector. The Rayleigh wave field at point a excited by transducer is

$$\overline{\boldsymbol{F}}^{ai}=\overline{\boldsymbol{G}}_\mathrm{T}^{r+}\overline{\boldsymbol{S}}^\mathrm{T} \tag{8}$$

This is the incident wave field for the reflector. $\boldsymbol{G}_\mathrm{T}^{r+}$ denotes the function of Rayleigh wave excitation in $+x$ direction by transducer. The reflective wave field $\boldsymbol{F}^{ar}$ at point a from reflector is

$$\overline{\boldsymbol{F}}^{ar}=\overline{\boldsymbol{G}}_\mathrm{R}^{r-}\overline{\boldsymbol{S}}^\mathrm{R} \tag{9}$$

The transmission wave field $\boldsymbol{F}^{bt}$ at point b is

$$\overline{\boldsymbol{F}}^{bt}=\overline{\boldsymbol{G}}^{r+}(\overline{\boldsymbol{S}}^\mathrm{T}+\overline{\boldsymbol{S}}^\mathrm{R}) \tag{10}$$

$\boldsymbol{G}_\mathrm{R}^{r+}$ and $\boldsymbol{G}_\mathrm{R}^{r-}$ denote the excitation in $+x$ and $-x$ direction by reflector. For convenience, the reflector

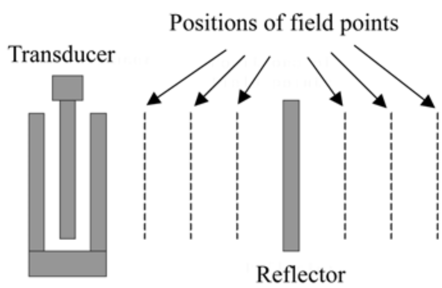

Fig.1 Schedule diagram of structure

center is taken as reference point for all the phases of wave field, either at point a, or at point b. The reflective coefficient r is

$$r=(\bar{\boldsymbol{F}}^{ar})_j\big/(\bar{\boldsymbol{F}}^{ai})_j \tag{11}$$

and the transmission coefficient t is

$$t=(\bar{\boldsymbol{F}}^{bt})_j\big/(\bar{\boldsymbol{F}}^{ai})_j \tag{12}$$

In Eqs.(11) and (12) the subscript j represents jth component of field vector. Taking any value of j (j=1,2,3 or 4), the result for r and t should be the same, because the ratios among components are fixed for Rayleigh wave. However, Eqs.(11) and (12) are not exact. Relevant corrections are given in Section 2.4.

Scattering coefficient

The scattering of Rayleigh wave into bulk wave could be considered as two processes: firstly, the incident Rayleigh wave excites a source $\boldsymbol{S}^{\mathrm{R}r}$ on the reflector, and secondly, the scattered bulk wave can be considered as the bulk wave emitted by source $\boldsymbol{S}^{\mathrm{R}r}$. The excited power of bulk waves by source on reflector $\boldsymbol{S}^{\mathrm{R}r}$ can be obtained by Eq.(63) in reference (Milsom *et al.*, 1977).

Correction of reflective coefficient and transmission coefficient

$\boldsymbol{S}^{\mathrm{R}}$ in Eq.(9) is obtained by solving Eq.(4). $\boldsymbol{G}$ in Eq.(4) includes three parts as shown in Eq.(6). Therefore, the solution $\boldsymbol{S}^{\mathrm{R}}$ can be considered as the result of action by source of transducer $\boldsymbol{S}^{\mathrm{T}}$ through all of $\boldsymbol{G}^0$, $\boldsymbol{G}^r$ and $\boldsymbol{G}^b$. Due to the linearity of the problem, we can take $\boldsymbol{S}^{\mathrm{R}}$ as composed of three parts.

$$\boldsymbol{S}^{\mathrm{R}}=\boldsymbol{S}^{\mathrm{R}0}+\boldsymbol{S}^{\mathrm{R}r}+\boldsymbol{S}^{\mathrm{R}b} \tag{13}$$

What we are studying is the reflection, transmission and scattering of Rayleigh wave. We should use the following formula instead of Eqs.(9) and (10)

$$\bar{\boldsymbol{F}}^{ar}=\bar{\boldsymbol{G}}_{\mathrm{R}}^{r-}\bar{\boldsymbol{S}}^{\mathrm{R}r} \tag{14}$$

$$\bar{\boldsymbol{F}}^{bt}=\bar{\boldsymbol{G}}^{r+}(\bar{\boldsymbol{S}}^{\mathrm{T}}+\bar{\boldsymbol{S}}^{\mathrm{R}r}) \tag{15}$$

The calculated values of reflective and transmission wave field are expressed as Eqs.(9) and (10). To obtain correct reflective and transmission coefficient, we must use Eqs.(14) and (15) to substitute into Eqs.(11) and (12) to make correction.

Bulk wave interaction $\boldsymbol{G}^b$ denotes the interaction between two points on the surface. With the propagation of bulk wave, the energy is continuously radiated into bulk wave, so it is short range energy rapidly attenuated with distance. In a computer experiment, under 2D assumption and assumption of no material loss (no propagation attenuation for plane Rayleigh wave), one can prolong the distance between transducer and reflector arbitrarily without any influence on the result. In such a structure, $\boldsymbol{S}^{\mathrm{R}b}$ vanishes, in this case, we can put it as zero in Eq.(13).

Interaction $\boldsymbol{G}^r$ has a time delay due to propagation of Rayleigh wave from transducer to reflector, interaction $\boldsymbol{G}^0$ is a kind of simultaneous action (due to ignoring of propagation time of electro-magnetic wave). In order to cancel out $\boldsymbol{S}^{\mathrm{R}0}$ in Eqs.(9) and (10), we calculate the field distribution twice. In the second calculation, we shift the location of reflector for half a wavelength of Rayleigh wave length (either close to or far from transducer). Sources $\boldsymbol{S}^{\mathrm{T}0}$ and $\boldsymbol{S}^{\mathrm{R}0}$ keep unchanged, but sources excited by Rayleigh wave will be changed. Because we fix the phase reference at the reflector center, whatever the reflector shift, the shift will cause all $\boldsymbol{G}_{\mathrm{R}}$ to change their sign.

We express quantities twice by A and B, the calculated fields are:

$$\begin{aligned}\boldsymbol{F}_i^A&=(\boldsymbol{S}^{\mathrm{T}0}+\boldsymbol{S}_A^{\mathrm{T}r})\boldsymbol{G}_{\mathrm{T}}^+\\ \boldsymbol{F}_r^A&=(\boldsymbol{S}^{\mathrm{R}0}+\boldsymbol{S}_A^{\mathrm{R}r})\boldsymbol{G}_{\mathrm{R}}^-\\ \boldsymbol{F}_t^A&=(\boldsymbol{S}^{\mathrm{T}0}+\boldsymbol{S}_A^{\mathrm{T}r})\boldsymbol{G}_{\mathrm{T}}^++(\boldsymbol{S}^{\mathrm{R}0}+\boldsymbol{S}_A^{\mathrm{Rr}})\boldsymbol{G}_{\mathrm{R}}^+\end{aligned} \tag{16}$$

$$\begin{aligned}\boldsymbol{F}_i^B&=(\boldsymbol{S}^{\mathrm{T}0}+\boldsymbol{S}_B^{\mathrm{T}r})\boldsymbol{G}_{\mathrm{T}}^+\\ \boldsymbol{F}_r^B&=(\boldsymbol{S}^{\mathrm{R}0}+\boldsymbol{S}_B^{\mathrm{R}r})\boldsymbol{G}_{\mathrm{R}}^-\\ \boldsymbol{F}_t^B&=(\boldsymbol{S}^{\mathrm{T}0}+\boldsymbol{S}_B^{\mathrm{T}r})\boldsymbol{G}_{\mathrm{T}}^++(\boldsymbol{S}^{\mathrm{R}0}+\boldsymbol{S}_B^{\mathrm{R}r})\boldsymbol{G}_{\mathrm{R}}^+\end{aligned} \tag{17}$$

According to the above discussion, the reflective coefficient and transmission coefficient must be

$$\begin{aligned}r^A&=\frac{\boldsymbol{S}_A^{\mathrm{R}r}\boldsymbol{G}_{\mathrm{R}}^-}{\boldsymbol{F}_i^A}=\frac{\boldsymbol{F}_r^A-\boldsymbol{S}^{\mathrm{R}0}\boldsymbol{G}_{\mathrm{R}}^-}{\boldsymbol{F}_i^A}\\ t^A&=\frac{\boldsymbol{F}_t^A-\boldsymbol{S}^{\mathrm{R}0}\boldsymbol{G}_{\mathrm{R}}^+}{\boldsymbol{F}_i^A}\end{aligned} \tag{18}$$

$$\begin{aligned} r^B &= \frac{\boldsymbol{S}_B^{\mathrm{R}r}\boldsymbol{G}_\mathrm{R}^-}{\boldsymbol{F}_i^B} = \frac{\boldsymbol{F}_r^B + \boldsymbol{S}^{\mathrm{R}0}\boldsymbol{G}_\mathrm{R}^-}{\boldsymbol{F}_i^B} \\ t^B &= \frac{\boldsymbol{F}_t^B + \boldsymbol{S}^{\mathrm{R}0}\boldsymbol{G}_\mathrm{R}^+}{\boldsymbol{F}_i^B} \end{aligned} \tag{19}$$

Reflection and transmission are physical phenomena independent of reflector location. We have

$$r^A = r^B,\ t^A = t^B \tag{20}$$

Finally we get

$$r = \frac{(\boldsymbol{F}_r^A + \boldsymbol{F}_r^B)_j}{(\boldsymbol{F}_i^A + \boldsymbol{F}_i^B)_j},\ t = \frac{(\boldsymbol{F}_t^A + \boldsymbol{F}_t^B)_j}{(\boldsymbol{F}_i^A + \boldsymbol{F}_i^B)_j} \tag{21}$$

r and t in Eq.(21) are complex numbers with amplitude and phase angle. $\boldsymbol{F}$ in Eq.(21) is a complex vector with mechanical and electrical components. The subscript 'j' in Eq.(21) means that any component, whether mechanical or electrical, can be used. One will obtain the same result if the same component (j=1, 2, 3 or 4) is taken in numerator and denominator. All the quantities on the right sides of Eqs.(20) and (21) are obtained from FEM/BEM result, we have achieved the correction, now.

RESULTS

The described above method can be applied to arbitrary combination of electrodes as reflector. We take one grounded electrode as the reflector to show the result as an example. In the calculation the substrate is 128° $LiNbO_3$. We set the electrode width and thickness as 1 μm and 0.04 μm. It corresponds to an electrode with metallized ratio of 0.5 in a single finger transducer with center frequency of 1000 MHz. In fact, for any width value one can get the corresponding curves by setting the frequency scale inversely proportional to electrode width.

Validity of the theory and calculation

(1) Since there is a time delay in Rayleigh wave propagation from transducer to reflector between the sources $\boldsymbol{S}^{\mathrm{R0}}$ and $\boldsymbol{S}^{\mathrm{Rr}}$, the Rayleigh waves excited by them will interfere with each other. The interference causes frequency oscillation in all the reflection, transmission and scattering curves with frequency. The period of such oscillation is the inverse of the propagation time from transducer to reflector. We looked at these curves in a very fine frequency scan. Fig.2 shows the curves for results before correction and after correction, (a) for the reflectivity; (b) for the reflection phase and (c) for the transmission coefficient.

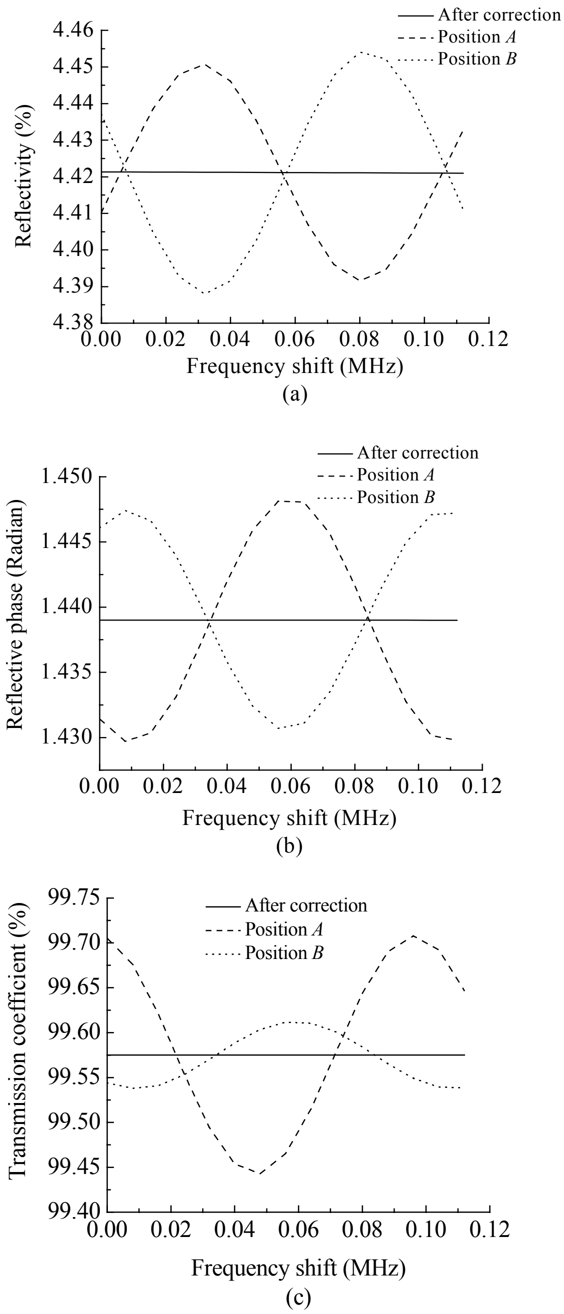

Fig.2 The results before correction and after correction, (a) for the reflectivity; (b) for the reflection phase and (c) for the transmission coefficient

reflection phase and (c) for the transmission coefficient. The abscissa is the frequency shift from 2000 MHz. The dash and dot lines are for results by Eqs.(9)~(12), solid line is for result after correction according to Eq.(21). The oscillation in corrected curves disappears.

(2) In calculation experiment, one must have a concrete transducer, from which the reflection and transmission coefficient should be independent. To verify this, we calculated twice, in which the transducers are 11 fingers and 31 fingers, respectively. Fig.3 shows the calculated reflectivity as comparison.

(3) The calculation results are also independent of the positions of field points A and B. In the calculation, different positions of A and B give the same result.

(4) In calculation we can use different component (different j) in Eq.(21) with results being the same.

(5) We calculated the results by transferring the data from frequency domain to time domain, then use time gating to separate incident signal from reflective echo. By this method we can get accurate reflectivity, but the transmission coefficient and reflection phase have large error. Fig.4 compares results for reflectivity between the two methods.

Results

Fig.5 shows the calculated reflectivity for various electrode thicknesses. Fig.6 shows the calculated reflection phase for various electrode thicknesses. Fig.7 shows transmission coefficient versus electrode thickness. Fig.8 shows transmission phase versus electrode thickness. Fig.9 shows bulk wave scattering coefficient versus electrode thickness. In Fig.2~8 the coefficients are ratio of amplitude; in Fig.9, the value is given as ratio of energy. In calculation, we can obtain the scattering of slow shear wave, fast shear wave and longitudinal wave, respectively. In Fig.10, we fix the electrode thickness as 0.04 μm and give the scattering coefficient for each wave mode. The scattering coefficient is the ratio between amplitude, the square root of the coefficients in Fig.9.

According to calculation, whether the electrode

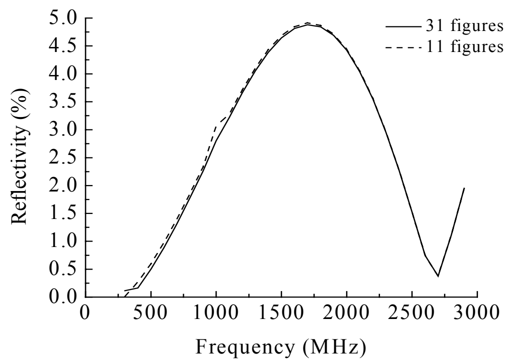

Fig.3 Comparison between calculated reflectivity by different transducers

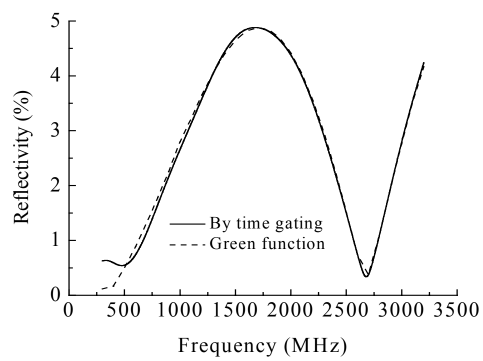

Fig.4 The comparison between calculated reflectivity by two methods

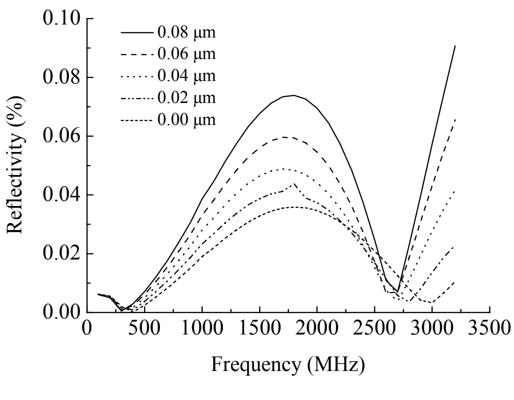

Fig.5 Reflectivity with various thicknesses

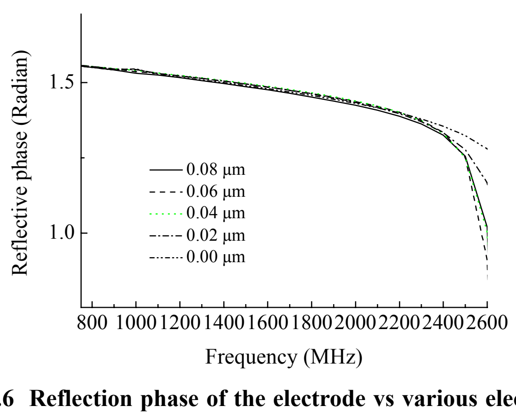

Fig.6 Reflection phase of the electrode vs various electrode thicknesses

is grounded or floating, the result is about the same. Fig.11 gives their curves as an example, in which the electrode thickness is 0.04 μm.

Checking

Because Eq.(1) should be theoretically satisfactory, we can take the remainder Δ for checking the result.

$$\Delta=1-|r|^2-|t|^2-|s|^2 \quad (22)$$

Fig.12 shows the result of checking. In most parts of the figure, the error is within ±0.0005.

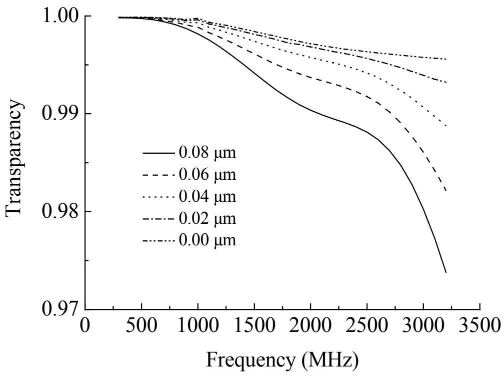

Fig.7 Transmission coefficient versus electrode thickness

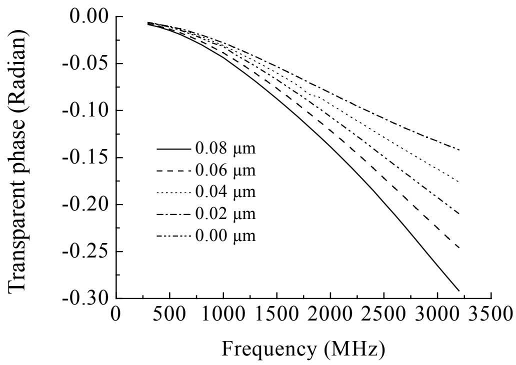

Fig.8 Transmission phase versus electrode thickness

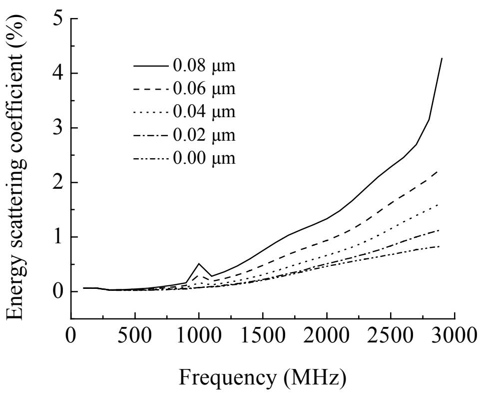

Fig.9 Energy scattering coefficient versus electrode thickness

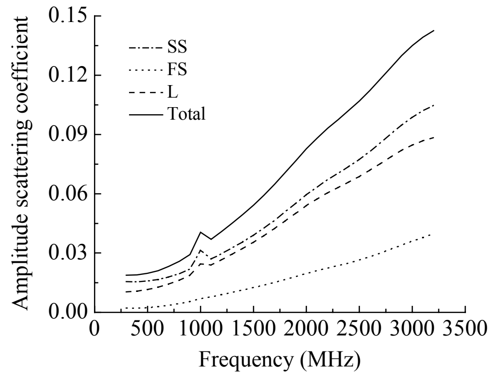

Fig.10 Bulk wave scattering coefficient (amplitude) for various wave modes: SS-slow shear wave; FS-fast shear wave; L-longitudinal wave. Thickness=0.04 μm

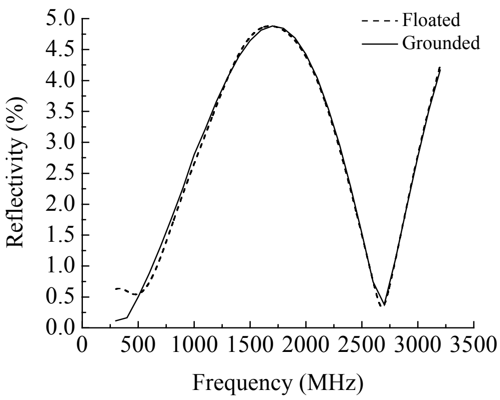

Fig.11 Comparison of reflectivity for grounded and floating electrode; thickness=0.04 μm

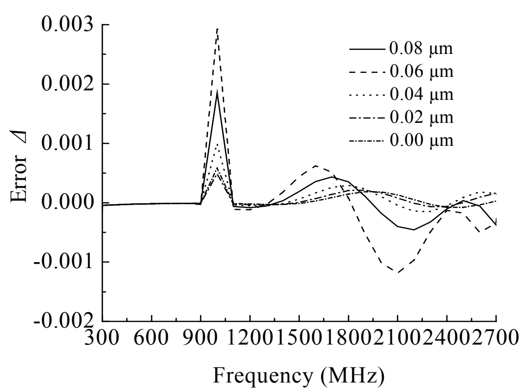

Fig.12 Checking of the error

DISCUSSION

A new method, which utilizes Green's function and its linearity, is introduced for analysis and calculation of Rayleigh wave reflection, transmission and scattering. It is shown that this is a powerful tool which can be applied to various kinds of problems theoretically and in precise quantitative calculation, not limited to SAW reflection, once it is combined with 2D FEM/BEM software. Such problems include propagation, reflection and scattering of SSBW, leaky shear wave, leaky longitudinal wave, etc.

Calculation of Rayleigh wave reflection, transmission and scattering, is fast and accurate. Previously, we tried to use time gating in order to have high accuracy, our calculation of 20000 frequency points, took about 10 d on a high speed PC, and the accuracy was still not satisfactory enough; by the present method, on the same PC, we could get a curve in about 40 min. Each frequency point needed only about 1 min. From calculation on every frequency point, we obtained all the coefficients at this frequency immediately. The accuracy was satisfactory. The obtained reflection phase and scattering coefficients had high enough precision.

128° $LiNbO_3$ was used in this paper, but in fact this method can be used for any SAW substrate.

This method has no limitation on the combination of electrodes into a reflector, and can also yield result in a wide frequency range. Therefore, it is a powerful tool for designing SAW ID tags or other devices.

In this work we focused on the reflection, transmission and scattering by pure Rayleigh wave, we put the reflector far from other electrodes to limit the influence of bulk wave by neighbor fingers, and we corrected the influence of static charge. However, in practical device, for example, in ID tag, all those effects (bulk wave interaction and static charge influence) exist in addition to Rayleigh wave interaction. Such effects did not have designer's attention in general.

According to the structure used here, there is multi-reflection of Rayleigh wave between transducer and reflector. However, in our theoretical derivation, there is no assumption on the source. It means that in above formula, source $\boldsymbol{S}^{\mathrm{T}}$ may be arbitrary, not necessarily a pure harmonic or a δ impulse. The above derivation includes multi-reflection of Rayleigh wave.

We corrected the effect of static charge on reflection and transmission, but have not successfully corrected the effect of static charge on bulk wave scattering. It remains a difficulty for us, so far, and causes errors in scattering coefficients, which are about 5%~10% of the coefficient themselves.

In the results, there is a particular frequency, 1000 MHz, the resonant frequency of the reflective electrode. The checking result shows we have error there. It remains to be improved.

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