Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

A dynamic decision model for portfolio investment and assets management

QIAN Edward Y. (钱彦敏)[†1], FENG Ying (冯 颖)[†‡2], HIGGISION James[3]

([1]*Center for Private Economy Research, Zhejiang University, Hangzhou 310027, China*)

([2]*Department of Science Technology Research, City College, Zhejiang University, Hangzhou 310007, China*)

([3]*Faculty of Engineering, the University of Waterloo, Waterloo, N2L 3G1, Canada*)

[†]E-mail: yanminqian_ca@yahoo.com.cn; hanqi_feng@msn.com

Received Apr. 13, 2005; revision accepted June 27, 2005

Abstract: This paper addresses a dynamic portfolio investment problem. It discusses how we can dynamically choose candidate assets, achieve the possible maximum revenue and reduce the risk to the minimum level. The paper generalizes Markowitz's portfolio selection theory and Sharpe's rule for investment decision. An analytical solution is presented to show how an institutional or individual investor can combine Markowitz's portfolio selection theory, generalized Sharpe's rule and Value-at-Risk (VaR) to find candidate assets and optimal level of position sizes for investment (dis-investment). The result shows that the generalized Markowitz's portfolio selection theory and generalized Sharpe's rule improve decision making for investment.

Key words: Portfolio investment, Value-at-Risk (VaR), Generalized Sharpe's rule
doi:10.1631/jzus.2005.AS0163 **Document code:** A **CLC number:** F833.5

INTRODUCTION

In investment, investors face a decision problem of choosing among many assets. How do we choose assets to construct an optimal portfolio? After constructing the optimal portfolio, how can we adjust it dynamically, by acquiring a new asset or some assets into the portfolio, dis-investing an asset or some assets from the portfolio? Should we adjust the portfolio based on risk or return? These are the practical questions facing the investors. For a financial institution, or even a personal investor, it is very important to know how to dynamically choose financial assets contingent on real situation in order to achieve the maximum revenue and reduce the risk to the minimum level within the investor's budget constraint. This paper addresses this type of investment problems. Theoretically, this is a dynamical optimization problem focusing on investment in a group of assets, and adjustment of the composition of the assets to add value and avoid loss contingent on time. The volatility of asset prices implies that the investment cannot be just a single decision (or action) forever; that is, the investment should not be just fixed on some assets. It must be adjustable in order to seize the opportunity to realize potential gains and avoid possible losses due to the fluctuations in financial market. Therefore, the problem is how to carry out the process of selecting candidate assets for investment, then constructing a portfolio, and monitoring its risk, dynamically adjusting the portfolio.

Markowitz (1952; 1959) proposed a portfolio selection theory that was widely accepted by both the academic circle and investment practitioners. Markowitz's portfolio theory is based on mean-variance and utility maximization theory. It seems to have solved the problem of investing a group of assets. It provides a good recommendation on choosing assets; however, it cannot be applied to the

[‡]Corresponding author

dynamic investment and dis-investment problem. Suppose that some candidate assets are given and an optimal portfolio has been constructed from this given package of assets. The next step is to acquire new assets or dis-invest existing assets dynamically to make the portfolio a better investment. This is a dynamic improvement to the optimized portfolio, but Markowitz model cannot help in this respect. As Litterman (1996) points out, the basic issues in the development of portfolio analytical tools to guide investment and risk management in the decision making still have not been well sorted out. The Markowitz model deals with a static and closed investment problem. It cannot be adjusted as the risk or price changes; Also it cannot be used in the dynamic setting in an open investment case, such as the acquisition new assets or dis-investment of the existing assets in real world (Hodges and Brealey, 1978; Wilcox, 2001; Dynkin *et al*., 2000); further it does not keep track of the Value-at-Risk for the portfolio (Ho *et al*., 1996); hence it could not solve the asset investment and risk management problem in practice. In middle and late of 1990s, following several major financial cases, the risk management became a major focus of research and researches on "Value-at-Risk (VaR)" receive much interest (Linsmeier *et al*., 1996; Best, 2001). But these contributions are still not well incorporated into the whole picture of the dynamic investment and risk management for the decision-making. The paper is based on Dowd (1999)'s idea of using the generalized Sharpe's rule to implement a practical investment decision rule, and develops an optimization method for the above practical portfolio investment problem and risk management. The remaining parts of the paper is organized as follows: Section 2 derives the decision model and critical rules based on the generalised Sharpe's rule; Section 3 tests the model and discusses analytical results with cases for acquiring and dis-investing assets; Section 4 concludes the paper.

DECISION MODEL

Sharpe's rule and its generalization

In practice, investors face the investment decision problem of choosing between two assets. The well-known Sharpe's rule tells us how to choose the asset among the several alternatives based on the Sharpe's ratio (*SR*): the expected return on the relevant asset divided by the standard deviation of its return (Sharpe, 1963; 1994; 1978). Suppose that we have two assets, *A* and *B*. The Sharpe's rule tells us to choose *A*, if $SR_A>SR_B$ and choose *B*, if $SR_A<SR_B$, where SR_A is the Sharpe's ratio of asset *A* and SR_B is the Sharpe's ratio of asset *B* (Sharpe, 1994). But the traditional Sharpe's rule cannot deal with a non-zero correlation asset with the existing portfolio. To solve this problem, Dowd (1999; 2000) propose the following idea: instead of considering the Sharpe's ratio applied to each asset on its own, consider a new asset position relative to the existing portfolio that the investor holds. For example, we can take an asset *A*, and use the Sharpe's ratio to decide which portfolio to choose between the existing portfolio and the new portfolio composed of the existing portfolio plus the new asset *A*. Dowd's idea provides a way to solve the problem that the traditional Sharpe's rule and portfolio selection theory cannot solve, but he does not provide the procedure for investment decision on how to select the portfolio and how to dynamically adjust the existing portfolio. The rest of this section will exploit this idea and incorporate it into the above generalized Markowitz's portfolio selection theory, and find a practical procedure and decision rule for the optimal investment.

Required rate of return of the new candidate asset

The new rule is that we should invest in the new asset *A* if it increases the Sharpe's ratio on our adjusted existing portfolio (that is, the existing portfolio plus the new assets) compared to the existing portfolio before adjusting, otherwise we do not invest this new asset (Dowd, 1999). The rule for this choice can be expressed as the following:

$$SR^{\text{new}} = R_p^{\text{new}} / \sigma_{R_p^{\text{new}}} \geq R_p^{\text{old}} / \sigma_{R_p^{\text{old}}} = SR^{\text{old}} \quad (1)$$

where SR^{new} is the Sharpe's ratio of the new portfolio; SR^{old} is the Sharpe's ratio of old portfolio; R_p^{new} is the expected rate of return on the new constructed portfolio; R_p^{old} is the expected rate of return on the old portfolio; $\sigma_{R_p^{\text{new}}}$ is the standard deviation of the rate of return to the new portfolio; $\sigma_{R_p^{\text{old}}}$ is the standard deviation of the rate of return to the old portfolio.

Next, we show how to obtain these parameters. Let us construct the new portfolio using the old portfolio that the investor has already held and the new asset A. The proportion of the amount invested in A is ω, and the proportion of the amount in old portfolio is $(1-\omega)$. Therefore the new portfolio has the expected return R_p^{new}.

$$R_p^{\text{new}} = \omega R_A + (1-\omega) R_p^{\text{old}} \tag{2}$$

where R_A is the expected rate of return on the new asset A.

Substitute Eq.(2) into Eq.(1) and get R_A:

$$R_A \geq R_p^{\text{old}} + [\sigma_{R_p^{\text{new}}} / \sigma_{R_p^{\text{old}}} - 1] R_p^{\text{old}} / \omega \tag{3}$$

Eq.(3) is the new condition for choosing the new asset A:

Define: $R_{\text{required}} = R_p^{\text{old}} + [\sigma_{R_p^{\text{new}}} / \sigma_{R_p^{\text{old}}} - 1] R_p^{\text{old}} / \omega$

Then, the condition can be expressed as:

Rule I

$$\begin{cases} R_A \geq R_{\text{required}} \\ R_{\text{required}} = R_p^{\text{old}} + [\sigma_{R_p^{\text{new}}} / \sigma_{R_p^{\text{old}}} - 1] R_p^{\text{old}} / \omega \end{cases}$$

This new rule means that there is a required rate of return for choosing new asset A to add to the existing portfolio. The investor should choose the new asset A if its expected return is at least as great as the required rate of return for the new candidate asset A.

Value-at-Risk and the required rate of return

After each adjustment on the existing portfolio, the risk level is altered. So we need to keep track of the risk level of the new adjusted portfolio. We use Value-at-Risk to estimate the risk level of the portfolio. Value-at-Risk is an approach for measuring the risk of an asset or a portfolio. It has been widely used by financial institutions (Linsmeier and Pearson, 2000). Many scholars discuss various VaR methodologies (Linsmeier and Pearson, 1996; Johansson *et al*., 1999). This approach uses historical data to find the volatility (variance) of the key factor(s) of the asset, and assigns a confidence level α to estimate the risk. If the return of a portfolio is assumed to be normally distributed, the VaR of the portfolio is calculated as $n_\alpha \sigma_{R_p} W$, where n_α is the confidence parameter on which the VaR is predicated, σ_{R_p} is the standard deviation of the portfolio return, and W is a scale parameter reflecting the overall size of the portfolio. So if α=99%, the VaR is $2.33\sigma_{R_p} W$.

Given the expression of the VaR which is based on the normal distribution assumption, for the potential new portfolio at a confidence level of α and an existing portfolio, we have:

$$\begin{aligned} & VaR^{\text{new}} / VaR^{\text{old}} \\ & = (n_a W^{\text{new}} \sigma_{R_p^{\text{new}}}) / (n_a W^{\text{old}} \sigma_{R_p^{\text{old}}}) \\ & = (W^{\text{new}} \sigma_{R_p^{\text{new}}}) / (W^{\text{old}} \sigma_{R_p^{\text{old}}}) \end{aligned} \tag{4}$$

where VaR^{new} is the Value-at-Risk of the new portfolio, and VaR^{old} is the Value-at-Risk of the existing portfolio at the confidence level α. We can use Eq.(4) to replace the standard deviation in Eq.(3), to obtain the resulting expression:

$$R_A \geq R_p^{\text{old}} + [VaR^{\text{new}} / VaR^{\text{old}} - 1] R_p^{\text{old}} / \omega \tag{5}$$

Eq.(5) is the decision rule for acquiring the candidate new asset A using VaR.

We can also define:

$$R_{\text{required}} = R_p^{\text{old}} + [VaR^{\text{new}} / VaR^{\text{old}} - 1] R_p^{\text{old}} / \omega$$

Then, the condition can be expressed as:

Rule II

$$\begin{cases} R_A \geq R_{\text{required}} \\ R_{\text{required}} = R_p^{\text{old}} + \left[VaR^{\text{new}} / VaR^{\text{old}} - 1 \right] R_p^{\text{old}} / \omega \end{cases}$$

Rule II implies that there is a required rate of return for deciding whether to add new asset A to the existing portfolio when considering the Value-at-Risk of the potential new portfolio and the existing portfolio. This required rate of return of the new candidate asset A consists of the expected return on the existing portfolio plus an adjustment factor that depends on the Value-at-Risk associated with both the potential new portfolio and the existing portfolio. The higher the risk, the higher the adjustment factor is, and the higher the required rate of return for this candidate new asset should be. This implication is similar to what we can obtain from the Markowitz's portfolio selection models discussed in the previous chapters.

Define the incremental VaR as $IVaR$,

$$IVaR = VaR^{\text{new}} - VaR^{\text{old}}$$

and define $\eta_A(VaR)$ as the percentage increase in VaR caused by the acquisition of the position in asset A divided by the relative size of the new position,

$$\eta_A(VaR) = (VaR^{\text{new}} - VaR^{\text{old}}) / VaR^{\text{old}}$$

Thus, Rule II can be rearranged as:

Rule II′

$$\begin{cases} R_A \geq R_{\text{required}} \\ R_{\text{required}} = R_p^{\text{old}} + \left[VaR^{\text{new}} / VaR^{\text{old}} - 1 \right] R_p^{\text{old}} / \omega \\ \qquad\qquad = \left[1 + \eta_A(VaR) \right] R_p^{\text{old}} \end{cases} \quad (6)$$

$\eta_A(VaR)$ can be interpreted as the elasticity of the VaR with respect to α, which is the proportion of amount invested in new asset A. This elasticity can be used to measure the increase in the risk of the portfolio, adjusted for the size of α. The implication of Rule II in Eq.(6) is that the required rate of return of the candidate asset is equal to the expected rate of return to the exiting portfolio times one plus the VaR elasticity. It is obvious that the greater the elasticity (or the higher the $IVaR$), the greater the risk associated with the new investment, and the higher the required rate of return of the new candidate asset.

Solving the optimal problem again

1. Behind the problem

The above generalisation of the Sharpe's rule now can be applied to any investment or dis-investment involving a new candidate asset or existing assets, or taking short position we do not have. The decision rule indicates whether we would be better off making an investment (or dis-investment) decision. It seems that it is quite easy to make investment (or dis-investment) decision using the generalised Sharpe's rule, just by plugging the related data into a computer program to calculate R_A, and the problem is solved simply by choosing the size of the new candidate asset.

2. Optimal size of new candidate asset

In actual investment decision-making, it is often the case that we do not know if the candidate asset is a proper choice and what proportion of it should be if we decide to go with it, that is, we must decide the optimal position size of the new asset. To solve this problem, we need to find that satisfies Eq.(3) or Eq.(5). There is a level of ω, which can guarantee the minimum required rate of return of the new candidate asset that satisfies Eq.(3) and Eq.(5).

The relationship between the required rate of return and the position size ω mainly depends on the ratio of the variance of the new portfolio corresponding to the position size and the variance of the old portfolio, or the $\eta_A(VaR)$, the VaR elasticity. From Eqs.(3) and (4) or Eq.(5), we can find that the required rate of the return R_A increases with VaR^{new}, $IVaR$ and $\eta_A(VaR)$. All these values reflect the degree of risk; this is why they will push the required rate of return to a higher level. The relationship between the required rate of the return and the position size ω forms a curve which reflects the rate of change of $\eta_A(VaR)$ and the incremental VaR. The shape of the required return curve reflects the shape of an underlying $IVaR$ curve. The curve showing the relationship between required return curve and the position size will either fall initially and then start to rise (Fig.1), or rise indefinitely (Fig.2).

Whether the curve initially falls or rises depends on the degree of the correlation of the return on the asset with the return on the rest of the portfolio. The

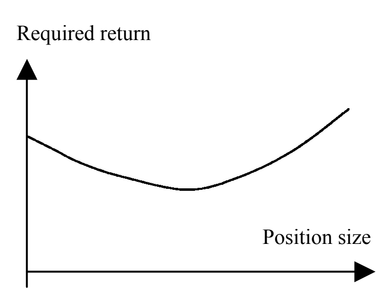

Fig.1 Required return and the position size

curve rises after it passes the lowest point because after a certain point the investment in the new asset would become so large relative to the portfolio that it would start to dominate it. Further increases in the size of the position in the new asset would add to overall risk and push up the *IVaR*, therefore pushing up the required rate of return.

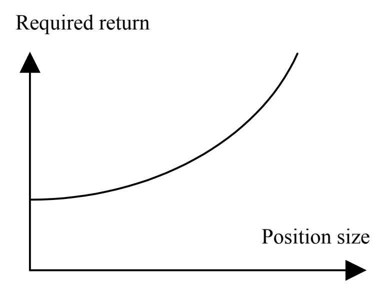

Fig.2 Required return and the position size

3. Answer to the decision problem

The answer to the investment decision problem is based on the relationship between the required rate of the return of the new asset and the expected return on the asset. The method used to solve this problem is to find the optimal level of the investment at which the required return just cuts the expected return curve from below. This criterion is necessary to ensure that the investment level ω^* is optimal. This level of investment is optimal because it maximizes the risk-adjust expected return. At any investment level above ω^*, the marginal increase in investment has a required return that exceeds the return expected from the asset, which means that we would be better off by reducing our investment. At any level below, marginal increases in investment have a required return that is less than the return expected from the investment, which means we are better off increasing our investment. Thus, the optimal level of the investment is that at which the investment level equals ω^*. As can be seen from Fig.4, it is not optimal to invest where the required return cuts the expected asset return from the above, because we can always increase the investment surplus (the excess of the expected asset return over the required return) by investing more, and could not do so if the initial position is optimal.

(1) No optimal investments level with new asset (no acquisition)

In Fig.3, the expected return level to the new candidate asset is always lower than the required return regardless of the change of size of the position. For this case, no amount of the new asset is worth adding to the existing portfolio according to Rule I or Rule II.

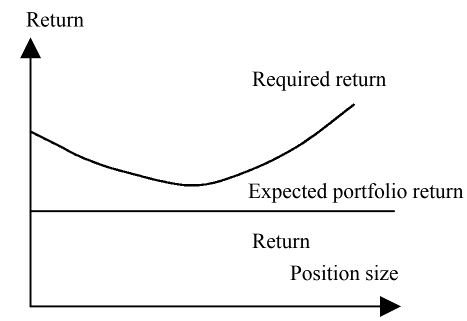

Fig.3 Required return and position size

(2) Optimal investment level with ω^* proportion in new asset (acquisition)

In Fig.4, the two curves meet at two points; thus we only need to examine which points indicate the optimal level. By the criterion above, we should invest in the new asset with the size of investment (given by ω^* in Fig.4) determined by the point at which the required return cuts the expected return curve from below.

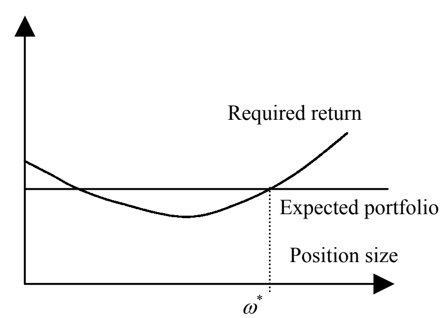

Fig.4 Required return and position size

TESTING AND ANALYTICAL RESULTS: TWO PORTFOLIO SELECTION CASES

Data source

We constructed two portfolios using five years of daily data of the price of several stocks collected from the Bloomberg[1]. The time period is from December 5, 1997 to December 5, 2000. Construct portfolio as the followings:

Portfolio 1: It is composed of TD (CN Equity), AOL (US Equity) and Coca-Cola (US Equity). New candidate asset: NT (US Equity).

Portfolio 2: Composed of TD (CN Equity), AOL (US Equity) and NT (US Equity). New candidate asset: Coca-Cola (US Equity).

Methodology of constructing the portfolios with the proper position size

The procedure involved in the generalized Sharpe's rule assumes that the investor already holds the existing portfolio. We do not have information about how it is constructed; so we assume that the position for each asset in existing portfolios is at the optimal level. If the position sizes for the assets in the portfolios are not optimal, the general Sharpe's rule is not powerful enough to demonstrate that the decision result can determine the optimized portfolio. Hence, we use the Markowitz model in the portfolio selection to construct the portfolios, and assume that the investor selects the minimum-variance portfolio (Elton *et al*., 1998a; 1998b; Panjer *et al*., 1990). So, we have:

$$\boldsymbol{X}^{\text{MIN}} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}{\boldsymbol{e}^{\text{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{e}} \tag{7}$$

where $\boldsymbol{X}^{\text{MIN}}=(x_1, x_2, x_3)$ is the minimum variance portfolio, $\boldsymbol{\Sigma}^{-1}$ is the covariance matrix, and $\boldsymbol{e}^{\text{T}}=(1,1,1)$ is the proportion of the position size for each asset.

Analytical results

Based on the weekly data, we calculated the weekly return, the variance of each asset, the covariance between assets in the old and new portfolio, and the correlation between the new candidate assets and the existing portfolio. Then, we use the minimum-variance method in Markowitz model to obtain the $\boldsymbol{X}^{\text{MIN}}$ for each existing portfolio.

A problem in applying the generalized Sharpe's rule is how to obtain the variance for the prospective new portfolio after acquiring the new asset. Although there are different approaches, the following is recommended as an estimator. This paper uses the position size as weights for the standard deviation. Thus, the weights for variance should be the square of the position size.

$$\begin{aligned}\sigma^2_{R_p^{\text{new}}} &= \omega^2\sigma^2_{\text{new-asset}} + (1-\omega)^2\sigma^2_{R_p^{\text{old}}} \\ &\quad + 2\omega(1-\omega)\rho\sigma_{\text{new-asset}}\sigma_{R_p^{\text{old}}}\end{aligned} \tag{8}$$

where ω is the position size for the new asset, ρ is the correlation coefficient between the existing portfolio and the new asset, and $\sigma^2_{\text{new-asset}}$ is the variance of the new asset α. Then, using the equation: $\min\{\boldsymbol{X}^{\text{T}}\boldsymbol{\Sigma}\boldsymbol{X} \mid \boldsymbol{U}^{\text{T}}\boldsymbol{X} = \boldsymbol{U}_p, \boldsymbol{e}^{\text{T}}\boldsymbol{X} = 1\}$, we obtain the required return for the new assets:

$$R_{\text{required}} = R_p^{\text{old}} + [\sigma_{R_p^{\text{new}}} / \sigma_{R_p^{\text{old}}} - 1]R_p^{\text{old}} / \omega$$

We use different ω to try to obtain the optimal level ω^* and the required return for the candidate asset (see Appendix I: Matlab programs and results).

Case 1:

We use a Portfolio composed of TD (CN Equity), AOL (US Equity) and Coca-Cola (US Equity) as the existing portfolio. Choose NT (US Equity) as the new candidate asset. Using equation:

$$\min\{\boldsymbol{X}^{\text{T}}\boldsymbol{\Sigma}\boldsymbol{X} \mid \boldsymbol{U}^{\text{T}}\boldsymbol{X} = \boldsymbol{U}_p, \boldsymbol{e}^{\text{T}}\boldsymbol{X} = 1\}$$

we get the minimum variance optimal portfolio as the existing portfolio:

$$\boldsymbol{X}^{\text{MIN}} = \begin{bmatrix} 0.3932 \\ 0.0637 \\ 0.5431 \end{bmatrix}$$

The existing portfolio consists of: X_{TD}=0.3932, X_{AOL}=0.0637, $X_{\text{Coca-Cola}}$=0.5431.

When NT is selected as the new asset, we find the expected rate of return of the new candidate asset NT, R_4=0.005459515 is always greater than the required return (Fig.5). There is no specific optimal solution of position size for the candidate asset NT. This means choosing any ω to construct a new portfolio would be better than the existing portfolio. It

[1]Bloomberg is an information network that permits instantaneous access to real-time financial data. This network is run by Bloomberg L.P. Company.

implies that using NT to replace all assets in the old portfolio is the best choice. We notice that the required rate of return is not described as the theoretical one in Figs.5 or 6. The variances contribute to this deviation as explained in the next paragraph.

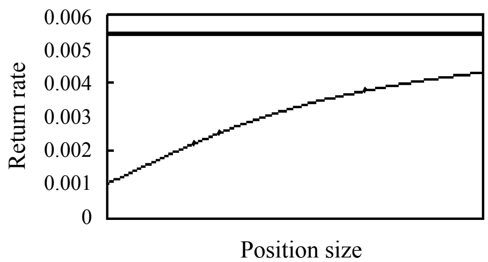

Fig.5 Optimal investment levels for Case 1 (Portfolio with TD, AOL & Coca-Cola vs NT)

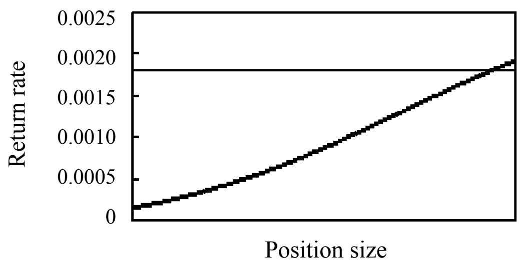

Fig.6 Optimal investment level for Case 2 (Portfolio with TD, AOL & NT vs Coca-Cola)

From the above covariance matrix, we can easily see that σ_{TD}^2(=0.002246745) and $\sigma_{Coca\text{-}Cola}^2$ (=0.001792329) are much less than σ_{AOL}^2(= 0.006616914), just one third or less of σ_{AOL}^2. But when covariance matrix for the assets TD, AOL, Coca-Cola, NT is:

$$\begin{bmatrix} 0.002246745 & 0.001279223 & 0.000152322 & 0.001044738 \\ 0.001279223 & 0.006616914 & 0.000226959 & 0.00261825 \\ 0.000152322 & 0.000226959 & 0.001792329 & 4.09346\text{E-}05 \\ 0.001044738 & 0.00261825 & 4.09346\text{E-}05 & 0.006374525 \end{bmatrix}$$

NT is chosen as the new asset, it has higher variance, σ_{NT}^2=0.006374525, similar to the AOL. So, when specifying the $\boldsymbol{X}^{MIN}$, we choose the minimum-variance optimal portfolio, and set a greater position size for TD and Coca-Cola and less for AOL to minimize the variance of the portfolio. However, when we choose the candidate asset for the new portfolio, the higher variance implies possibly higher expected return, and we consider the higher expected return from the new asset.

Case 2:

First, we use TD (CN Equity), AOL (US Equity) and NT (US Equity) to construct a portfolio as the existing portfolio by Eq.(7). We get the minimum variance optimal portfolio as:

$$\boldsymbol{X}^{MIN} = \begin{bmatrix} 0.7585 \\ 0.0948 \\ 0.1474 \end{bmatrix}$$

The existing portfolio consists of: X_{TD}=0.7585, X_{AOL}=0.0948, X_{NT}=0.1474.

We choose Coca-Cola (US Equity) as the new candidate asset. When Coca-Cola is selected as the candidate asset for the new portfolio, the required return curve cuts the expected return curve of Coca-Cola with value R_4=0.001775643, from the below (Fig.6). This means that there is an optimal level ω^* at which the new portfolio will be better off. This optimal ω^*=0.9203. The optimal position sizes for the assets in the new portfolio $\boldsymbol{X}_{new}$ is:

$$\boldsymbol{X}_{new} = \begin{bmatrix} 0.0604 \\ 0.0075 \\ 0.0117 \\ 0.9203 \end{bmatrix}$$

That is X_{TD}=0.0604, X_{AOL}=0.0075, X_{NT}=0.0117, X_{AOL}=0.9203.

This result verifies the situation described in Fig.4. As in Case 2, the variances, their covariances and rates of return contribute to this deviation, as explained in the next paragraph.

When covariance matrix for the assets TD, AOL, NT, Coca-Cola is:

$$\begin{bmatrix} 0.002246745 & 0.001279223 & 0.001044738 & 0.000152322 \\ 0.001279223 & 0.006616914 & 0.002618250 & 0.000226959 \\ 0.001044738 & 0.002618250 & 0.006374525 & 4.09346\text{E-}05 \\ 0.000152322 & 0.000226959 & 4.09346\text{E-}05 & 0.001792329 \end{bmatrix}$$

we can easily see that σ_{TD}^2(=0.002246745) is much less than σ_{AOL}^2(=0.006616914) and σ_{NT}^2(=

0.006374525), in fact it was just one third or less of them. But when Coca-Cola is selected as the candidate new asset, it has a lower variance, $\sigma_{\text{Coca-Cola}}^2$ (=0.001792329). When specifying the $\boldsymbol{X}^{\text{MIN}}$, we choose the minimum-variance optimal portfolio, and choose a greater size for TD and less for AOL and NT to minimize the variance of the portfolio. When we choose the candidate asset for the new portfolio, we consider the higher expected return. Coca-Cola does not have very higher return, but there exists a point at which its expected return meets the required return from the below (Fig.6). This determines an optimal level for the new portfolio.

CONCLUSION

By solving the Markowitz's minimum-variance portfolio selection problem and by operationalizing the decision rules, we have verified the validity of the new decision rule based on Dowd's idea of generalizing Sharpe's rule. This new method combines Markowitz's portfolio selection theory and the Shape's rule, and develops a very practical decision rule which can be used to make investment decision, assets acquisition, and dis-investment starting from any existing portfolio. Also, the method for solving the optimal level of investment for the new asset in the new portfolio is developed in this paper for implementing the decision rule. It provides a solution to the real investment decision related to portfolio management. The main conclusions can be summarized as the followings:

Advantages and potential use

1. Assets investment and the portfolio management

The method developed above is useful in asset investment and portfolio management. The most important use perhaps is to evaluate the efficiency of the current and new portfolios. The efficiency of evaluation relies on Rule I and Rule II based on Eqs.(3) and (6), which state that any new asset, or any included asset in the portfolio, should have an expected return at least as great as the required return. Similarly, any asset excluded from the portfolio should have an expected return that is less than the required return. Using these guidelines to invest or dis-invest, or to exclude an asset, therefore is straightforward.

2. Risk monitoring and hedge decision

Dynamically evaluating the existing portfolio will change the value of the risk of the portfolio. Thus, dynamically monitoring the risk and hedging the portfolio is another important aspect of asset management. The new method overcomes the shortcoming of the traditional hedge and dynamic hedge which could not afford an efficient way to monitor the dynamic risk–possibly incorrect risk prediction and high cost (Hull, 2000; Wilcox, 2001; Ahn *et al*., 1999). It can be used to guide the risk monitoring and hedging decision. This is because hedging decisions are investment decisions with the objective of reducing the overall risk. Rule II based on Eq.(2) is related to the Value-at-Risk of both the new and old portfolio. To apply Rule II to reduce overall *VaR*, we need a negative *IVaR*. By Eq.(6), the hedge position must have a required return less than R_p^{old}. This VaR approach to hedge tells us whether to hedge, and if we do hedge, the size of position. It is a great advantage that this VaR approach hedges the exposure of the portfolio as a whole, not the exposure of some particular part of it. This allows for the interaction of the hedge position with all the risk in the portfolio, and overcomes the shortcoming of the traditional approach of hedge in some standard textbooks.

Limitations of the approach

(1) The approach can be used efficiently based on the assumption that the returns of the assets are jointly normally distributed. However, if the distribution is not normal, Rules II and I based on Eq.(3) cannot be used to provide a good guidance. Some problems such as skewness, excess kurtosis, or other non-normal features need to be considered in the investment decision. In such conditions, calculating the VaRs and IVaRs would be quite complex and the adjustment of them is quite difficult. A standard example is the risk of the large market move. Market returns often show fat tails that indicate that large losses are more likely than predicted by the normality assumption. Reliance on the normal distribution therefore means it can lead to a dramatic underestimation of true VaR.

(2) The approach relies on the historical data of the asset; so all the important parameters are historical.

However, financial markets are dynamic, changing not just the price but the key factors determining the pro. Any new policy, change of preference of the investor, or the customer of the listed companies could easily change the market. Using the historical data for decision-making may lack reliability due to structural changes in the market. This is the main drawback of the statistical methods used in the prediction. This is why many investors are fundamental and not technical, although there are many theories for risk management.

(3) This approach still needs lots of work on data processing the data and requires an explicit modelling if the optimal portfolio is large. Hence, it is not easy to use it for the short-term dynamic hedging, say, weekly and daily hedging because we cannot solve a dynamic optimal portfolio selection problem by using Markowitz theory and these new decision rules (Rules I and II) quickly enough. Developing investment decision support system based on software and database would solve this problem for the short-term dynamic hedging with high frequency.

ACKNOWLEDGEMENT

We are grateful to CIDA and CPER ZJU funding to support this research. We thank Professor Tony Wirjanto, at the Centre of Advanced Finance Research, University of Waterloo, and Professor Kevin Lee at the Business School, University of Windsor, Canada for their kind help and discussion with authors. Thanks also go to Dr. Deming Lu and Dr. Xiaoquan Zhang for their helpful comments.

References

Best, M.J., 2001. Portfolio Optimization, ACT/SAT 991p Course Notes. University of Waterloo.

Dowd, K., 1999. A Value-at-Risk approach to risk-return analysis. *The Journal of Portfolio Management*, **25**(summer):60-67.

Dowd, K., 2000. Estimating Value-at-Risk: A subjective approach. *The Journal of Risk Finance*, **1**(4):43-46.

Dynkin, L., Hyman, J., Wu, W., 2000. Value of skill in security selection versus asset allocation in credit markets. *The Journal of Portfolio Management*, **27**(1):20-41.

Elton, E.J., Gruber, M.J., Padberg, M.W., 1978a. Simple criteria for optimal portfolio selection: tracing out the efficient frontier. *The Journal of Finance*, **33**(1):296-302.

Elton, E.J., Gruber, M.J., Padberg, M.W., 1978b. Simple criteria for optimal portfolio. *The Journal of Finance*, **31**(5):1341-1357.

Hodges, S.D., Brealey, R.A., 1978. Portfolio Selection in a Dynamic and Uncertain World, Modern Development in Investment Management, Second Edition. Dryden Press, Hinsdale, Illinois, p.348-364.

Ho, T.S.Y., Chen, M.Z.H., Eng, F.H.T., 1996. VaR analytics: portfolio structure, key rate convexities, and VaR betas–A new approach to determining the VaR of a portfolio. *The Journal of Portfolio Management*, **23**:89-98.

Johansson, F., Sieler, M.J., Tjarberg, M., 1999. Measuring downside portfolio risk. *The Journal of Finance of Portfolio Management*, **26**(1):96-107.

Litterman, R., 1996. Hot Sports™ and hedges. *Journal of Portfolio Management*, **Special Issue**:52-75.

Linsmeier, T.J., Pearson, N.D., 1996. Risk Measurement: An Introduction to Value-at-Risk. Working Paper, University of Illinois at Urban-Champaign.

Linsmeir, T.J., Pearson, N.D., 2000. Value-at-Risk. *Financial Analysts Journal*, **56**(2):47-67.

Markowitz, H.M., 1952. Portfolio selection. *The Journal of Finance*, **VII**(1):77-91.

Markowitz, H.M., 1959. Portfolio Selection: Efficient Diversification of Investments. Yale University Press.

Sharpe, W.F., 1963. A simplified model for portfolio analysis. *Management Science*, **9**:277-293.

Sharpe, W.F., 1994. The Sharpe ratio. *Journal of Portfolio Management*, **21**(1):49-58.

Wilcox, J., 2001. Better dynamic hedging. *The Journal of Risk Finance*, **2**:5-15.