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Synthesis of Petri net supervisors enforcing general constraints

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Abstract: This paper deals with the synthesis of Petri net supervisor enforcing the more expressive constraints including marking terms, firing vector terms and Parikh vector terms. The method is developed to handle uncontrollable and unobservable transitions existing in the constraints. The "greater-than or equal" general constraints can also be transformed into "less-than or equal" Parikh constraints. An example is analyzed to show how the problem is solved. General constraint is first transformed into Parikh vector constraints, and Matrix-Transformation is proposed to obtain the admissible constraints without uncontrollable and unobservable transitions. Then the supervisor can be constructed based on constraints only consisting of Parikh vector terms. The method is proved to be more concise and effective than the method presented by Iordache and Moody especially when applied to large scale systems.

Key words: Petri net, Supervisor control, Parikh vector, Uncontrollable, Unobservable
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INTRODUCTION

Petri net (PN) (Murata, 1989) is a powerful tool due to its graphical and algebraic representation ability, especially when applied in the Discrete Event System (DES). As supervisor synthesis is an essential issue in DES, efficient methods (Giua *et al*., 1992; Giua and Dicesare, 1994; Holloway and Krogh, 1990; Li and Wonham, 1994; Yamalidou and Moody, 1996) have been proposed for the design of supervisory controller enforcing the marking terms $\boldsymbol{\mu}$ of a Petri net satisfying the constraints of the form

$$\boldsymbol{L\mu} \le \boldsymbol{b}, \tag{1}$$

where $\boldsymbol{L}\in\mathbb{Z}^{m_c\times m}, \boldsymbol{\mu}\in\mathbb{Z}^{m}, \boldsymbol{b}\in\mathbb{Z}^{+m_c}$, m is the number of places, and m_c is the number of constraints. The constraints of Eq.(1) can describe forbidden marking problems, modeling of finite resources condition for liveness and deadlock prevention, and so on.

Eq.(1) has been extended in (Yamalidou and Moody, 1996) to the form

$$\boldsymbol{L\mu}+\boldsymbol{Hq} \le \boldsymbol{b}, \tag{2}$$

where $\boldsymbol{H}\in\mathbb{N}^{m_c\times n}$ ($\boldsymbol{H}$ is assumed to have nonnegative elements without losing generality), n is the number of transitions. The firing vector $\boldsymbol{q}$ consists of elements q_i, where q_i is defined as follows

$$q_i=\begin{cases}1, & \text{if } t_i \text{ is to be fired next from } \boldsymbol{\mu},\\ 0, & \text{else.}\end{cases}$$

Eq.(2) concerns about enabling of transitions and the states, which can increase the expressive ability.

Li and Wonham (1994) first proposed the Parikh vector term and which has been added to the linear constraints to compose the generalized constraints

$$\boldsymbol{L\mu}+\boldsymbol{Hq}+\boldsymbol{Cv} \le \boldsymbol{b}, \tag{3}$$

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where $\boldsymbol{C}\in\mathbb{Z}^{m_c\times n}$, v_i is the ith element of $\boldsymbol{v}$, and stands for how many times t_i has fired since the system's initialization, and Eq.(3) has greatly increased the expressive power of the constraints. As marking terms in Eq.(3) do not make Eq.(3) more expressive, only make it easy to understand and construct the constraints, Eq.(3) can be simplified to

$$\boldsymbol{Hq}+\boldsymbol{Cv}\le\boldsymbol{b}. \tag{4}$$

If there are no self-loops, Eq.(4) can be rewritten as

$$\boldsymbol{Cv}\le\boldsymbol{b}. \tag{5}$$

While the Parikh vector terms can be used to describe the difference between the numbers of firing of two transitions, future research will focus on the properties of the Parikh vector term in order to achieve more expressive power and convenient analysis method.

The Parikh vector terms are viewed as the marking of a sink place added to the transitions, and Iordache and Antsaklis (2002; 2003) transformed the constraints of Eq.(3) into Eq.(1) that are enforced on the transformed PN, and then obtained the supervisory controller by using the method presented by Moody and Antsaklis (1996). The method for the constraints of Eq.(1) is based on the concept of place invariants. Moody and Antsaklis (2000) and Wu *et al*. (2003) proposed methods to deal with uncontrollable and unobservable transitions. As the methods are quite complex in matrix multiplication for a large system, Wang and Yan (2005) presented a novel method to solve this problem which can greatly ease the computation burden. But Wang only considers the "less-than or equal" constraints without uncontrollable and unobservable transitions. In practical issues, "greater-than or equal" constraints and uncontrollable and unobservable issues are very common, thus cannot be eliminated. For this case, the method to transform "greater-than or equal" general constraints into "less-than or equal" Parikh constraints, and constraints with uncontrollable and unobservable transitions in Parikh vector term is proposed in this paper.

This paper first introduces the basic concepts of generalized constraint, uncontrollable and unobservable transitions and the method of transformation. How to deal with the "greater-than or equal" constraints is discussed in Section 3. Section 4 will provide the details of how to obtain the admissible constraints involving uncontrollable and unobservable transitions. An example of AVs system borrowed from (Iordache and Antsaklis, 2003) is used to illustrate the procedure of handling the problem and comparison is also given to show the effectiveness of the method in Section 5. At last, conclusion is given in Section 6.

BACKGROUND

Constraints of Eqs.(1) and (2) have been fully discussed and are easily used to represent the PN models. This section discusses how to describe the PN operation by constraints of Eq.(4). For a Petri net, we denote $\boldsymbol{D}_\mathrm{p}$ as the incidence matrix without controller ($\boldsymbol{D}_\mathrm{p}^+$, $\boldsymbol{D}_\mathrm{p}^-$ stand for the input and output matrix respectively), t_i as the transition corresponding to the column i of $\boldsymbol{D}_\mathrm{p}$. Thus, we have the state equation

$$\boldsymbol{\mu}=\boldsymbol{\mu}_0+\boldsymbol{D}_\mathrm{p}\boldsymbol{v}, \tag{6}$$

where $\boldsymbol{\mu}_0$ is the initial marking. From Eq.(6), we have $-\boldsymbol{D}_\mathrm{p}\boldsymbol{v}\le\boldsymbol{\mu}_0$, let $\boldsymbol{C}=-\boldsymbol{D}_\mathrm{p}$. If Petri net has self-loops, the incidence matrix $\boldsymbol{D}_\mathrm{p}$ is insufficient to determine whether a transition is enabled or not. Term $\boldsymbol{Hq}$, where $\boldsymbol{H}=\boldsymbol{D}_\mathrm{p}^-$, is introduced into the following constraint

$$\boldsymbol{Cv}+\boldsymbol{Hq}\le\boldsymbol{\mu}_0, \tag{7}$$

which can describe the operation of PN even with self-loops.

Definition 1 Petri net is a 5-tuple PN ($\boldsymbol{P},\boldsymbol{T},\boldsymbol{F},\boldsymbol{W},\boldsymbol{\mu}_0$), where $\boldsymbol{P}=\{p_1, p_2, \dots, p_m\}$ is a finite set of places; $\boldsymbol{T}=\{t_1, t_2, \dots, t_n\}$ is a finite set of transitions; $\boldsymbol{F}\subseteq(\boldsymbol{P}\times\boldsymbol{T})\cup(\boldsymbol{T}\times\boldsymbol{P})$; $\boldsymbol{W}:\boldsymbol{F}\rightarrow\{1,2,3,\dots\}$ is a weight function; $\boldsymbol{\mu}_0:\boldsymbol{P}\rightarrow\{0,1,2,\dots\}$ is the initial marking.

Definition 2 $\bullet x=\{y|(y,x)\in\boldsymbol{F}\}$ is denoted as the pre-set of x, $x\bullet=\{z|(x,z)\in\boldsymbol{F}\}$ is the post-set of x.

The pre-set and post-set of a place $p\in\boldsymbol{P}$ are

$$\bullet p=\{t\in\boldsymbol{T}|(t,p)\in\boldsymbol{F}\},\ \ p\bullet=\{t\in\boldsymbol{T}|(p,t)\in\boldsymbol{F}\},$$

then we have

$$\mu(p)=\mu_0(p)+\sum_{t\in\bullet p}W(t,p)v(t)-\sum_{t=p\bullet}W(p,t)v(t). \quad (8)$$

As $\mu(p)$ is a non-negative integer, we have

$$\mu(p)=\mu_0(p)+\sum_{t\in\bullet p}W(t,p)v(t)-\sum_{t=p\bullet}W(p,t)v(t)\geq 0,$$

which means the number of tokens consumed must be less than the tokens received plus the initial tokens. Therefore, the following proposition can be obtained.

Proposition 1 Each place can be regarded as a control place enforcing a Parikh constraint and vice versa.

Thus the constraint of Eq.(4) can be enforced by adding a control place to the plant iff $\boldsymbol{b}\geq 0$.

Example 1 Considering the Petri net in Fig.1, each place can be regarded as an inequality as follows:

$$q_1+v_2\leq 3;\ v_2-v_3\leq 0;\ -2v_1-v_2+v_3\leq 1.$$

In a reverse way, if Parikh constraint $-v_1+v_2+v_3\leq 1$ is enforced on the Petri net, the supervisory controller can be directly obtained as shown in Fig.2.

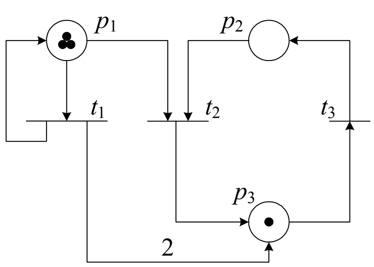

Fig.1 Petri net model

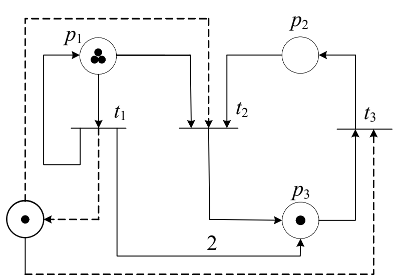

Fig.2 Petri net model with controller

"GREATER-THAN OR EQUAL" CONSTRAINTS

Some systems may require resource reserve constraints, and then constraints will be applied. For simplicity and without loss of generality, we assume there are self-loops in the Petri net, and only one constraint is discussed here.

The "greater-than or equal" general constraint is as follows

$$\boldsymbol{l}\boldsymbol{\mu}+\boldsymbol{c}\boldsymbol{v}\geq \boldsymbol{b}, \quad (9)$$

where $\boldsymbol{l}\in\mathbb{Z}^{1\times m}$, $\boldsymbol{\mu}\in\mathbb{Z}^{m}$, $\boldsymbol{c}\in\mathbb{Z}^{1\times n}$, $\boldsymbol{v}\in\mathbb{Z}^{+n}$, m and n stand for the number of places and transitions respectively, and we denote $\|\boldsymbol{l}\|$ as the support of $\boldsymbol{l}$, where $\|\boldsymbol{l}\|=\{p\in\boldsymbol{P}|\ l(p)\neq 0\}$, similarly, $\|\boldsymbol{c}\|$ is the support of $\boldsymbol{c}$, $\|\boldsymbol{c}\|=\{t\in\boldsymbol{T}|\ c(t)\neq 0\}$.

Theorem 1 If "greater-than or equal" general constraints satisfy the initial condition, it can be transformed into a "less-than or equal" Parikh constraints, i.e., if $\boldsymbol{l}\boldsymbol{\mu}_0+\boldsymbol{c}\boldsymbol{v}_0\geq\boldsymbol{b}$, then $\boldsymbol{l}\boldsymbol{\mu}+\boldsymbol{c}\boldsymbol{v}\geq\boldsymbol{b}$ can be transformed into $\boldsymbol{c}'\boldsymbol{v}\geq\boldsymbol{b}'$.

Proof $\boldsymbol{l}\boldsymbol{\mu}+\boldsymbol{c}\boldsymbol{v}$ can be rewritten as

$$\sum_{p\in\|\boldsymbol{l}\|}l(p)\mu(p)+\sum_{t\in\|\boldsymbol{c}\|}c(t)v(t).$$

According to Eq.(8), we have

$$\begin{aligned}\boldsymbol{l}\boldsymbol{\mu}&=\sum_{p\in\|\boldsymbol{l}\|}l(p)\mu(p)\\&=\sum_{p\in\|\boldsymbol{l}\|}l(p)\left[\mu_0(p)+\sum_{t\in\bullet p}W(t,p)v(t)-\sum_{t=p\bullet}W(p,t)v(t)\right]\\&=\sum_{p\in\|\boldsymbol{l}\|}l(p)\mu_0(p)+\sum_{p\in\|\boldsymbol{l}\|,t\in\bullet p}l(p)W(t,p)v(t)\\&\quad-\sum_{p\in\|\boldsymbol{l}\|,t=p\bullet}l(p)W(p,t)v(t)\\&=\boldsymbol{l}\boldsymbol{\mu}_0+\sum_{p\in\|\boldsymbol{l}\|,t\in\bullet p}l(p)W(t,p)v(t)-\sum_{p\in\|\boldsymbol{l}\|,t=p\bullet}l(p)W(p,t)v(t).\end{aligned}$$

Eq.(9) can be rewritten as

$$\begin{aligned}&\sum_{p\in\|\boldsymbol{l}\|,t=p\bullet}l(p)W(p,t)v(t)-\sum_{p\in\|\boldsymbol{l}\|,t\in\bullet p}l(p)W(t,p)v(t)\\&\quad-\sum_{t\in\|\boldsymbol{c}\|}c(t)v(t)\leq\boldsymbol{l}\boldsymbol{\mu}_0-\boldsymbol{b}.\end{aligned} \quad (10)$$

The left part of Eq.(10) is in the form of $\boldsymbol{c}'\boldsymbol{v}$, the right part we denote as $\boldsymbol{b}'$, then Eq.(10) can be rewritten in the form of $\boldsymbol{c}'\boldsymbol{v}\le\boldsymbol{b}'$.

As $\boldsymbol{l}\boldsymbol{\mu}_0+\boldsymbol{c}\boldsymbol{v}_0\ge\boldsymbol{b}$, $\boldsymbol{v}_0=0$ for an initialized Petri net, thus we have $\boldsymbol{b}'=\boldsymbol{l}\boldsymbol{\mu}_0-\boldsymbol{b}\ge0$, which proves the theorem.

UNCONTROLLABLE AND UNOBSERVABLE TRANSITONS

In supervisory control of Petri net, there should be no arcs from control places to the uncontrollable transitions, as the uncontrollable transitions cannot be controlled. While for the unobservable transitions, which cannot be detected directly, there should be no arcs between control places and the unobservable transitions, which are also assumed uncontrollable. In the constraints of Eq.(1), let $\boldsymbol{D}_{\mathrm{uc}}$ stand for the uncontrollable part of $\boldsymbol{D}_{\mathrm{p}}$, and $\boldsymbol{D}_{\mathrm{uo}}$ the unobservable part, the enforceable set of constraints should satisfy

$$\boldsymbol{L}\boldsymbol{D}_{\mathrm{uc}}\le0, \qquad \boldsymbol{L}\boldsymbol{D}_{\mathrm{uo}}=0.$$

Lemma 1 in (Moody and Antsakis, 2000) shows how to obtain the enforceable constraints.

Lemma 1 Let $\boldsymbol{R}_1\in\mathbb{Z}^{n_{\mathrm{c}}\times m}$ satisfy $\boldsymbol{R}_1\boldsymbol{\mu}_{\mathrm{p}}\ge0\,\forall\boldsymbol{\mu}_{\mathrm{p}}$, $\boldsymbol{R}_2\in\mathbb{Z}^{n_{\mathrm{c}}\times n_{\mathrm{c}}}$ as diagonal matrix. If $\boldsymbol{L}'\boldsymbol{\mu}_{\mathrm{p}}\le\boldsymbol{b}'$ where $\boldsymbol{L}'=\boldsymbol{R}_1+\boldsymbol{R}_2\boldsymbol{L}$, $\boldsymbol{b}'=\boldsymbol{R}_2(\boldsymbol{b}+\boldsymbol{1})-\boldsymbol{1}$ and $\boldsymbol{1}$ is an n_{c} dimensional vector of 1's, then $\boldsymbol{L}\boldsymbol{\mu}_{\mathrm{p}}\le\boldsymbol{b}$.

Lemma 1 was proved in (Moody and Antsaklis, 2000), and a lot of results have been achieved by Moody in the synthesis of supervisor enforcing the constraints of Eqs.(1) and (2) with the uncontrollable and unobservable transitions.

As for the PN enforcing constraints of Eq.(5) with uncontrollable and unobservable transitions, a method is proposed to modify $\boldsymbol{C}$ and $\boldsymbol{b}$ to get the admissible constraints.

Let $\boldsymbol{C}_{\mathrm{uc}}$, $\boldsymbol{C}_{\mathrm{uo}}$ be the uncontrollable and unobservable part of $\boldsymbol{C}$ respectively. Regarding the properties of uncontrollable and unobservable transitions, the admissible Parikh vector term constraints should satisfy

$$\boldsymbol{C}_{\mathrm{uc}}\le0, \tag{11}$$
$$\boldsymbol{C}_{\mathrm{uo}}=0. \tag{12}$$

If Eqs.(11) and (12) are not satisfied, $\boldsymbol{C}$ and $\boldsymbol{b}$ have to be modified to ensure the constraints enforceable.

Theorem 2 Let $\boldsymbol{R}\in\mathbb{Z}^{m_{\mathrm{c}}\times m}$ satisfy $\boldsymbol{R}\ge0$, $\boldsymbol{R}\boldsymbol{\mu}_0\le\boldsymbol{b}$ and if $\boldsymbol{C}'\boldsymbol{v}\le\boldsymbol{b}'$ where $\boldsymbol{C}'=\boldsymbol{R}\boldsymbol{D}_{\mathrm{p}}+\boldsymbol{C}$, $\boldsymbol{b}'=\boldsymbol{b}-\boldsymbol{R}\boldsymbol{\mu}_0\ge0$, then $\boldsymbol{C}\boldsymbol{v}\le\boldsymbol{b}$.

Proof The transformed constraint is $(\boldsymbol{R}\boldsymbol{D}_{\mathrm{p}}+\boldsymbol{C})\boldsymbol{v}\le\boldsymbol{b}-\boldsymbol{R}\boldsymbol{\mu}_0$, which can be rewritten as $\boldsymbol{R}(\boldsymbol{D}_{\mathrm{p}}\boldsymbol{v}+\boldsymbol{\mu}_0)+\boldsymbol{C}\boldsymbol{v}\le\boldsymbol{b}$, because $\boldsymbol{\mu}=\boldsymbol{\mu}_0+\boldsymbol{D}_{\mathrm{p}}\boldsymbol{v}\ge0$, $\boldsymbol{R}(\boldsymbol{D}_{\mathrm{p}}\boldsymbol{v}+\boldsymbol{\mu}_0)\ge0$, therefore $\boldsymbol{C}\boldsymbol{v}\le\boldsymbol{b}$.

By Theorem 2, if there exists such $\boldsymbol{R}$ that satisfies all the conditions, there must exist a transformed constraint which guarantees $\boldsymbol{C}\boldsymbol{v}\le\boldsymbol{b}$ without violating the characterization of uncontrollable and unobservable transitions.

$\boldsymbol{R}$ in Theorem 1 has to satisfy the following conditions

$$\begin{cases}\boldsymbol{R}\ge0,\\ \boldsymbol{R}\boldsymbol{\mu}_0\le\boldsymbol{b},\\ \boldsymbol{C}'_{\mathrm{uc}}=\boldsymbol{R}\boldsymbol{D}_{\mathrm{uc}}+\boldsymbol{C}_{\mathrm{uc}}\le0,\\ \boldsymbol{C}'_{\mathrm{uo}}=\boldsymbol{R}\boldsymbol{D}_{\mathrm{uo}}+\boldsymbol{C}_{\mathrm{uo}}=0.\end{cases} \tag{13}$$

$\boldsymbol{R}$ can be calculated through the following algorithm.

Algorithm 1

Let $\boldsymbol{M}=\begin{bmatrix}\boldsymbol{D}_{\mathrm{uc}} & \boldsymbol{D}_{\mathrm{uo}} & \boldsymbol{\mu}_0 & \boldsymbol{I}_1\\ \boldsymbol{C}_{\mathrm{uc}} & \boldsymbol{C}_{\mathrm{uo}} & -\boldsymbol{b} & \boldsymbol{0}\end{bmatrix}$,

where $\boldsymbol{M}\in\mathbb{Z}^{(m\times n_{\mathrm{uc}})\times(n_{\mathrm{uc}}+n_{\mathrm{uo}}+m+1)}$, $\boldsymbol{D}_{\mathrm{uc}}\in\mathbb{Z}^{m\times n_{\mathrm{uc}}}$, $\boldsymbol{D}_{\mathrm{uo}}\in\mathbb{Z}^{m\times n_{\mathrm{uo}}}$, $\boldsymbol{C}_{\mathrm{uc}}\in\mathbb{Z}^{m_{\mathrm{c}}\times n_{\mathrm{uc}}}$, $\boldsymbol{C}_{\mathrm{uo}}\in\mathbb{Z}^{m_{\mathrm{c}}\times n_{\mathrm{uo}}}$, $\boldsymbol{\mu}_0\in\mathbb{Z}^{m}$, $\boldsymbol{b}\in\mathbb{Z}^{+m_{\mathrm{c}}}$.

Through row operations, we can obtain $\boldsymbol{M}'$

$$\boldsymbol{M}'=\begin{bmatrix}\boldsymbol{R}\boldsymbol{D}_{\mathrm{uc}} & \boldsymbol{R}\boldsymbol{D}_{\mathrm{uo}} & \boldsymbol{R}\boldsymbol{\mu}_0 & \boldsymbol{R}\\ \boldsymbol{R}\boldsymbol{D}_{\mathrm{uc}}+\boldsymbol{C}_{\mathrm{uc}} & \boldsymbol{R}\boldsymbol{D}_{\mathrm{uo}}+\boldsymbol{C}_{\mathrm{uo}} & \boldsymbol{R}\boldsymbol{\mu}_0-\boldsymbol{b} & \boldsymbol{R}\end{bmatrix},$$

in which conditions Eq.(13) are satisfied, and $\boldsymbol{R}$ is used to achieve $\boldsymbol{C}'\boldsymbol{v}\le\boldsymbol{b}'$, the admissible constraints.

By Algorithm 1, $\boldsymbol{R}$ can be easily obtained if the controller exists, and we should note that only add operations can be executed in the matrix row operation to ensure $\boldsymbol{R}\ge0$.

EXAMPLE

This section gives an application of Theorem 2 and describes the whole procedure of the supervisor design for comparison with the method mentioned in (Iordache and Antsaklis, 2002; 2003) to show the advantages. The example is borrowed from (Iordache and Antsaklis, 2003) shown in Fig.3, in which PN models a manufacturing cell. Autonomous vehicles (AVs) can enter a restricted area (RA) from the left or from the right. The left AVs enter the RA via t_2 and exit via t_{13}; the right AVs enter via t_5 and exit via t_{14}. The number of AVs in the RA is limited to m. Moreover, left and right AVs should not appear in the RA at the same time. These constraints can be written as

$$mq_2 \le m - v_5 + v_{14}, \quad (14)$$

$$mq_5 \le m - v_2 + v_{13}. \quad (15)$$

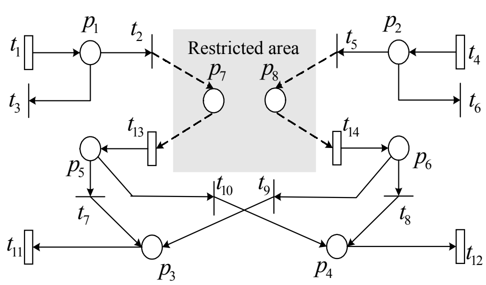

Fig.3 Petri net model of a manufacturing cell

The marking of p_1 (p_2) represents the number of left (right) AVs that wait to enter the RA, such an AV may be rerouted to another RA via t_3 (t_6). The left AVs should stay in line if there are no right AVs in p_2 or in the RA. Then the constraint can be written as

$$q_3 \le \mu_2 + v_3 - v_{14}. \quad (16)$$

For some fairness and firing restrictions which can be referred to (Iordache and Antsaklis, 2003), we write the following constraints

$$v_{11} - v_{12} \le n, \quad (17)$$

$$v_{12} - v_{11} \le n. \quad (18)$$

Because there are q_2, q_3, q_5 in the constraints, (t_2', p_2', t_2''), (t_3', p_3', t_3''), (t_5', p_5', t_5'') are constructed to replace t_2, t_3, t_5, which are shown in Fig.4. q_i' is replaced by μ_i', and then can be transformed into v_i form. The transforming procedure is as follows.

The constraints can be rewritten as follows to eliminate q_i:

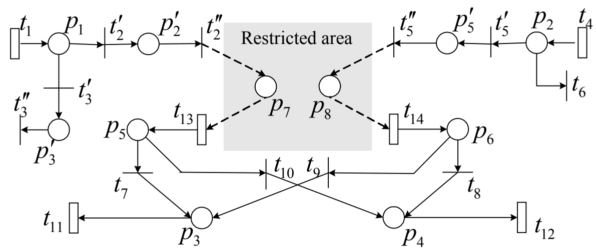

Fig.4 Modified Petri net model

$$m\mu_2' + v_5 - v_{14} \le m, \quad (14.1)$$

$$m\mu_5' + v_2 - v_{13} \le m, \quad (15.1)$$

$$-\mu_2 + \mu_3' - v_5 + v_{14} \le 0, \quad (16.1)$$

$$v_{11} - v_{12} \le n, \quad (17.1)$$

$$v_{12} - v_{11} \le n. \quad (18.1)$$

When we apply Wang and Yan (2005)'s method, μ_i can be replaced by v_i. We have the following constraints involving only Parikh vector term

$$m(v_2' - v_2'') + v_5' - v_{14} \le m, \quad (14.2)$$

$$m(v_5' - v_5'') + v_2' - v_{13} \le m, \quad (15.2)$$

$$-v_4 + v_6 + v_3' - v_3'' + v_{14} \le 0, \quad (16.2)$$

$$v_{11} - v_{12} \le n, \quad (17.2)$$

$$v_{12} - v_{11} \le n. \quad (18.2)$$

Assume the uncontrollable transitions to be t_1, t_4, t_{11}, t_{12}, t_{13} and t_{14}, and the unobservable transitions to be t_{13} and t_{14}.

According to Theorem 1 and Algorithm 1, the modified constraints are as follows

$$m(v_2' - v_2'') + v_5' - v_8 - v_9 \le m, \quad (14.3)$$

$$m(v_5' - v_5'') + v_2' - v_7 - v_{10} \le m, \quad (15.3)$$

$$v_3' - v_3'' - v_4 + v_5 + v_6 \le 0, \quad (16.3)$$

$$v_7 + v_9 - v_{12} \le n, \quad (17.3)$$

$$v_8 + v_{10} - v_{11} \le n. \quad (18.3)$$

According to Proposition 1, the controlled PN can be synthesized, then (t_2', p_2', t_2'') can be changed to t_2 and the same operation can be applied to the other two. The controlled PN is shown in Fig.5. For comparison with the example in (Iordache and Antsaklis, 2003), only three controllers are presented here,

C_1, C_2, C_3 corresponding to the constraints Eqs.(15), (16), (18) respectively.

Fig.5 shows that the method used in this paper is more efficient and concise than that of Iordache (2003).

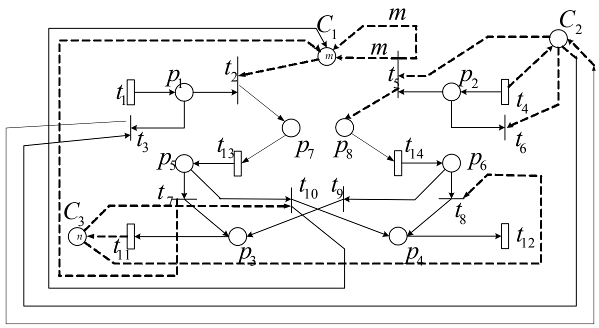

Fig.5 Petri net model with controllers

CONCLUSION

This paper presents the problem of supervisor synthesis involving general constraints with uncontrollable and unobservable transitions. General constraints, which have strong expressive ability, can be transformed to Parikh vector terms to make the supervisor synthesis easier and more effective. A method to obtain admissible constraints to avoid the existence of unobservable and unobservable transition in the constraints is proposed here. An example is given to detail the procedure of solving problem. Our future work will focus on properties of the transformed PN to obtain the optimal and maximally permissive controller, as well as the deadlock and liveness.

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