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Interval standard neural network models for nonlinear systems*

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Abstract: A neural-network-based robust control design is suggested for control of a class of nonlinear systems. The design approach employs a neural network, whose activation functions satisfy the sector conditions, to approximate the nonlinear system. To improve the approximation performance and to account for the parameter perturbations during operation, a novel neural network model termed standard neural network model (SNNM) is proposed. If the uncertainty is bounded, the SNNM is called an interval SNNM (ISNNM). A state-feedback control law is designed for the nonlinear system modelled by an ISNNM such that the closed-loop system is globally, robustly, and asymptotically stable. The control design equations are shown to be a set of linear matrix inequalities (LMIs) that can be easily solved by available convex optimization algorithms. An example is given to illustrate the control design procedure, and the performance of the proposed approach is compared with that of a related method reported in literature.

Key words: Interval standard neural network model (ISNNM), Linear matrix inequality (LMI), Nonlinear system, Asymptotic stability, Robust control
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INTRODUCTION

Neural networks have been successfully employed for controlling nonlinear systems since the 1990's (Narendra and Parthasarathy 1990; Hunt *et al*., 1992; Suykens *et al*., 1996). In these nonlinear control systems, neural networks have been used either for modelling the system to be controlled, or for designing a controller, or both. Recently, the robustness issue has been an important focus of research in neuro-control circles (Suykens *et al*., 1996; Wams *et al*., 1999; Ayala Botto *et al*., 2000; Xu and Ioannou, 2003; Lin and Lin, 2001). It is well known that the performance of a carefully designed system may degrade seriously due to the unavoidable uncertainties resulting from modelling error, external disturbance and parameter fluctuation during on-line operation, if the robustness of the controller is not taken into account during design. Thus, it is essential to introduce robustness measures in designing controllers in the presence of uncertainties. If deviations and perturbations in system parameters are the main sources of uncertainty, and are all bounded, the system is called an interval system. Recently, based on the linear matrix inequality (LMI) approach (Boyd *et al*., 1994), several global and robust stability criteria for interval recurrent neural networks have been proposed (Li *et al*., 2004; Xu *et al*., 2004). In this paper, we follow the ideas of Li *et al.*(2004) and Xu *et al.*(2004) and extend the method to controllers design for nonlinear systems. Robustness of controllers is the focus of this paper.

Several neural network models have been employed for modelling nonlinear systems, and the approaches to controller synthesis are also quite diversified. In this paper, we propose a new neural network model termed standard neural network model (SNNM) which is the extension of Lur'e system (Boyd *et al*., 1994) to provide a general framework for describing various neural networks. Based on this unified model, we first design a state-feedback controller for the SNNM with known parameters using the Lyapunov

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method and S-procedure such that the close-loop system is globally asymptotically stable, then we extend the design approach to the interval SNNM (ISNNM). The resulting design equations turn out to be a set of LMIs which can be solved conveniently using existing optimization software, e.g., MATLAB LMI Control Toolbox (Gahinet *et al*., 1995). Next, we train neural networks to approximate nonlinear systems and transform the neural networks into SNNMs or ISNNMs when we consider the influence of approximation errors and the deviations and perturbations of parameters of neural networks on nonlinear systems. Then, based on the design methodology for SNNMs and ISNNMs, we design state-feedback controllers to stabilize or robustly stabilize the nonlinear systems.

The following notations are used throughout the paper. $\mathbb{R}^n$ denotes n dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices, $\boldsymbol{I}$ denotes identity matrix of appropriate order, $\|\boldsymbol{x}\|$ denotes the Euclidean norm of the vector $\boldsymbol{x}$, $*$ denotes the symmetric parts. The notations $\boldsymbol{X}>\boldsymbol{Y}$ and $\boldsymbol{X}\geq\boldsymbol{Y}$, respectively, where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are matrices of the same dimensions, mean that the matrix $(\boldsymbol{X}-\boldsymbol{Y})$ is positive definite and positive semi-definite, respectively. If $\boldsymbol{X}\in\mathbb{R}^p$ and $\boldsymbol{Y}\in\mathbb{R}^q$, $\boldsymbol{C}(\boldsymbol{X};\ \boldsymbol{Y})$ denotes the space of all continuous functions mapping $\mathbb{R}^p\rightarrow\mathbb{R}^q$.

STANDARD NEURAL NETWORK MODEL

In linear robust control theory, a system with uncertainties can be transformed into a standard form known as linear fractional transformation (LFT) (Chandrasekharan, 1996). Similar to the LFT, as shown in (Moore *et al*., 1968; Rios-Patron, 2000), we can synthesize controllers for nonlinear systems composed of a neural network by transforming them into SNNMs. The SNNM represents a neural network model in form of a linear dynamic system coupled with static nonlinear operators consisting of bounded activation functions. Here, we discuss only the discrete-time SNNM, though similar architecture and results for continuous-time SNNMs can also be achieved (Zhang and Liu, 2005). A discrete-time SNNM with inputs and outputs is shown in Fig.1. The block $\boldsymbol{\Phi}$ is a block diagonal operator composed of nonlinear activation functions $\phi_i(\xi_i(k))$, which are typically continuous, differentiable, monotonically increasing, slope-restricted, and bounded. The matrix $\boldsymbol{N}$ represents a linear mapping between the inputs and outputs with a time delay operator $z^{-1}\boldsymbol{I}$ in the discrete-time case (or the integrator $\int$ in the continuous-time case) and the operator $\boldsymbol{\Phi}$. The vectors $\xi(k)$ and $\boldsymbol{\phi}(\xi(k))$ are the input and output of the nonlinear operator $\boldsymbol{\Phi}$, respectively. The vectors $\boldsymbol{u}(k)$ and $\boldsymbol{y}(k)$ are the inputs and outputs of the SNNM, respectively.

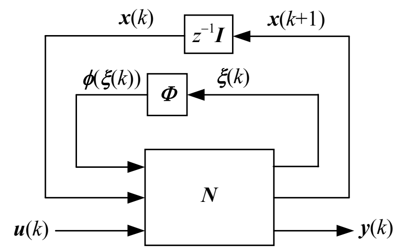

Fig.1 Discrete-time standard neural network model (SNNM) with inputs and outputs

If $\boldsymbol{N}$ in Fig.1 is partitioned as

$$\boldsymbol{N}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}_p & \boldsymbol{B}_u\\ \boldsymbol{C}_q & \boldsymbol{D}_p & \boldsymbol{D}_{qu}\\ \boldsymbol{C}_y & \boldsymbol{D}_{yp} & \boldsymbol{D}_u\end{bmatrix}, \tag{1}$$

then the input-output SNNM can be depicted as an input-output linear difference inclusion (LDI):

$$\begin{cases}\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}_p\boldsymbol{\phi}(\xi(k))+\boldsymbol{B}_u\boldsymbol{u}(k),\\ \xi(k)=\boldsymbol{C}_q\boldsymbol{x}(k)+\boldsymbol{D}_p\boldsymbol{\phi}(\xi(k))+\boldsymbol{D}_{qu}\boldsymbol{u}(k),\\ \boldsymbol{y}(k)=\boldsymbol{C}_y\boldsymbol{x}(k)+\boldsymbol{D}_{yp}\boldsymbol{\phi}(\xi(k))+\boldsymbol{D}_u\boldsymbol{u}(k),\end{cases} \tag{2}$$

where $\boldsymbol{x}\in\mathbb{R}^n$ is the state vector, $\boldsymbol{A}=(a_{ij})\in\mathbb{R}^{n\times n}$, $\boldsymbol{B}_p=(b_{p,ij})\in\mathbb{R}^{n\times L}$, $\boldsymbol{B}_u=(b_{u,ij})\in\mathbb{R}^{n\times m}$, $\boldsymbol{C}_q=(c_{q,ij})\in\mathbb{R}^{L\times n}$, $\boldsymbol{C}_y=(c_{y,ij})\in\mathbb{R}^{l\times n}$, $\boldsymbol{D}_p=(d_{p,ij})\in\mathbb{R}^{L\times L}$, $\boldsymbol{D}_u=(d_{u,ij})\in\mathbb{R}^{l\times m}$, $\boldsymbol{D}_{qu}=(d_{qu,ij})\in\mathbb{R}^{L\times m}$, and $\boldsymbol{D}_{yp}=(d_{yp,ij})\in\mathbb{R}^{l\times L}$ are the corresponding state-space matrices, $\xi\in\mathbb{R}^L$ are the inputs of nonlinear operator $\boldsymbol{\Phi}$, $\boldsymbol{\phi}\in\boldsymbol{C}(\mathbb{R}^L;\ \mathbb{R}^L)$ are the outputs of nonlinear operator $\boldsymbol{\Phi}$ satisfying $\boldsymbol{\phi}(0)=0$, $\boldsymbol{u}\in\mathbb{R}^m$ are the inputs, $\boldsymbol{y}\in\mathbb{R}^l$ are the outputs, and $L\in\mathbb{R}$ is the number of nonlinear activation functions (that is, the total number of neurons in the hidden layers and output layer of the neural network).

In this paper, we assume that the activation functions in the SNNM satisfy the sector conditions $\phi_i(\xi_i(k))/\xi_i(k)\in[0, u_i]$, i.e., $\phi_i(\xi_i(k))\cdot[\phi_i(\xi_i(k)-u_i\xi_i(k))]\leq0$.

In the following sections, we will design state-feedback controller for the SNNM Eq.(2) whose parameters are known such that the overall closed-loop system is globally asymptotically stable. Based on the above results, we then discuss the design approach of the SNNM with interval parameters. The controller is of the form

$$\boldsymbol{u}(k)=\boldsymbol{K}\boldsymbol{x}(k), \tag{3}$$

where $\boldsymbol{K}\in\mathbb{R}^{m\times n}$ is the feedback gain. The overall closed-loop system of the SNNM Eq.(2) and the feedback controller Eq.(3) is described by

$$\begin{cases}\boldsymbol{x}(k+1)=\tilde{\boldsymbol{A}}\boldsymbol{x}(k)+\boldsymbol{B}_p\boldsymbol{\phi}(\boldsymbol{\xi}(k)),\\ \boldsymbol{\xi}(k)=\tilde{\boldsymbol{C}}_q\boldsymbol{x}(k)+\boldsymbol{D}_p\boldsymbol{\phi}(\boldsymbol{\xi}(k)),\\ \boldsymbol{y}(k)=\tilde{\boldsymbol{C}}_y\boldsymbol{x}(k)+\boldsymbol{D}_{yp}\boldsymbol{\phi}(\boldsymbol{\xi}(k)),\end{cases} \tag{4}$$

where

$$\tilde{\boldsymbol{A}}=\boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{K},\ \tilde{\boldsymbol{C}}_q=\boldsymbol{C}_q+\boldsymbol{D}_{qu}\boldsymbol{K},\ \tilde{\boldsymbol{C}}_y=\boldsymbol{C}_y+\boldsymbol{D}_u\boldsymbol{K}.$$

FEEDBACK STABILIZATION OF THE SNNM WITH CONSTANT PARAMETERS

Theorem 1 There exists a state-feedback control law $\boldsymbol{u}(k)=\boldsymbol{K}\boldsymbol{x}(k)$ such that the closed-loop system Eq.(4) with constant parameters is globally asymptotically stable provided that there exist symmetric positive definite matrices $\boldsymbol{X}$, a matrix $\boldsymbol{Y}$, diagonal positive definite matrix $\boldsymbol{S}$, and diagonal semi-positive definite matrix $\boldsymbol{\Psi}$ that satisfy the following LMI

$$\begin{bmatrix}-\boldsymbol{X} & \boldsymbol{AX}+\boldsymbol{B}_u\boldsymbol{Y} & \boldsymbol{B}_p\boldsymbol{S}\\ (\boldsymbol{AX}+\boldsymbol{B}_u\boldsymbol{Y})^{\mathrm{T}} & -\boldsymbol{X} & (\boldsymbol{C}_q\boldsymbol{X}+\boldsymbol{D}_{qu}\boldsymbol{Y})^{\mathrm{T}}\\ \boldsymbol{SB}_p^{\mathrm{T}} & \boldsymbol{C}_q\boldsymbol{X}+\boldsymbol{D}_{qu}\boldsymbol{Y} & \boldsymbol{D}_p\boldsymbol{S}+\boldsymbol{SD}_p^{\mathrm{T}}-2\boldsymbol{\Psi}\end{bmatrix}<0. \tag{5}$$

Furthermore, the feedback gain $\boldsymbol{K}$ is obtained as $\boldsymbol{K}=\boldsymbol{Y}\boldsymbol{X}^{-1}$.

Proof For simplicity, we denote $\boldsymbol{x}(k)$ as $\boldsymbol{x}_k$, $\xi_i(k)$ as $\xi_{k,i}$, $\phi_i(\xi_i(k))$ as $\phi_{k,i}$, $\boldsymbol{\phi}(\boldsymbol{\xi}(k))$ as $\boldsymbol{\phi}_k$. For the closed-loop system Eq.(4), we choose the following positive definite Lyapunov function

$$V(\boldsymbol{x}_k)=\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_k+2\sum_{i=1}^{L}\lambda_i\sum_{j=0}^{k-1}\phi_i(\xi_i(j))\xi_i(j),$$

where $\boldsymbol{P}>0$, $\lambda_i\geq0$. Thus, $\forall\boldsymbol{x}_k\neq0$, $V(\boldsymbol{x}_k)>0$ and $V(\boldsymbol{x}_k)=0$ iff $\boldsymbol{x}_k=0$. The difference along the solution of Eq.(4) is

$$\begin{aligned}\Delta V(\boldsymbol{x}_k)&=V(\boldsymbol{x}_{k+1})-V(\boldsymbol{x}_k)\\
&=\boldsymbol{x}_{k+1}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_{k+1}-\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_k+2\sum_{i=1}^{L}\lambda_i\phi_{k,i}\xi_{k,i}\\
&=(\tilde{\boldsymbol{A}}\boldsymbol{x}_k+\boldsymbol{B}_p\boldsymbol{\phi}_k)^{\mathrm{T}}\boldsymbol{P}(\tilde{\boldsymbol{A}}\boldsymbol{x}_k+\boldsymbol{B}_p\boldsymbol{\phi}_k)\\
&\quad-\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_k+2\sum_{i=0}^{L}\lambda_i\phi_{k,i}(\tilde{\boldsymbol{C}}_{q,i}\boldsymbol{x}_k+\boldsymbol{D}_{p,i}\boldsymbol{\phi}_k)\\
&=\boldsymbol{x}_k^{\mathrm{T}}(\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\tilde{\boldsymbol{A}}-\boldsymbol{P})\boldsymbol{x}_k+\boldsymbol{x}_k^{\mathrm{T}}(\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}_p+\tilde{\boldsymbol{C}}_q^{\mathrm{T}}\boldsymbol{\Lambda})\boldsymbol{\phi}_k+\boldsymbol{\phi}_k^{\mathrm{T}}(\boldsymbol{B}_p^{\mathrm{T}}\boldsymbol{P}\tilde{\boldsymbol{A}}\\
&\quad+\boldsymbol{\Lambda}\tilde{\boldsymbol{C}}_q)\boldsymbol{x}_k+\boldsymbol{\phi}_k^{\mathrm{T}}(\boldsymbol{B}_p^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}_p+\boldsymbol{\Lambda}\boldsymbol{D}_p+\boldsymbol{D}_p^{\mathrm{T}}\boldsymbol{\Lambda})\boldsymbol{\phi}_k\\
&=\begin{bmatrix}\boldsymbol{x}_k\\ \boldsymbol{\phi}_k\end{bmatrix}^{\mathrm{T}}\underbrace{\begin{bmatrix}\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\tilde{\boldsymbol{A}}-\boldsymbol{P} & \tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}_p+\tilde{\boldsymbol{C}}_q^{\mathrm{T}}\boldsymbol{\Lambda}\\ \boldsymbol{B}_p^{\mathrm{T}}\boldsymbol{P}\tilde{\boldsymbol{A}}+\boldsymbol{\Lambda}\tilde{\boldsymbol{C}}_q & \boldsymbol{B}_p^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}_p+\boldsymbol{\Lambda}\boldsymbol{D}_p+\boldsymbol{D}_p^{\mathrm{T}}\boldsymbol{\Lambda}\end{bmatrix}}_{\boldsymbol{T}_0}\begin{bmatrix}\boldsymbol{x}_k\\ \boldsymbol{\phi}_k\end{bmatrix},\end{aligned}$$

where $\tilde{\boldsymbol{C}}_{q,i}$ is the ith row of matrix $\tilde{\boldsymbol{C}}_q$, $\boldsymbol{D}_{p,i}$ is the ith row of matrix $\boldsymbol{D}_p$, $\boldsymbol{\Lambda}=\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_L)$ and $\boldsymbol{\Lambda}\geq0$.

The sector conditions, $\phi_{k,i}(\phi_{k,i}-u_i\xi_{k,i})\leq0$, can be rewritten as follows

$$\phi_{k,i}(\phi_{k,i}-u_i\tilde{\boldsymbol{C}}_{q,i}\boldsymbol{x}_k-u_i\boldsymbol{D}_{p,i}\boldsymbol{\phi}_k)\leq0,$$

which is equivalent to

$$2\phi_{k,i}^2-2\phi_{k,i}u_i\tilde{\boldsymbol{C}}_{q,i}\boldsymbol{x}_k-2\phi_{k,i}u_i\boldsymbol{D}_{p,i}\boldsymbol{\phi}_k\leq0. \tag{6}$$

Rewrite Eq.(6) in the matrix form

$$\begin{bmatrix}\boldsymbol{x}_k\\ \phi_{k,1}\\ \vdots\\ \phi_{k,i-1}\\ \phi_{k,i}\\ \phi_{k,i+1}\\ \vdots\\ \phi_{k,L}\end{bmatrix}^{\mathrm{T}}\underbrace{\begin{bmatrix}0&0&\cdots&0&-\tilde{\boldsymbol{C}}_{q,i}^{\mathrm{T}}u_i&0&\cdots&0\\ 0&0&\cdots&0&-d_{p,i,1}u_i&0&\cdots&0\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 0&0&\cdots&0&-d_{p,i,i-1}u_i&0&\cdots&0\\ -u_i\tilde{\boldsymbol{C}}_{q,i}&-u_id_{p,i,1}&\cdots&-u_id_{p,i,i-1}&2-2u_id_{p,i,i}&-u_id_{p,i,i+1}&\cdots&-u_id_{p,i,L}\\ 0&0&\cdots&0&-d_{p,i,i+1}u_i&0&\cdots&0\\ \vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\ 0&0&\cdots&0&-d_{p,i,L}u_i&0&\cdots&0\end{bmatrix}}_{T_i}\begin{bmatrix}\boldsymbol{x}_k\\ \phi_{k,1}\\ \vdots\\ \phi_{k,i-1}\\ \phi_{k,i}\\ \phi_{k,i+1}\\ \vdots\\ \phi_{k,L}\end{bmatrix}\leq0,$$

where $d_{p,i,j}$ is the element of matrix $\boldsymbol{D}_p$ at ith row and jth column. By the S-procedure (Boyd *et al*., 1994), if there exist $\tau_i \geq 0$ (i=1, …, L), such that the following inequality holds

$$\boldsymbol{T}_0 - \sum_{i=1}^{L} \tau_i \boldsymbol{T}_i = \begin{bmatrix} \tilde{\boldsymbol{A}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{A}} - \boldsymbol{P} & \tilde{\boldsymbol{A}}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{B}_p + \tilde{\boldsymbol{C}}_q^{\mathrm{T}} \boldsymbol{\Lambda} \\ \boldsymbol{B}_p^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{A}} + \boldsymbol{\Lambda} \tilde{\boldsymbol{C}}_q & \boldsymbol{B}_p^{\mathrm{T}} \boldsymbol{P} \boldsymbol{B}_p + \boldsymbol{\Lambda} \boldsymbol{D}_p + \boldsymbol{D}_p^{\mathrm{T}} \boldsymbol{\Lambda} \end{bmatrix} - \begin{bmatrix} \boldsymbol{0} & -\tilde{\boldsymbol{C}}_q^{\mathrm{T}} \boldsymbol{U} \boldsymbol{T} \\ -\boldsymbol{T} \boldsymbol{U} \tilde{\boldsymbol{C}}_q & 2\boldsymbol{T} - \boldsymbol{D}_p^{\mathrm{T}} \boldsymbol{U} \boldsymbol{T} - \boldsymbol{T} \boldsymbol{U} \boldsymbol{D}_p \end{bmatrix}$$

$$= \begin{bmatrix} \tilde{\boldsymbol{A}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{A}} - \boldsymbol{P} & \tilde{\boldsymbol{A}}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{B}_p + \tilde{\boldsymbol{C}}_q^{\mathrm{T}} \boldsymbol{\Lambda} + \tilde{\boldsymbol{C}}_q^{\mathrm{T}} \boldsymbol{U} \boldsymbol{T} \\ * & \begin{pmatrix} \boldsymbol{B}_p^{\mathrm{T}} \boldsymbol{P} \boldsymbol{B}_p + \boldsymbol{\Lambda} \boldsymbol{D}_p + \boldsymbol{D}_p^{\mathrm{T}} \boldsymbol{\Lambda} \\ -2\boldsymbol{T} + \boldsymbol{D}_p^{\mathrm{T}} \boldsymbol{U} \boldsymbol{T} + \boldsymbol{T} \boldsymbol{U} \boldsymbol{D}_p \end{pmatrix} \end{bmatrix} < 0, \quad (7)$$

where $\boldsymbol{U}$=diag($u_1, u_2, \ldots, u_L$), $\boldsymbol{T}$=diag($\tau_1, \tau_2, \ldots, \tau_L$) and $\boldsymbol{T} \geq 0$, then $\boldsymbol{T}_0 < 0$, that is, $\forall \boldsymbol{x}_k \neq 0$, $\Delta V(\boldsymbol{x}_k) < 0$ and $\Delta V(\boldsymbol{x}_k) = 0$ iff $\boldsymbol{x}_k = 0$. So, if there exist a symmetric positive definite matrix $\boldsymbol{P}$, and diagonal semi-positive definite matrices $\boldsymbol{\Lambda}$ and $\boldsymbol{T}$, such that the LMI Eq.(7) holds, then the origin of the closed-loop system Eq.(4) is globally asymptotically stable. Study of the structure of the parameters in Eq.(7) shows that Eq.(7) is a nonlinear matrix inequality over $\boldsymbol{P}$, $\boldsymbol{T}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{K}$. Since no efficient algorithms are available for solving Eq.(7), we must convert Eq.(7) into LMI which can be solved by the MATLAB LMI Control Toolbox (Gahinet *et al*., 1995). Using the well-known Schur complements (Boyd *et al*., 1994), Eq.(7) can be expressed as:

$$\begin{bmatrix} -\boldsymbol{P} & \boldsymbol{P} \tilde{\boldsymbol{A}} & \boldsymbol{P} \boldsymbol{B}_p \\ * & -\boldsymbol{P} & \tilde{\boldsymbol{C}}_q^{\mathrm{T}} \boldsymbol{\Lambda} + \tilde{\boldsymbol{C}}_q^{\mathrm{T}} \boldsymbol{U} \boldsymbol{T} \\ * & * & \boldsymbol{\Lambda} \boldsymbol{D}_p + \boldsymbol{D}_p^{\mathrm{T}} \boldsymbol{\Lambda} - 2\boldsymbol{T} + \boldsymbol{D}_p^{\mathrm{T}} \boldsymbol{U} \boldsymbol{T} + \boldsymbol{T} \boldsymbol{U} \boldsymbol{D} \end{bmatrix} < 0. \quad (8)$$

Pre- and post-multiplying the left-hand side matrix of Eq.(8) by the diagonal matrix ($\boldsymbol{P}^{-1}$, $\boldsymbol{P}^{-1}$, $(\boldsymbol{\Lambda}+\boldsymbol{U}\boldsymbol{T})^{-1}$) makes Eq.(8) equivalent to

$$\begin{bmatrix} -\boldsymbol{P}^{-1} & \tilde{\boldsymbol{A}} \boldsymbol{P}^{-1} & \boldsymbol{B}_p (\boldsymbol{\Lambda} + \boldsymbol{U} \boldsymbol{T})^{-1} \\ * & -\boldsymbol{P}^{-1} & \boldsymbol{P}^{-1} \tilde{\boldsymbol{C}}_q^{\mathrm{T}} \\ * & * & \begin{pmatrix} \boldsymbol{D}_p (\boldsymbol{\Lambda} + \boldsymbol{U} \boldsymbol{T})^{-1} + (\boldsymbol{\Lambda} + \boldsymbol{U} \boldsymbol{T})^{-1} \boldsymbol{D}_p^{\mathrm{T}} \\ -2(\boldsymbol{\Lambda} + \boldsymbol{U} \boldsymbol{T})^{-1} \boldsymbol{T} (\boldsymbol{\Lambda} + \boldsymbol{U} \boldsymbol{T})^{-1} \end{pmatrix} \end{bmatrix} < 0. \quad (9)$$

By letting $\boldsymbol{X}=\boldsymbol{P}^{-1}$, $\boldsymbol{Y}=\boldsymbol{K}\boldsymbol{X}$, $\boldsymbol{S}=(\boldsymbol{\Lambda}+\boldsymbol{U}\boldsymbol{T})^{-1}$, and $\boldsymbol{\Psi}=(\boldsymbol{\Lambda}+\boldsymbol{U}\boldsymbol{T})^{-1}\boldsymbol{T}(\boldsymbol{\Lambda}+\boldsymbol{U}\boldsymbol{T})^{-1}$, Eq.(9) can be rewritten as Eq.(5).

FEEDBACK STABILIZATION OF THE INTERVAL SNNM

Theorem 1 is obtained under the assumption that all the parameters in the SNNM Eq.(2) are known. However, deviations and perturbations may occur to the parameters; such deviations and perturbations are usually bounded. Under this assumption, we may intervalize the parameters in Eq.(2). Assume the parameters of the SNNM in Eq.(2) satisfy the following constraints:

$$\underline{a}_{ij} \leq a_{ij} \leq \bar{a}_{ij}, \ \underline{b}_{p,ij} \leq b_{p,ij} \leq \bar{b}_{p,ij}, \ \underline{b}_{u,ij} \leq b_{u,ij} \leq \bar{b}_{u,ij},$$
$$\underline{c}_{q,ij} \leq c_{q,ij} \leq \bar{c}_{q,ij}, \ \underline{d}_{p,ij} \leq d_{p,ij} \leq \bar{d}_{p,ij},$$
$$\underline{d}_{qu,ij} \leq d_{qu,ij} \leq \bar{d}_{qu,ij}, \ \underline{c}_{y,ij} \leq c_{y,ij} \leq \bar{c}_{y,ij},$$
$$\underline{d}_{yp,ij} \leq d_{yp,ij} \leq \bar{d}_{yp,ij}, \ \underline{d}_{u,ij} \leq d_{u,ij} \leq \bar{d}_{u,ij}.$$

For convenience, let

$$\underline{\boldsymbol{A}} = (\underline{a}_{ij})_{n\times n}, \ \bar{\boldsymbol{A}} = (\bar{a}_{ij})_{n\times n}, \ \underline{\boldsymbol{B}}_p = (\underline{b}_{p,ij})_{n\times L},$$
$$\bar{\boldsymbol{B}}_p = (\bar{b}_{p,ij})_{n\times L}, \ \underline{\boldsymbol{B}}_u = (\underline{b}_{u,ij})_{n\times m}, \ \bar{\boldsymbol{B}}_u = (\bar{b}_{u,ij})_{n\times m},$$
$$\underline{\boldsymbol{C}}_q = (\underline{c}_{q,ij})_{L\times n}, \ \bar{\boldsymbol{C}}_q = (\bar{c}_{q,ij})_{L\times n}, \ \underline{\boldsymbol{D}}_p = (\underline{d}_{p,ij})_{L\times L},$$
$$\bar{\boldsymbol{D}}_p = (\bar{d}_{p,ij})_{L\times L}, \ \underline{\boldsymbol{D}}_{qu} = (\underline{d}_{qu,ij})_{L\times m}, \ \bar{\boldsymbol{D}}_{qu} = (\bar{d}_{qu,ij})_{L\times m},$$
$$\underline{\boldsymbol{C}}_y = (\underline{c}_{y,ij})_{l\times n}, \ \bar{\boldsymbol{C}}_y = (\bar{c}_{y,ij})_{l\times n}, \ \underline{\boldsymbol{D}}_{yp} = (\underline{d}_{yp,ij})_{l\times L},$$
$$\bar{\boldsymbol{D}}_{yp} = (\bar{d}_{yp,ij})_{l\times L}, \ \underline{\boldsymbol{D}}_u = (\underline{d}_{u,ij})_{l\times m}, \ \bar{\boldsymbol{D}}_u = (\bar{d}_{u,ij})_{l\times m},$$
$$\boldsymbol{A}^0 = \frac{1}{2}(\underline{\boldsymbol{A}} + \bar{\boldsymbol{A}}), \ \boldsymbol{A}^* = \frac{1}{2}(\bar{\boldsymbol{A}} - \underline{\boldsymbol{A}}) = (a_{ij}^*)_{n\times n},$$
$$\boldsymbol{B}_p^0 = \frac{1}{2}(\underline{\boldsymbol{B}}_p + \bar{\boldsymbol{B}}_p), \ \boldsymbol{B}_p^* = \frac{1}{2}(\bar{\boldsymbol{B}}_p - \underline{\boldsymbol{B}}_p) = (b_{p,ij}^*)_{n\times L},$$
$$\boldsymbol{B}_u^0 = \frac{1}{2}(\underline{\boldsymbol{B}}_u + \bar{\boldsymbol{B}}_u), \ \boldsymbol{B}_u^* = \frac{1}{2}(\bar{\boldsymbol{B}}_u - \underline{\boldsymbol{B}}_u) = (b_{u,ij}^*)_{n\times m},$$
$$\boldsymbol{C}_q^0 = \frac{1}{2}(\underline{\boldsymbol{C}}_q + \bar{\boldsymbol{C}}_q), \ \boldsymbol{C}_q^* = \frac{1}{2}(\bar{\boldsymbol{C}}_q - \underline{\boldsymbol{C}}_q) = (c_{q,ij}^*)_{L\times n},$$
$$\boldsymbol{D}_p^0 = \frac{1}{2}(\underline{\boldsymbol{D}}_p + \bar{\boldsymbol{D}}_p), \ \boldsymbol{D}_p^* = \frac{1}{2}(\bar{\boldsymbol{D}}_p - \underline{\boldsymbol{D}}_p) = (d_{p,ij}^*)_{L\times L},$$
$$\boldsymbol{D}_{qu}^0 = \frac{1}{2}(\underline{\boldsymbol{D}}_{qu} + \bar{\boldsymbol{D}}_{qu}), \ \boldsymbol{D}_{qu}^* = \frac{1}{2}(\bar{\boldsymbol{D}}_{qu} - \underline{\boldsymbol{D}}_{qu}) = (d_{qu,ij}^*)_{L\times m},$$
$$\boldsymbol{C}_y^0 = \frac{1}{2}(\underline{\boldsymbol{C}}_y + \bar{\boldsymbol{C}}_y), \ \boldsymbol{C}_y^* = \frac{1}{2}(\bar{\boldsymbol{C}}_y - \underline{\boldsymbol{C}}_y) = (c_{y,ij}^*)_{l\times n},$$

$$\boldsymbol{D}_{yp}^{0}=\frac{1}{2}(\underline{\boldsymbol{D}}_{yp}+\bar{\boldsymbol{D}}_{yp}),\ \boldsymbol{D}_{yp}^{*}=\frac{1}{2}(\bar{\boldsymbol{D}}_{yp}-\underline{\boldsymbol{D}}_{yp})=(d_{yp,ij}^{*})_{l\times L},$$

$$\boldsymbol{D}_{u}^{0}=\frac{1}{2}(\underline{\boldsymbol{D}}_{u}+\bar{\boldsymbol{D}}_{u}),\ \boldsymbol{D}_{u}^{*}=\frac{1}{2}(\bar{\boldsymbol{D}}_{u}-\underline{\boldsymbol{D}}_{u})=(d_{u,ij}^{*})_{l\times m},$$

where $\boldsymbol{A}^0$, $\boldsymbol{B}_p^0$, $\boldsymbol{B}_u^0$, $\boldsymbol{C}_q^0$, $\boldsymbol{D}_p^0$, $\boldsymbol{D}_{qu}^0$, $\boldsymbol{C}_y^0$, $\boldsymbol{D}_{yp}^0$, and $\boldsymbol{D}_u^0$ are called reference matrices of interval matrices $\boldsymbol{A}$, $\boldsymbol{B}_p$, $\boldsymbol{B}_u$, $\boldsymbol{C}_q$, $\boldsymbol{D}_p$, $\boldsymbol{D}_{qu}$, $\boldsymbol{C}_y$, $\boldsymbol{D}_{yp}$, and $\boldsymbol{D}_u$, respectively. Consequently, the system

$$\begin{cases}\boldsymbol{x}(k+1)=\boldsymbol{A}^{0}\boldsymbol{x}(k)+\boldsymbol{B}_{p}^{0}\boldsymbol{\phi}(\boldsymbol{\xi}(k))+\boldsymbol{B}_{u}^{0}\boldsymbol{u}(k),\\ \boldsymbol{\xi}(k)=\boldsymbol{C}_{q}^{0}\boldsymbol{x}(k)+\boldsymbol{D}_{p}^{0}\boldsymbol{\phi}(\boldsymbol{\xi}(k))+\boldsymbol{D}_{qu}^{0}\boldsymbol{u}(k),\\ \boldsymbol{y}(k)=\boldsymbol{C}_{y}^{0}\boldsymbol{x}(k)+\boldsymbol{D}_{yp}^{0}\boldsymbol{\phi}(\boldsymbol{\xi}(k))+\boldsymbol{D}_{u}^{0}\boldsymbol{u}(k),\end{cases}\quad(10)$$

is called the reference system of the SNNM Eq.(2).

Note that each element of matrices $\boldsymbol{A}^*$, $\boldsymbol{B}_p^*$, $\boldsymbol{B}_u^*$, $\boldsymbol{C}_q^*$, $\boldsymbol{D}_p^*$, $\boldsymbol{D}_{qu}^*$, $\boldsymbol{C}_y^*$, $\boldsymbol{D}_{yp}^*$, and $\boldsymbol{D}_u^*$ is nonnegative. So we can define

$$\boldsymbol{E}_1=\left[\sqrt{a_{11}^*}\boldsymbol{I}_1,\cdots,\sqrt{a_{1n}^*}\boldsymbol{I}_1,\cdots,\sqrt{a_{n1}^*}\boldsymbol{I}_n,\cdots,\sqrt{a_{nn}^*}\boldsymbol{I}_n\right]_{n\times n^2},$$

$$\boldsymbol{F}_1=\left[\sqrt{a_{11}^*}\boldsymbol{I}_1,\cdots,\sqrt{a_{1n}^*}\boldsymbol{I}_n,\cdots,\sqrt{a_{n1}^*}\boldsymbol{I}_1,\cdots,\sqrt{a_{nn}^*}\boldsymbol{I}_n\right]_{n^2\times n}^{\mathrm{T}},$$

$$\boldsymbol{E}_2=\left[\sqrt{b_{p,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{p,1L}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{p,n1}^*}\boldsymbol{I}_L,\cdots,\sqrt{b_{p,nL}^*}\boldsymbol{I}_L\right]_{n\times nL},$$

$$\boldsymbol{F}_2=\left[\sqrt{b_{p,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{p,1L}^*}\boldsymbol{I}_L,\cdots,\sqrt{b_{p,n1}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{p,nL}^*}\boldsymbol{I}_L\right]_{nL\times L}^{\mathrm{T}},$$

$$\boldsymbol{E}_3=\left[\sqrt{b_{u,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{u,1m}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{u,n1}^*}\boldsymbol{I}_m,\cdots,\sqrt{b_{u,nm}^*}\boldsymbol{I}_m\right]_{n\times nm},$$

$$\boldsymbol{F}_3=\left[\sqrt{b_{u,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{u,1m}^*}\boldsymbol{I}_m,\cdots,\sqrt{b_{u,n1}^*}\boldsymbol{I}_1,\cdots,\sqrt{b_{u,nm}^*}\boldsymbol{I}_m\right]_{nm\times m}^{\mathrm{T}},$$

$$\boldsymbol{E}_4=\left[\sqrt{c_{q,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{q,1n}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{q,L1}^*}\boldsymbol{I}_n,\cdots,\sqrt{c_{q,Ln}^*}\boldsymbol{I}_n\right]_{L\times Ln},$$

$$\boldsymbol{F}_4=\left[\sqrt{c_{q,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{q,1n}^*}\boldsymbol{I}_n,\cdots,\sqrt{c_{q,L1}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{q,Ln}^*}\boldsymbol{I}_n\right]_{Ln\times n}^{\mathrm{T}},$$

$$\boldsymbol{E}_5=\left[\sqrt{d_{p,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{p,1L}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{p,L1}^*}\boldsymbol{I}_L,\cdots,\sqrt{d_{p,LL}^*}\boldsymbol{I}_L\right]_{L\times L^2},$$

$$\boldsymbol{F}_5=\left[\sqrt{d_{p,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{p,1L}^*}\boldsymbol{I}_L,\cdots,\sqrt{d_{p,L1}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{p,LL}^*}\boldsymbol{I}_L\right]_{L^2\times L}^{\mathrm{T}},$$

$$\boldsymbol{E}_6=\left[\sqrt{d_{qu,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{qu,1m}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{qu,L1}^*}\boldsymbol{I}_m,\cdots,\sqrt{d_{qu,Lm}^*}\boldsymbol{I}_m\right]_{L\times Lm},$$

$$\boldsymbol{F}_6=\left[\sqrt{d_{qu,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{qu,1m}^*}\boldsymbol{I}_m,\cdots,\sqrt{d_{qu,L1}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{qu,Lm}^*}\boldsymbol{I}_m\right]_{Lm\times m}^{\mathrm{T}},$$

$$\boldsymbol{E}_7=\left[\sqrt{c_{y,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{y,1n}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{y,l1}^*}\boldsymbol{I}_n,\cdots,\sqrt{c_{y,ln}^*}\boldsymbol{I}_n\right]_{l\times ln},$$

$$\boldsymbol{F}_7=\left[\sqrt{c_{y,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{y,1n}^*}\boldsymbol{I}_n,\cdots,\sqrt{c_{y,l1}^*}\boldsymbol{I}_1,\cdots,\sqrt{c_{y,ln}^*}\boldsymbol{I}_n\right]_{ln\times n}^{\mathrm{T}},$$

$$\boldsymbol{E}_8=\left[\sqrt{d_{yp,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{yp,1L}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{yp,l1}^*}\boldsymbol{I}_L,\cdots,\sqrt{d_{yp,lL}^*}\boldsymbol{I}_L\right]_{l\times lL},$$

$$\boldsymbol{F}_8=\left[\sqrt{d_{yp,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{yp,1L}^*}\boldsymbol{I}_L,\cdots,\sqrt{d_{yp,l1}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{yp,lL}^*}\boldsymbol{I}_L\right]_{lL\times L}^{\mathrm{T}},$$

$$\boldsymbol{E}_9=\left[\sqrt{d_{u,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{u,1m}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{u,l1}^*}\boldsymbol{I}_m,\cdots,\sqrt{d_{u,lm}^*}\boldsymbol{I}_m\right]_{l\times lm},$$

$$\boldsymbol{F}_9=\left[\sqrt{d_{u,11}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{u,1m}^*}\boldsymbol{I}_m,\cdots,\sqrt{d_{u,l1}^*}\boldsymbol{I}_1,\cdots,\sqrt{d_{u,lm}^*}\boldsymbol{I}_m\right]_{lm\times m}^{\mathrm{T}},$$

where $\boldsymbol{I}_i$ denotes the ith column vector of the identity matrix. Obviously, we have

$$\boldsymbol{E}_1\boldsymbol{E}_1^{\mathrm{T}}=\mathrm{diag}\left(\sum_{j=1}^{n}a_{1j}^*,\cdots,\sum_{j=1}^{n}a_{nj}^*\right)_{n\times n},$$

$$\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{F}_1=\mathrm{diag}\left(\sum_{j=1}^{n}a_{j1}^*,\cdots,\sum_{j=1}^{n}a_{jn}^*\right)_{n\times n},$$

$$\boldsymbol{E}_2\boldsymbol{E}_2^{\mathrm{T}}=\mathrm{diag}\left(\sum_{j=1}^{L}b_{p,1j}^*,\cdots,\sum_{j=1}^{L}b_{p,nj}^*\right)_{n\times n},$$

$$\boldsymbol{F}_2^{\mathrm{T}}\boldsymbol{F}_2=\mathrm{diag}\left(\sum_{j=1}^{n}b_{p,j1}^*,\cdots,\sum_{j=1}^{n}b_{p,jL}^*\right)_{L\times L},$$

$$\boldsymbol{E}_3\boldsymbol{E}_3^{\mathrm{T}}=\mathrm{diag}\left(\sum_{j=1}^{m}b_{u,1j}^*,\cdots,\sum_{j=1}^{m}b_{u,nj}^*\right)_{n\times n},$$

$$\boldsymbol{F}_3^{\mathrm{T}}\boldsymbol{F}_3=\mathrm{diag}\left(\sum_{j=1}^{n}b_{u,j1}^*,\cdots,\sum_{j=1}^{n}b_{u,jm}^*\right)_{m\times m},$$

$$\boldsymbol{E}_4\boldsymbol{E}_4^{\mathrm{T}}=\mathrm{diag}\left(\sum_{j=1}^{n}c_{q,1j}^*,\cdots,\sum_{j=1}^{n}c_{q,Lj}^*\right)_{L\times L},$$

$$\boldsymbol{F}_4^{\mathrm{T}}\boldsymbol{F}_4=\mathrm{diag}\left(\sum_{j=1}^{L}c_{q,j1}^*,\cdots,\sum_{j=1}^{L}c_{q,jn}^*\right)_{n\times n},$$

$$\boldsymbol{E}_5\boldsymbol{E}_5^{\mathrm{T}}=\mathrm{diag}\left(\sum_{j=1}^{L}d_{p,1j}^*,\cdots,\sum_{j=1}^{L}d_{p,Lj}^*\right)_{L\times L},$$

$$\boldsymbol{F}_5^{\mathrm{T}}\boldsymbol{F}_5=\mathrm{diag}\left(\sum_{j=1}^{L}d_{p,j1}^*,\cdots,\sum_{j=1}^{L}d_{p,jL}^*\right)_{L\times L},$$

$$\boldsymbol{E}_6\boldsymbol{E}_6^{\mathrm{T}}=\mathrm{diag}\left(\sum_{j=1}^{m}d_{qu,1j}^*,\cdots,\sum_{j=1}^{m}d_{qu,Lj}^*\right)_{L\times L},$$

$$\boldsymbol{F}_6^{\mathrm{T}}\boldsymbol{F}_6=\mathrm{diag}\left(\sum_{j=1}^{L}d_{qu,j1}^*,\cdots,\sum_{j=1}^{L}d_{qu,jm}^*\right)_{m\times m},$$

and define

$$\boldsymbol{R}_1=(\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{F}_1+\boldsymbol{F}_4^{\mathrm{T}}\boldsymbol{F}_4)^{1/2}$$

$$=\mathrm{diag}\left(\sqrt{\sum_{j=1}^{n}a_{j1}^*+\sum_{j=1}^{L}c_{q,j1}^*},\cdots,\sqrt{\sum_{j=1}^{n}a_{jn}^*+\sum_{j=1}^{L}c_{q,jn}^*}\right)_{n\times n},$$

$$\boldsymbol{R}_2=(\boldsymbol{F}_2^{\mathrm{T}}\boldsymbol{F}_2+\boldsymbol{F}_5^{\mathrm{T}}\boldsymbol{F}_5)^{1/2}$$
$$=\mathrm{diag}\left(\sqrt{\sum_{j=1}^{n}b_{p,j1}^{*}+\sum_{j=1}^{L}d_{p,j1}^{*}},\cdots,\sqrt{\sum_{j=1}^{n}b_{p,jL}^{*}+\sum_{j=1}^{L}d_{p,jL}^{*}}\right)_{L\times L},$$
$$\boldsymbol{R}_3=(\boldsymbol{F}_3^{\mathrm{T}}\boldsymbol{F}_3+\boldsymbol{F}_6^{\mathrm{T}}\boldsymbol{F}_6)^{1/2}$$
$$=\mathrm{diag}\left(\sqrt{\sum_{j=1}^{n}b_{u,j1}^{*}+\sum_{j=1}^{L}d_{qu,j1}^{*}},\ldots,\sqrt{\sum_{j=1}^{n}b_{u,jm}^{*}+\sum_{j=1}^{L}d_{qu,jm}^{*}}\right)_{m\times m}.$$

From Lemma 1 in (Li *et al*., 2004), the SNNM Eq.(2) with interval parameters is equivalent to the following system:

$$\begin{cases}\boldsymbol{x}(k+1)=(\boldsymbol{A}^0+\boldsymbol{E}_1\boldsymbol{\Sigma}_1\boldsymbol{F}_1)\boldsymbol{x}(k)+(\boldsymbol{B}_p^0+\boldsymbol{E}_2\boldsymbol{\Sigma}_2\boldsymbol{F}_2)\phi(\boldsymbol{\xi}(k))\\ \qquad +(\boldsymbol{B}_u^0+\boldsymbol{E}_3\boldsymbol{\Sigma}_3\boldsymbol{F}_3)\boldsymbol{u}(k),\\ \boldsymbol{\xi}(k)=(\boldsymbol{C}_q^0+\boldsymbol{E}_4\boldsymbol{\Sigma}_4\boldsymbol{F}_4)\boldsymbol{x}(k)+(\boldsymbol{D}_p^0+\boldsymbol{E}_5\boldsymbol{\Sigma}_5\boldsymbol{F}_5)\phi(\boldsymbol{\xi}(k))\\ \qquad +(\boldsymbol{D}_{qu}^0+\boldsymbol{E}_6\boldsymbol{\Sigma}_6\boldsymbol{F}_6)\boldsymbol{u}(k),\\ \boldsymbol{y}(k)=(\boldsymbol{C}_y^0+\boldsymbol{E}_7\boldsymbol{\Sigma}_7\boldsymbol{F}_7)\boldsymbol{x}(k)+(\boldsymbol{D}_{yp}^0+\boldsymbol{E}_8\boldsymbol{\Sigma}_8\boldsymbol{F}_8)\boldsymbol{\phi}(\boldsymbol{\xi}(k))\\ \qquad +(\boldsymbol{D}_u^0+\boldsymbol{E}_9\boldsymbol{\Sigma}_9\boldsymbol{F}_9)\boldsymbol{u}(k),\end{cases} \tag{11}$$

where $\boldsymbol{\Sigma}_1$, $\boldsymbol{\Sigma}_2$, $\boldsymbol{\Sigma}_3$, $\boldsymbol{\Sigma}_4$, $\boldsymbol{\Sigma}_5$, $\boldsymbol{\Sigma}_6$, $\boldsymbol{\Sigma}_7$, $\boldsymbol{\Sigma}_8$, and $\boldsymbol{\Sigma}_9$ are diagonal matrices of appropriate dimension, and absolute values of their diagonal elements are not larger than 1.

Next, we develop a design procedure for the interval SNNM Eq.(11). The following lemma is necessary to derive Theorem 2 below.

Lemma 1 (Khargonekar *et al*., 1990) Let $\boldsymbol{D}$ and $\boldsymbol{E}$ be real matrices of appropriate dimensions. Then, for any scalar δ>0,

$$\boldsymbol{DE}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\le\delta\boldsymbol{DD}^{\mathrm{T}}+\delta^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}.$$

Theorem 2 There exists a state-feedback control law $\boldsymbol{u}(k)=\boldsymbol{Kx}(k)$ such that the closed-loop system Eq.(4) with interval parameters is globally robustly asymptotically stable provided that there exist symmetric positive definite matrices $\boldsymbol{X}$, a matrix $\boldsymbol{Y}$, diagonal positive definite matrix $\boldsymbol{S}$, diagonal semi-positive definite matrix $\boldsymbol{\Psi}$, and a scalar α>0 that satisfy the following LMI

$$\begin{bmatrix}\boldsymbol{G}_1 & \boldsymbol{G}_2\\ \boldsymbol{G}_2^{\mathrm{T}} & \boldsymbol{G}_3\end{bmatrix}<0, \tag{12}$$

where

$$\boldsymbol{G}_1=\begin{bmatrix}-\boldsymbol{X}+\alpha(\boldsymbol{E}_1\boldsymbol{E}_1^{\mathrm{T}}+\boldsymbol{E}_2\boldsymbol{E}_2^{\mathrm{T}}+\boldsymbol{E}_3\boldsymbol{E}_3^{\mathrm{T}}) & \boldsymbol{A}^0\boldsymbol{X}+\boldsymbol{B}_u^0\boldsymbol{Y} & \boldsymbol{B}_p^0\boldsymbol{S}\\ (\boldsymbol{A}^0\boldsymbol{X}+\boldsymbol{B}_u^0\boldsymbol{Y})^{\mathrm{T}} & -\boldsymbol{X} & (\boldsymbol{C}_q^0\boldsymbol{X}+\boldsymbol{D}_{qu}^0\boldsymbol{Y})^{\mathrm{T}}\\ \boldsymbol{S}(\boldsymbol{B}_p^0)^{\mathrm{T}} & \boldsymbol{C}_q^0\boldsymbol{X}+\boldsymbol{D}_{qu}^0\boldsymbol{Y} & \boldsymbol{D}_p^0\boldsymbol{S}+\boldsymbol{S}(\boldsymbol{D}_p^0)^{\mathrm{T}}-2\boldsymbol{\Psi}+\alpha(\boldsymbol{E}_4\boldsymbol{E}_4^{\mathrm{T}}+\boldsymbol{E}_5\boldsymbol{E}_5^{\mathrm{T}}+\boldsymbol{E}_6\boldsymbol{E}_6^{\mathrm{T}})\end{bmatrix},$$

$$\boldsymbol{G}_2=\begin{bmatrix}\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{X}^{\mathrm{T}}\boldsymbol{R}_1&\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{R}_3&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{SR}_2\end{bmatrix},$$

$$\boldsymbol{G}_3=\mathrm{diag}(-\alpha\boldsymbol{I},-\alpha\boldsymbol{I},-\alpha\boldsymbol{I},-\alpha\boldsymbol{I}).$$

Furthermore, the feedback gain $\boldsymbol{K}$ is obtained as $\boldsymbol{K}=\boldsymbol{YX}^{-1}$.

Proof Let

$$\boldsymbol{V}=\begin{bmatrix}\boldsymbol{0}&\boldsymbol{E}_1\boldsymbol{\Sigma}_1\boldsymbol{F}_1\boldsymbol{X}+\boldsymbol{E}_3\boldsymbol{\Sigma}_3\boldsymbol{F}_3\boldsymbol{Y}&\boldsymbol{E}_2\boldsymbol{\Sigma}_2\boldsymbol{F}_2\boldsymbol{S}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{E}_4\boldsymbol{\Sigma}_4\boldsymbol{F}_4\boldsymbol{X}+\boldsymbol{E}_6\boldsymbol{\Sigma}_6\boldsymbol{F}_6\boldsymbol{Y}&\boldsymbol{E}_5\boldsymbol{\Sigma}_5\boldsymbol{F}_5\boldsymbol{S}\end{bmatrix}$$
$$=\begin{bmatrix}\boldsymbol{E}_1\boldsymbol{\Sigma}_1&\boldsymbol{E}_3\boldsymbol{\Sigma}_3&\boldsymbol{E}_2\boldsymbol{\Sigma}_2&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{E}_4\boldsymbol{\Sigma}_4&\boldsymbol{E}_6\boldsymbol{\Sigma}_6&\boldsymbol{E}_5\boldsymbol{\Sigma}_5\end{bmatrix}\times\begin{bmatrix}\boldsymbol{0}&\boldsymbol{F}_1\boldsymbol{X}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{F}_3\boldsymbol{Y}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{F}_2\boldsymbol{S}\\ \boldsymbol{0}&\boldsymbol{F}_4\boldsymbol{X}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{F}_6\boldsymbol{Y}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{F}_5\boldsymbol{S}\end{bmatrix}.$$

Using Lemma1 1, we can deduce the following inequality:

$$\boldsymbol{V}+\boldsymbol{V}^{\mathrm{T}}\le\alpha\mathrm{diag}(\boldsymbol{E}_1\boldsymbol{E}_1^{\mathrm{T}}+\boldsymbol{E}_2\boldsymbol{E}_2^{\mathrm{T}}+\boldsymbol{E}_3\boldsymbol{E}_3^{\mathrm{T}},0,\boldsymbol{E}_4\boldsymbol{E}_4^{\mathrm{T}}+\boldsymbol{E}_5\boldsymbol{E}_5^{\mathrm{T}}+\boldsymbol{E}_6\boldsymbol{E}_6^{\mathrm{T}})+\alpha^{-1}\mathrm{diag}(0,\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{F}_1+\boldsymbol{F}_4^{\mathrm{T}}\boldsymbol{F}_4)\boldsymbol{X}+\boldsymbol{Y}^{T}(\boldsymbol{F}_3^{\mathrm{T}}\boldsymbol{F}_3+\boldsymbol{F}_6^{\mathrm{T}}\boldsymbol{F}_6)\boldsymbol{Y},\boldsymbol{S}(\boldsymbol{F}_2^{\mathrm{T}}\boldsymbol{F}_2+\boldsymbol{F}_5^{\mathrm{T}}\boldsymbol{F}_5)\boldsymbol{S}). \tag{13}$$

On the other hand, for system Eq.(11), by Eq.(5) in Theorem 1 together with Eq.(13)

$$\begin{bmatrix} -\boldsymbol{X} & (\boldsymbol{A}^0+\boldsymbol{E}_1\boldsymbol{\Sigma}_1\boldsymbol{F}_1)\boldsymbol{X}+(\boldsymbol{B}_u^0+\boldsymbol{E}_3\boldsymbol{\Sigma}_3\boldsymbol{F}_3)\boldsymbol{Y} & (\boldsymbol{B}_p^0+\boldsymbol{E}_2\boldsymbol{\Sigma}_2\boldsymbol{F}_2)\boldsymbol{S} \\ * & -\boldsymbol{X} & ((\boldsymbol{C}_q^0+\boldsymbol{E}_4\boldsymbol{\Sigma}_4\boldsymbol{F}_4)\boldsymbol{X}+(\boldsymbol{D}_{qu}^0+\boldsymbol{E}_6\boldsymbol{\Sigma}_6\boldsymbol{F}_6)\boldsymbol{Y})^{\mathrm{T}} \\ * & * & (\boldsymbol{D}_p^0+\boldsymbol{E}_5\boldsymbol{\Sigma}_5\boldsymbol{F}_5)\boldsymbol{S}+\boldsymbol{S}(\boldsymbol{D}_p^0+\boldsymbol{E}_5\boldsymbol{\Sigma}_5\boldsymbol{F}_5)^{\mathrm{T}}-2\boldsymbol{\Psi} \end{bmatrix}$$

$$=\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{A}^0\boldsymbol{X}+\boldsymbol{B}_u^0\boldsymbol{Y} & \boldsymbol{B}_p^0\boldsymbol{S} \\ (\boldsymbol{A}^0\boldsymbol{X}+\boldsymbol{B}_u^0\boldsymbol{Y})^{\mathrm{T}} & -\boldsymbol{X} & (\boldsymbol{C}_q^0\boldsymbol{X}+\boldsymbol{D}_{qu}^0\boldsymbol{Y})^{\mathrm{T}} \\ \boldsymbol{S}(\boldsymbol{B}_p^0)^{\mathrm{T}} & \boldsymbol{C}_q^0\boldsymbol{X}+\boldsymbol{D}_{qu}^0\boldsymbol{Y} & \boldsymbol{D}_p^0\boldsymbol{S}+\boldsymbol{S}(\boldsymbol{D}_p^0)^{\mathrm{T}}-2\boldsymbol{\Psi} \end{bmatrix}$$

$$+\boldsymbol{V}+\boldsymbol{V}^{\mathrm{T}} \le \boldsymbol{G}_1+\alpha^{-1}\mathrm{diag}(0,\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{F}_1^{\mathrm{T}}\boldsymbol{F}_1+\boldsymbol{F}_4^{\mathrm{T}}\boldsymbol{F}_4)\boldsymbol{X}$$
$$+\boldsymbol{Y}^T(\boldsymbol{F}_3^{\mathrm{T}}\boldsymbol{F}_3+\boldsymbol{F}_6^{\mathrm{T}}\boldsymbol{F}_6)\boldsymbol{Y},\boldsymbol{S}(\boldsymbol{F}_2^{\mathrm{T}}\boldsymbol{F}_2+\boldsymbol{F}_5^{\mathrm{T}}\boldsymbol{F}_5)\boldsymbol{S})$$

$$=\boldsymbol{G}_1+\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X}^{\mathrm{T}}\boldsymbol{R}_1 & \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{R}_3 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S}\boldsymbol{R}_2 \end{bmatrix} \times\mathrm{diag}(\alpha^{-1}\boldsymbol{I},\alpha^{-1}\boldsymbol{I},\alpha^{-1}\boldsymbol{I},\alpha^{-1}\boldsymbol{I}) \times\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X}^{\mathrm{T}}\boldsymbol{R}_1 & \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{R}_3 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S}\boldsymbol{R}_2 \end{bmatrix}^{\mathrm{T}} < 0. \tag{14}$$

By applying the Schur complements (Boyd *et al.*, 1994), Eq.(14) can be expressed as Eq.(12).

CONTROLLER SYNTHESIS FOR THE NONLINEAR SYSTEM

Before employing Theorems 1 and 2 to synthesize the state-feedback controller to stabilize the neural-network-based system, we first re-express the system in the form of the SNNM described in Eq.(2).

Now we consider a class of discrete-time nonlinear systems in the following form:

$$\boldsymbol{y}(k)=\boldsymbol{F}(\boldsymbol{y}(k-1),\ldots,\boldsymbol{y}(k-d_y),\boldsymbol{u}(k-1),\ldots,\boldsymbol{u}(k-d_u)), \tag{15}$$

where $\{\boldsymbol{y}(k)\},\{\boldsymbol{u}(k)\}$ are the n_y-dimensional system output and the n_u-dimensional system input at time instance k, respectively, and $\boldsymbol{F}\in\boldsymbol{C}(\mathbb{R}^{n_y d_y+n_u d_u};\mathbb{R}^{n_y})$ is a continuous function, which is assumed to be unknown. Assume a neural network $NN(\boldsymbol{y}(k-1), \ldots, \boldsymbol{y}(k-d_y), \boldsymbol{u}(k-1), \boldsymbol{u}(k-d_u))$ is used to approximate the nonlinear system Eq.(15), which can be written as:

$$\boldsymbol{y}(k)=NN(\boldsymbol{y}(k-1),\ldots,\boldsymbol{y}(k-d_y),\boldsymbol{u}(k-1), \ldots,\boldsymbol{u}(k-d_u))+\boldsymbol{E}(k),$$

where $\boldsymbol{E}(k)$ is the approximation error vector. We assume that $\boldsymbol{E}(k)$ satisfies

$$\|\boldsymbol{E}(k)\|<\varepsilon$$

for all $\boldsymbol{y}(k-1), \ldots, \boldsymbol{y}(k-d_y), \boldsymbol{u}(k-1), \boldsymbol{u}(k-d_u)$, where ε is a given positive scalar. We can then transform the neural network $NN(\boldsymbol{y}(k-1), \ldots, \boldsymbol{y}(k-d_y), \boldsymbol{u}(k-1), \boldsymbol{u}(k-d_u))$ into the SNNM Eq.(2), and design a state-feedback control law by using Theorems 1 and 2.

Assumption on the approximation error is common in neural-network-based control design literature (Lin and Lin, 2001; Limanond and Si, 1998). This assumption ensures that a controller designed for a neural-network-based approximation model can maintain its performance when applied to the actual nonlinear system, which, as we will show afterwards, plays an important role in establishing global uniform ultimate boundedness of the closed-loop system.

Now we consider a simple example to illustrate that the ISNNM-based feedback controller can guarantee the robust stability of the closed-loop system. In particular, consider the following single-input single-output nonlinear system (Limanond and Si, 1998):

$$\boldsymbol{y}(k+1)=f_1\boldsymbol{y}(k)\boldsymbol{y}(k-1)/[1+\boldsymbol{y}^2(k)+\boldsymbol{y}^2(k-1)] +f_2\sin[0.5(\boldsymbol{y}(k)+\boldsymbol{y}(k-1))]\cos[0.5(\boldsymbol{y}(k) +\boldsymbol{y}(k-1))]+f_3\boldsymbol{u}(k), \tag{16}$$

where $\boldsymbol{u}(k)$ and $\boldsymbol{y}(k)$ denote the scalar input and output, the parameters f_1, f_2, and f_3 in (Limanond and Si, 1998) are set to 1.5, 0.7, and 1.2, respectively. Similar to the work by Limanond and Si (1998), we employ a multi-layer perceptions (MLP) network for approximating the system Eq.(16). The MLP with hyperbolic tangent activation functions has the following form:

$$\boldsymbol{y}(k+1)=a\tanh\left(\boldsymbol{W}_2\tanh\left(\boldsymbol{W}_1\cdot\begin{bmatrix}\boldsymbol{y}(k)\\ \boldsymbol{y}(k-1)\\ \boldsymbol{u}(k)\end{bmatrix}\right)\right), \tag{17}$$

where a is a scalar adjusting the range of the output, $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are 3×3 and 1×3 matrices that denote the weight matrix of the hidden and the output layer, respectively. These weight matrices are trained off-line using the backpropagation learning algorithm. The training data are obtained by sampling the input space in the interval [−1.8, 1.8] randomly and uniformly. After 5639 training steps, we obtain the weights of the MLP as follows:

$$a=2.7468,$$
$$\boldsymbol{W}_1=[\boldsymbol{W}_{11(3\times2)} \quad \boldsymbol{W}_{12(3\times1)}]=\begin{bmatrix}0.4811 & -0.1843 & 0.2316\\ 0.5554 & -0.1529 & 0.6986\\ -0.2718 & 0.0846 & 0.3220\end{bmatrix},$$
$$\boldsymbol{W}_2=[1.1997 \quad -0.4027 \quad 1.2829].$$

Converting MLP Eq.(17) into the SNNM Eq.(2) where $\boldsymbol{x}(k)=[\boldsymbol{y}(k) \quad \boldsymbol{y}(k-1)]^{\mathrm{T}}$, $\boldsymbol{A}=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$, $\boldsymbol{B}_p=\begin{bmatrix}0_{2\times3} & \begin{bmatrix}a\\0\end{bmatrix}\end{bmatrix}$, $\boldsymbol{B}_u=0$, $\boldsymbol{C}_q=\begin{bmatrix}\boldsymbol{W}_{11}\\ 0_{1\times2}\end{bmatrix}$, $\boldsymbol{D}_p=\begin{bmatrix}0_{3\times3} & 0_{3\times1}\\ \boldsymbol{W}_2 & 0_{1\times1}\end{bmatrix}$, $\boldsymbol{D}_{qu}=\begin{bmatrix}\boldsymbol{W}_{12}\\ 0_{1\times1}\end{bmatrix}$, $\boldsymbol{C}_y=[1 \quad 0]$, $\boldsymbol{D}_{yp}=0$, $\boldsymbol{D}_u=0$, $\boldsymbol{U}=\boldsymbol{I}$, and adopting a state-feedback controller described by Eq.(3), we can design a stabilized controller according to Theorem 1. Solving the LMI Eq.(5) with respect to the parameters $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{S}$, and $\boldsymbol{\Psi}$ by the LMI Control Toolbox (Gahinet *et al*., 1995), we achieve the following solution:

$$\boldsymbol{X}=\begin{bmatrix}0.7028 & 0.0541\\ 0.0541 & 1.2999\end{bmatrix}, \boldsymbol{Y}=[-0.4383, 0.2118],$$
$$\boldsymbol{S}=\mathrm{diag}(0.3433, 1.0054, 0.3103, 0.0828),$$
$$\boldsymbol{\Psi}=\mathrm{diag}(0.7989, 0.7989, 0.7989, 0.7989),$$
$$\boldsymbol{K}=\boldsymbol{Y}\boldsymbol{X}^{-1}=[-0.6383, 0.1894].$$

When the state-feedback law $\boldsymbol{u}(k)=\boldsymbol{K}\boldsymbol{x}(k)$ is applied on the nonlinear system Eq.(16), the output of the closed-loop system $\boldsymbol{y}(k)$ converges to zero asymptotically. In order to demonstrate the advantage of our approach, we compare the controller designed using our approach to that designed according to the approach suggested by Limanond and Si (1998). The results are presented in Fig.2, where the solid and dashed lines denote the output of the system with controller designed by our approach and the approach suggested by Limanond and Si (1998), respectively. Fig.2 shows that although both closed-loop systems are finally stabilized, our controller stabilizes the system within a much shorter time period.

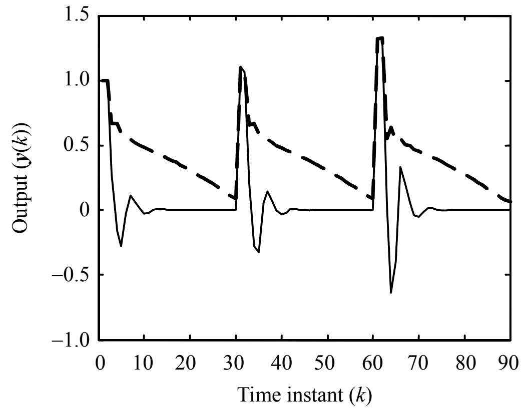

Fig.2 Output of the closed-loop system whose feedback gain *K* is [−0.6383, 0.1894] designed by our approach (the solid line), and [0.0278, −0.1522] designed by Limanond and Si (1998) (dashed line). *y*(*k*) is initialized arbitrarily at *k*=0, 30, 60, respectively

Next, we consider the case where the parameters f_1, f_2, and f_3 of nonlinear system Eq.(16) vary within a given range. In this case, the weights of the MLP can be expressed as follows with intervals:

$$\bar{a}=\underline{a}=2.7468,$$
$$\bar{\boldsymbol{W}}_1=[\bar{\boldsymbol{W}}_{11(3\times2)} \quad \bar{\boldsymbol{W}}_{12(3\times1)}]=\begin{bmatrix}0.6812 & 0.1843 & 0.8316\\ 0.9558 & -0.0529 & 1.6955\\ -0.0718 & 0.1284 & 0.5322\end{bmatrix},$$
$$\bar{\boldsymbol{W}}_2=[2.5679 \quad -0.2028 \quad 2.2900],$$
$$\underline{\boldsymbol{W}}_1=[\underline{\boldsymbol{W}}_{11(3\times2)} \quad \underline{\boldsymbol{W}}_{12(3\times1)}]=\begin{bmatrix}0.0811 & -1.1843 & -0.2356\\ -0.2556 & -1.1529 & 0.2887\\ -1.0018 & 0.0046 & -0.9927\end{bmatrix},$$
$$\underline{\boldsymbol{W}}_2=[0.1227 \quad -0.8824 \quad 0.2856].$$

Similar to the above design procedure, we can obtain the feedback gain $\boldsymbol{K}$=[−0.1365, −0.0685] using Theorem 2. We again compare our controller to the one designed using the approach suggested by Limanond and Si (1998) when the parameters are perturbed. It can be seen from Fig.3 that the controller designed by Limanond and Si (1998) fails to stabilize the system, while our controller succeeds. The difference in performance of the controllers can be attributed to the fact that the robustness of the system had been considered in the design approach proposed in this paper.

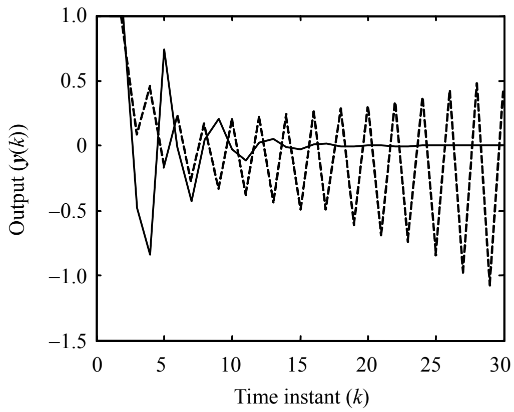

Fig.3 Output of the closed-loop system whose feedback gain *K* is [−0.1365, −0.0685] designed by our approach (solid line), and [0.0278, −0.1522] designed by Limanond and Si (1998) (dashed line). f_1, f_2 and f_3 are set to 1.5, 0.7 and 6.0, respectively

CONCLUSION

In this paper, we suggested an algorithm for designing robust controllers for a class of discrete-time nonlinear systems with bounded parameter uncertainty when neural networks are used to approximate the system. We proposed a novel interval standard neural network model (ISNNM) which describes a class of neural networks whose weights vary in a given range. One of the most important features of ISNNM is that the model can be represented as an interval LDI so that a controller for the ISNNM can be easily designed via the LMI approach. A state-feedback controller has been designed for the ISNNM such that the closed-loop system is globally robustly asymptotically stable. The robustness of controllers in the presence of approximation errors and parameter perturbations is guaranteed in our design approach which can be extended to the synthesis of any nonlinear control systems as long as the systems can be described in the form of the SNNM or interval SNNM. Note that no standard methods exist to convert non-SNNMs into the SNNMs, but the state transformation can be applied in general.

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