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Static analysis of synchronism deployable antenna*

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Abstract: A 3D synchronism deployable antenna was designed, analyzed, and manufactured by our research group. This antenna consists of tetrahedral elements from central element. Because there are springs at the ends of some of the rods, spider joints are applied. For analysis purpose, the structure is simplified and modelled by using 2D beam elements that have no bending stiffness. Displacement vectors are defined to include two translational displacements and one torsional displacement. The stiffness matrix derived by this method is relatively simple and well defined. The analysis results generated by using software developed by our research group agreed very well with available test data.

Key words: Space deployable structures, Analysis design, Reflector antenna
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INTRODUCTION

Large antennas for space applications have some differences as compared to their counterparts designed and manufactured for ground usage. For a larger space antenna, the applied loads in space can be several orders of magnitude less than the gravity loading it is subjected to when being fabricated and assembled on the ground. Since the effect of gravity cannot be removed from any experimental verification performed on the ground, we must rely on the use of efficient and highly accurate analysis methods for designing and verifying large space antennas. Tibert (2002) provided the state-of-the-art of deployable antenna. And many researches have been trying to analyze the beam with torsion moments.

Na and Kim (2006) analyzed the deployment of a multi-link flexible structure with the Timoshenko beam theory and determined the equations of motion using Hamilton's principle. Serna *et al*.(2006) presented a set of equivalent uniform moment factor values using both finite difference and finite element techniques. Armero and Ehrlich (2006) proposed some new finite elements for thin Euler-Bernoulli beams that incorporate the softening hinges to describe the beam's deformation. Yau (2006) suggested a finite element model for buckling analysis of tapered I-beams subjected to torsional moments. Ma (2006) analyzed the ideal displacement amplification ratio of a bridge-type flexure hinge based on the kinematic theory.

Two types of special beam elements developed specifically for modelling and analyzing large 3D-synchronism deployable space antennas will be studied in this paper. The first type is a one-beam element with torsion springs formed by two elements of the first type that are linked by a pin joint. As some loads are always zero, the stiffness matrices can be simplified and used to perform static and dynamic analyses of the antenna.

REDUCTION FORMULATION

Fig.1 shows the antenna structure to be analyzed.

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(a)

(b)

Fig.1 Antenna structure model. (a) Stowed state; (b) Deployable state

This is a beam-and-truss structure (Yue *et al*., 2001; Zhao and Guan, 2005). The equilibrium equation of this structure can be derived in terms of generalized stresses in a Cartesian coordinate system. Formulations are rewritten as follows (Pellegrino and Calladine, 1986; Zhang, 2001):

$$[\boldsymbol{A}]_{(n+p)(m+p)}\{\boldsymbol{\sigma}\}_{(m+p)}=\{\boldsymbol{l}\}_{m+p}, \tag{1}$$

where $\boldsymbol{A}$ is flexibility rectangular matrix, $\boldsymbol{\sigma}$ is generalized stress, $\boldsymbol{l}$ is generalized loads. The equilibrium equation can also be expressed in terms of generalized displacements

$$[\boldsymbol{A}^{\mathrm{T}}]_{(m+p)(n+p)}\{\boldsymbol{d}\}_{(m+p)}=\{\boldsymbol{\varepsilon}\}_{(n+p)}, \tag{2}$$

where $\boldsymbol{A}^{\mathrm{T}}$ is geometric compatibility $(m+p)(n+p)$ matrix, $\boldsymbol{d}$ is displacement of node $(m+p)$, $\boldsymbol{\varepsilon}$ is generalized strain. This equation is valid only for small deformation.

For a linear-elastic material, the generalized stresses $\boldsymbol{\sigma}$ are related to the generalized strains $\boldsymbol{\varepsilon}$ by the matrix of member $\boldsymbol{F}$, or just flexibility matrix by the relationship

$$[\boldsymbol{F}]_{(n+p)(n+p)}\{\boldsymbol{\sigma}\}_{(n+p)}=\{\boldsymbol{\varepsilon}\}_{(n+p)}. \tag{3}$$

If the number of zero load is p, i.e., $l_p=\mathbf{0}$, the system of equilibrium Eq.(1) can be split into the following two coupled systems of m and p linear equations, respectively

$$\boldsymbol{A}_{mn}\boldsymbol{\sigma}_n+\boldsymbol{A}_{mp}\boldsymbol{\sigma}_p=\boldsymbol{l}_m, \tag{4.1}$$

$$\boldsymbol{A}_{pn}\boldsymbol{\sigma}_n+\boldsymbol{A}_{pp}\boldsymbol{\sigma}_p=\mathbf{0}. \tag{4.2}$$

If $\boldsymbol{A}_{pp}$ is not singular, system Eq.(4.2) can be solved for $\boldsymbol{\sigma}_p$, in terms of $\boldsymbol{\sigma}_n$

$$\boldsymbol{\sigma}_p=-\boldsymbol{A}_{pp}^{-1}\boldsymbol{A}_{pn}\boldsymbol{\sigma}_n. \tag{5}$$

Substituting Eq.(5) into Eq.(4.1) yields

$$(\boldsymbol{A}_{mn}-\boldsymbol{A}_{mp}\boldsymbol{A}_{pp}^{-1}\boldsymbol{A}_{pn})\boldsymbol{\sigma}_n=\boldsymbol{l}_m. \tag{6}$$

That is

$$\boldsymbol{A}^*\boldsymbol{\sigma}^*=\boldsymbol{l}^*, \tag{7}$$

where

$$\boldsymbol{A}^*=\boldsymbol{A}_{mn}-\boldsymbol{A}_{mp}\boldsymbol{A}_{pp}^{-1}\boldsymbol{A}_{pn},\ \boldsymbol{\sigma}^*=\boldsymbol{\sigma}_n,\ \boldsymbol{l}^*=\boldsymbol{l}_m, \tag{8}$$

So that we can obtain (Zhang, 2001)

$$(\boldsymbol{A}_{mn}^{\mathrm{T}}-\boldsymbol{A}_{pn}^{\mathrm{T}}\boldsymbol{A}_{pp}^{-\mathrm{T}}\boldsymbol{A}_{mp}^{\mathrm{T}})\boldsymbol{d}_m=\boldsymbol{\varepsilon}_n-\boldsymbol{A}_{pn}^{\mathrm{T}}\boldsymbol{A}_{pp}^{-\mathrm{T}}\boldsymbol{\varepsilon}_p. \tag{9}$$

This is a new system of compatibility equations, with an $n\times m$ coefficient matrix. The internal work is

$$\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\varepsilon}=\boldsymbol{\sigma}_n^{\mathrm{T}}\boldsymbol{\varepsilon}_n+\boldsymbol{\sigma}_p^{\mathrm{T}}\boldsymbol{\varepsilon}_p. \tag{10}$$

The reduced flexibility matrix is

$$\boldsymbol{F}^*=\boldsymbol{F}_{nn}-\boldsymbol{F}_{np}\boldsymbol{A}_{pp}^{-1}\boldsymbol{A}_{pn}-\boldsymbol{A}_{pn}^{\mathrm{T}}\boldsymbol{A}_{pp}^{-\mathrm{T}}\boldsymbol{F}_{pn}+\boldsymbol{A}_{pn}^{\mathrm{T}}\boldsymbol{A}_{pp}^{-\mathrm{T}}\boldsymbol{F}_{pp}\boldsymbol{A}_{pp}^{-1}\boldsymbol{A}_{pn}. \tag{11}$$

The reduced stiffness matrix is

$$\boldsymbol{K}^*=(\boldsymbol{F}^*)^{-1}=\boldsymbol{F}_{nn}^{-1}-\boldsymbol{A}_{pn}^{-1}\boldsymbol{A}_{pp}\boldsymbol{F}_{np}^{-1}-\boldsymbol{F}_{pn}^{-1}\boldsymbol{A}_{pp}\boldsymbol{A}_{pn}^{-\mathrm{T}}+\boldsymbol{A}_{pn}^{-1}\boldsymbol{A}_{pp}\boldsymbol{F}_{pp}^{-1}\boldsymbol{A}_{pp}\boldsymbol{A}_{pn}^{-\mathrm{T}}. \tag{12}$$

MATRIX OF FIRST TYPE OF MACRO-ELEMENT

Fig.2 shows a one-beam element that has torsion spring joints at each end. The torsion spring joints can be connected to other elements or spider frictionless pins. A torsion spring joint transmits the force component and bending moment. When the joint revolves (turns), it also transmits shear force.

The beam is a 3D structure of a plane beam.

Fig.2 First type of macro-element

Equilibrium matrix

Macro-element 1 can be separated into a beam and two torsion springs as shown in Fig.3.

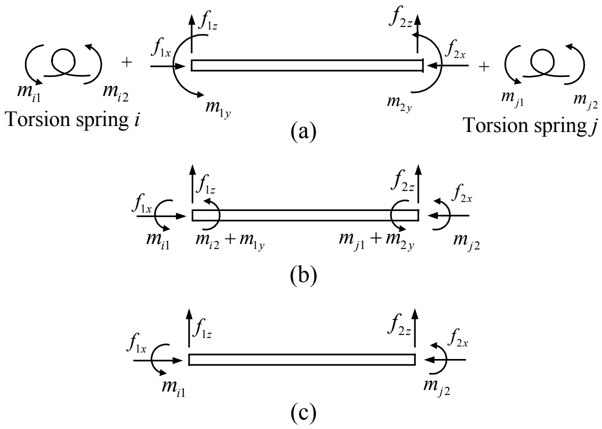

Fig.3 Reduced macro-element 1. (a) Separated force model; (b) Force combination; (c) Force simplification

In the state of Fig.3a, the equilibrium equation is

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1/L^1 & 1/L^1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/L^1 & -1/L^1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_i^1 \\ T^1 \\ M_1 \\ M_2 \\ M_j^1 \end{bmatrix} = \begin{bmatrix} m_{i1} \\ m_{i2} \\ f_{1x} \\ f_{1z} \\ m_{1y} \\ f_{2x} \\ f_{2z} \\ m_{2y} \\ m_{j1} \\ m_{j2} \end{bmatrix}, \quad (13)$$

where M_i^1, M_j^1 are torsion moments of the spring, m_{i1}, m_{i2}, m_{j1}, m_{j2} are generalized stresses loaded at the ends of torsion spring, T is axile force, L is length of the beam, superscript 1 represents the number of macro-element. At the nodes in Fig.3c, force equilibrium and displacement compatibility must be maintained. The equilibrium equation is

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1/L^1 & 1/L^1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/L^1 & -1/L^1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_i^1 \\ T^1 \\ M_1 \\ M_2 \\ M_j^2 \end{bmatrix} = \begin{bmatrix} m_{i1} \\ f_{1x} \\ f_{1z} \\ m_{i2}+m_{1y} \\ f_{2x} \\ f_{2z} \\ m_{j1}+m_{2y} \\ m_j^2 \end{bmatrix}. \quad (14)$$

As $m_{i2}+m_{1y}$ and $m_{j1}+m_{2y}$ are always zero, the equilibrium equation reduced to

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1/L^1 & 1/L^1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1/L^1 & -1/L^1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T^1 \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} f_{1x} \\ f_{1z} \\ m_{i1} \\ f_{2x} \\ f_{2z} \\ m_{j2} \end{bmatrix}. \quad (15)$$

The coefficient matrix of Eq.(15) is $\boldsymbol{A}^*$.

Geometric harmonious matrix

Let matrix $\boldsymbol{A}^*$ turn, get $(\boldsymbol{A}^*)^{\mathrm{T}}$, which is a reduced geometric harmonious matrix.

The reduced displacement vector is

$$\boldsymbol{d}^* = [d_{1x} \quad d_{1z} \quad \eta_{iy}^1 \quad d_{2x} \quad d_{2z} \quad \eta_{jy}^2], \quad (16)$$

where η_{iy}^1, η_{iy}^2 are generalized displacements of turn angle about torsion springs.

The reduced generalized strain vector is

$$\boldsymbol{\varepsilon} = [\phi_i^1 \quad e^1 \quad \phi_1 \quad \phi_2 \quad \phi_j^1]. \quad (17)$$

The generalized strain vector $\boldsymbol{\varepsilon}$ can be divided into $\boldsymbol{\varepsilon}_n$, $\boldsymbol{\varepsilon}_p$.

$$\boldsymbol{\varepsilon}^* = \boldsymbol{\varepsilon}_n - \boldsymbol{A}_{pn}^{\mathrm{T}}\boldsymbol{A}_{pp}^{-1}\boldsymbol{\varepsilon}_p = \begin{bmatrix} e^1 \\ \phi_1+\phi_i^1 \\ \phi_2+\phi_j^1 \end{bmatrix}, \quad (18)$$

where

$$\boldsymbol{\varepsilon}_p = \begin{bmatrix} \phi_i^1 \\ \phi_j^1 \end{bmatrix}, \quad \boldsymbol{\varepsilon}_n = \begin{bmatrix} e^1 \\ \phi_1+\phi_i^1 \\ \phi_2+\phi_j^1 \end{bmatrix}. \quad (19)$$

Flexibility matrix

For linear-elastic material behavior, flexibility matrix is given by

$$\begin{bmatrix} f_1 & 0 & 0 & 0 & 0 \\ 0 & L^1/EA^1 & 0 & 0 & 0 \\ 0 & 0 & 2i^1 & i^1 & 0 \\ 0 & 0 & i^1 & 2i^1 & 0 \\ 0 & 0 & 0 & 0 & f_2 \end{bmatrix} \begin{bmatrix} M_i^1 \\ T^1 \\ M_1 \\ M_2 \\ M_j^1 \end{bmatrix} = \begin{bmatrix} \phi_i^1 \\ e_1 \\ \phi_1 \\ \phi_2 \\ \phi_j^1 \end{bmatrix}, \quad (20)$$

where f_1, f_2 are flexibility of springs, $i^1=L^1/(6EI_y)$, A^1 is section area. Flexibility equation can be written as

$$\begin{bmatrix} L^1/EA^1 & 0 & 0 \\ 0 & f_1+2i^1 & i^1 \\ 0 & i^1 & f_2+2i^1 \end{bmatrix} \begin{bmatrix} T^1 \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} e^1 \\ \phi_1+\phi_i^1 \\ \phi_2+\phi_j^2 \end{bmatrix} = \boldsymbol{\varepsilon}^*, \quad (21)$$

where

$$\boldsymbol{F}^* = \begin{bmatrix} L^1/EA^1 & 0 & 0 \\ 0 & f_1+2i^1 & i^1 \\ 0 & i^1 & f_2+2i^1 \end{bmatrix}. \quad (22)$$

Stiffness matrix

From $\boldsymbol{A}^*\boldsymbol{\sigma}^*=\boldsymbol{1}^*$, $(\boldsymbol{A}^*)^{\mathrm{T}}\boldsymbol{d}^*=\boldsymbol{\varepsilon}^*$, $\boldsymbol{F}^*\boldsymbol{\sigma}^*=\boldsymbol{\varepsilon}^*$, the reduced stiffness matrix can be expressed as

$$\boldsymbol{K}^* = \begin{bmatrix} \frac{EA^1}{L^1} & 0 & 0 & -\frac{EA^1}{L^1} & 0 & 0 \\ 0 & \frac{P\Delta}{(L^1)^2} & \frac{Q_2\Delta}{L^1} & 0 & -\frac{P\Delta}{(L^1)^2} & \frac{Q_1}{L^1} \\ 0 & \frac{Q_2\Delta}{L^1} & R_2\Delta & 0 & -\frac{P\Delta}{(L^1)^2} & \frac{Q_1}{L^1} \\ -\frac{EA^1}{L^1} & 0 & 0 & \frac{EA^1}{L^1} & 0 & 0 \\ 0 & -\frac{P\Delta}{(L^1)^2} & -\frac{Q_2\Delta}{L^1} & 0 & \frac{P\Delta}{(L^1)^2} & -\frac{Q_1\Delta}{L^1} \\ 0 & \frac{Q_1\Delta}{L^1} & i^1\Delta & 0 & -\frac{Q_1\Delta}{L^1} & R_1\Delta \end{bmatrix}, \quad (23)$$

where A is area of the beam, L is length of the beam, $\Delta=1/[f_1f_2+2(f_1+f_2)i^1+3(i^1)^2]$, $P=f_1+f_2+6i^1$, $Q_1=f_1+3i^1$, $Q_2=f_2+3i^1$, $R_1=f_1+2i^1$, $R_2=f_2+2i^1$. If $f_1=f_2=f$, the stiffness matrix can be further simplified.

MATRIX OF SECOND TYPE OF MACRO-ELEMENT

As shown in Fig.4, macro-element 2 consists of two macro-element 1 jointed in the middle by a frictionless pin.

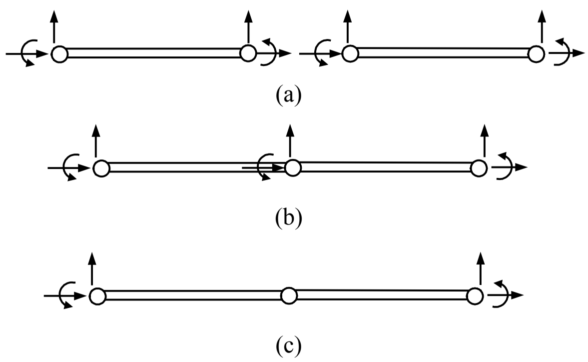

Fig.4 Macro-element 2. (a) Separated force model; (b) Force combination; (c) Force simplification

It is the key part of a deployable structure. The middle pin joint that links the two macro-elements 1 can turn by pin joint transfer axial force. The middle two torsion springs produce bending moment to macro-element 1. Fig.4a depicts the bending moment loaded on macro-element 1, Fig.4b shows all the forces on the macro and Fig.4c is the simplification. So that it has characteristics of a plane beam. Similarly, we can get the equilibrium equation of macro-element 2, that is

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1/L^1 & 1/L^1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1/L^1 & -1/L & 0 & -1/L^2 & 1/L^2 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/L^2 & 1/L^2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} T^1 \\ M_1 \\ M_3 \\ T^2 \\ M_4 \\ M_2 \end{bmatrix} = [f_{1x}\ f_{1z}\ m_{ij}^1\ f_{mx}\ f_{mz}\ m_{ijy}^m\ f_{2x}\ f_{2z}\ m_{jy}^2]^{\mathrm{T}}, \quad (24)$$

where m_{ijy}^m is torque of torsion spring in the middle two nodes, L^2, A^2 are length, area of the second beam respectively.

The flexibility equation can be written as

$$\begin{bmatrix} \frac{L^1}{EA^1} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_1 & i^1 & 0 & 0 & 0 \\ 0 & i^1 & f_3+2i^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{L^2}{EA^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & f_4+2i^2 & i^2 \\ 0 & 0 & 0 & 0 & i^2 & f_2+2i^2 \end{bmatrix} \times [T^1\ \ M_1\ \ M_3\ \ T^2\ \ M_4\ \ M_2]^{\mathrm{T}} = [e^1\ \ \phi_1+\phi_i^1\ \ \phi_3+\phi_j^1\ \ e^2\ \ \phi_4+\phi_i^2\ \ \phi_2+\phi_j^2]^{\mathrm{T}}, \quad (25)$$

where $i^1 = L^1/(6EI_y^1)$, $i^2 = L^2/(6EI_y^2)$.

Load components of middle node m are always equal to zero, so that the degrees of freedom can be reduced and Eq.(26) can be derived from Eq.(24) by primary transform

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1/L^1 & 0 & 0 & 1/L^1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1/L^2 & 0 & 0 & 1/L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1/L^1 & 1/L^2 & 0 & 1/L^1 & -1/L^2 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} T^1 \\ M_1 \\ M_2 \\ T^2 \\ M_3 \\ M_4 \end{bmatrix} = [f_{1x}\ f_{1z}\ m_{iy}^1\ f_{2x}\ f_{2z}\ m_{jy}^2\ f_{mx}\ f_{mz}\ m_{ijy}^m]^{\mathrm{T}}. \quad (26)$$

Since $f_{mx} = f_{my} = m_{ijy}^m = 0$, $\boldsymbol{A}^* = \boldsymbol{A}_{mn} - \boldsymbol{A}_{mp}\boldsymbol{A}_{pp}^{-1}\boldsymbol{A}_{pn}$, $\boldsymbol{A}^*$ is the reduced equilibrium matrix,

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{L^{1,2}} & \frac{1}{L^{1,2}} \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & \frac{1}{L^{1,2}} & -\frac{1}{L^{1,2}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} f_{1x} \\ f_{1z} \\ m_{iy} \\ f_{2x} \\ f_{2z} \\ m_{jy}^2 \end{bmatrix}, \quad (27)$$

where $L^{1,2}=L^1+L^2$.

The reduced geometrical harmonious matrix $\boldsymbol{B}^*=(\boldsymbol{A}^*)^{\mathrm{T}}$ corresponding to reduced load vector is

$$\boldsymbol{d}^* = [d_{1x}\quad d_{1z}\quad \eta_{iy}^1\quad d_{2x}\quad d_{2z}\quad \eta_{iy}^2]^{\mathrm{T}}. \quad (28)$$

The reduced generalized strain vector becomes

$$\boldsymbol{\varepsilon}^* = \begin{bmatrix} e^1 \\ \phi_1+\phi_i^1 \\ \phi_2+\phi_j^2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{L^2}{L^{1,2}} & -\frac{L^2}{L^{1,2}} \\ 0 & -\frac{L^1}{L^{1,2}} & -\frac{L^1}{L^{1,2}} \end{bmatrix} \begin{bmatrix} e^2 \\ \phi_3+\phi_j \\ \phi_4+\phi_i^2 \end{bmatrix} = \begin{bmatrix} e^1+e^2 \\ \phi_1+\phi_i^1+(L^2/L^{1,2})(\phi_3+\phi_j^1+\phi_4+\phi_i^2) \\ \phi_2+\phi_j^1+(L^1/L^{1,2})(\phi_3+\phi_j^1+\phi_4+\phi_i^2) \end{bmatrix}. \quad (29)$$

The stiffness matrix of macro-element 2 was assembled from macro-element 1. The inverse matrix of reduced flexibility matrix is shown as

$$(\boldsymbol{F}^*)^{-1}=\begin{bmatrix} EA/L & 0 & 0 \\ 0 & a & -b \\ 0 & -b & a \end{bmatrix}. \qquad (30)$$

The reduced stiffness matrix of macro-element 2 is

$$\boldsymbol{K}^*=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{2S}{L^2} & \frac{S}{L} & 0 & -\frac{2S}{L^2} & \frac{S}{L} \\ 0 & \frac{S}{L} & a & 0 & -\frac{S}{L} & b \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{2S}{L^2} & -\frac{S}{L} & 0 & \frac{2S}{L^2} & -\frac{S}{L} \\ 0 & \frac{S}{L} & b & 0 & -\frac{S}{L} & a \end{bmatrix}, \qquad (31)$$

where

$$S=a+b,$$

$$a=(f/2+4i)/(2f^2+10fi+12i^2), \qquad (32)$$

$$b=(f/2+2i)/(2f^2+10fi+12i^2). \qquad (33)$$

Obviously, the reduced stiffness matrix is of positive determinate symmetry and the linear reduced equilibrium equation can be solved.

STATIC ANALYSIS

This antenna consists of two types of elements, one is macro-element 2, consisting of upper and lower chord struts, the other is truss element, composed of abdominal struts. Stiffness matrix is assembled in the global coordinate system. Finally, the linear equations are solved.

Fig.5 shows a revolving parabolic deployable antenna, whose design parameters are: aperture diameter D=2.2 m, ratio of focal distance to aperture diameter f/D=0.4, area of strut cross section A=0.13345 cm^2, Young's modulus E=210 GPa, density of mass ρ=7.8 g/cm^3.

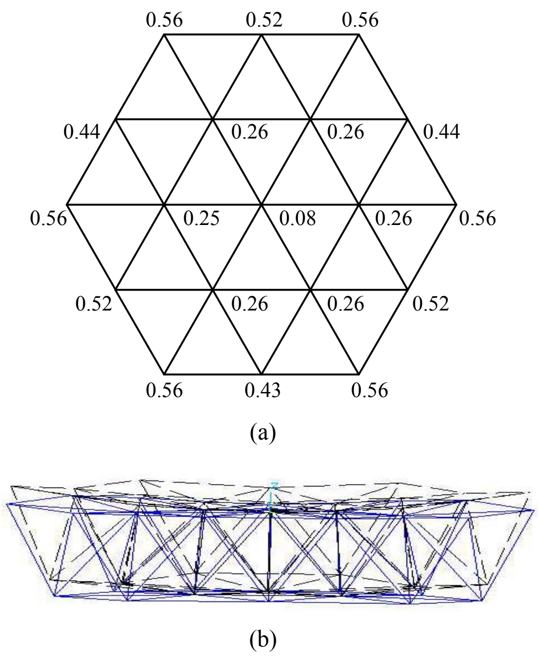

Fig.5 A revolving parabolic deployable antenna. (a) Orthographic drawing; (b) Plan drawing

CONCLUSION

We applied the reduction formula of equilibrium, compatibility and flexibility matrices in the force method (Pellegrino *et al.*, 1992) to derive the stiffness matrix of complicated new elements. In this paper two types of special beam elements are discussed. The first type consists of one beam with torsion spring installed at its end. The second type consists of two first type elements linked by pin joint. Since some loads are always zero, the stiffness matrix can be simplified. We analyze the static deformation of deployable structure by new elements. The count result is corrected compared with the experiment data.

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References

Armero, F., Ehrlich, D. 2006. Numerical modeling of softening hinges in thin Euler-Bernoulli beams. *Computers and Structures*, **84**:641-656. [doi:10.1016/j.compstruc.2005.11.010]

Ma, H.W., 2006. Analysis of the displacement amplification ratio of bridge-type flexure hinge. *Sensors and Actuators*, p.5118-5125.

Na, K.S., Kim, J.H., 2006. Deployment of a multi-link flexible structure. *Journal of Sound and Vibration*, **294**:298-313. [doi:10.1016/j.jsv.2005.11.018]

Pellegrino, S., Calladine, C.R., 1986. Matrix analysis of statically and kinematically indeterminate frameworks. *International Journal of Solids Structures*, **22**(4):409-428. [doi:10.1016/0020-7683(86)90014-4]

Pellegrino, S., Kwan, A.S.K., van Heerden, T.F., 1992. Reduction of equilibrium compatibility and flexibity matrices, in the force method. *International Journal of Methods in Engineering*, **35**(6):1219-1236. [doi:10.1002/nme.1620350605]

Serna, M.A., López, A., Puente, I., Yong, D.J., 2006. Equivalent uniform moment factors for lateral-torsional buckling of steel members. *Journal of Constructional Steel Research*, **62**(6):566-580. [doi:10.1016/j.jcsr.2005.09.001]

Tibert, G., 2002. Deployable Tensegrity Structures for Space Applications. Ph.D Dissertation, Royal Institute of Technology Department of Mechanics, Sweden.

Yau, J.D., 2006. Stability of tapered I-beams under torsional moments. *Finite Elements in Analysis and Design*, **42**(10):914-927. [doi:10.1016/j.finel.2006.01.008]

Yue, J.R., Guan, F.L., Hu, Q.B., 2001. Geometrical compatibility analysis for a kind of deployable space truss. *China Civil Engineering Journal*, **34**:15-21 (in Chinese).

Zhang, J.J., 2001. Design and Analysis Researches for Deployable Truss Structures Driven by Spring. Ph.D Dissertation, Zhejiang University, China (in Chinese).

Zhao, M.L., Guan, F.L., 2005. Kinematic analysis of deployable toroidal spatial truss structures for large mesh antenna. *Journal of the International Association for Shell and Spatial Sructures*, **46**:195-204.