Journal of Zhejiang University SCIENCE A
ISSN 1009-3095 (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Robust model predictive control for discrete uncertain nonlinear systems with time-delay via fuzzy model*

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Received Nov. 21, 2005; revision accepted Feb. 22, 2006

Abstract: An extended robust model predictive control approach for input constrained discrete uncertain nonlinear systems with time-delay based on a class of uncertain T-S fuzzy models that satisfy sector bound condition is presented. In this approach, the minimization problem of the "worst-case" objective function is converted into the linear objective minimization problem involving linear matrix inequalities (LMIs) constraints. The state feedback control law is obtained by solving convex optimization of a set of LMIs. Sufficient condition for stability and a new upper bound on robust performance index are given for these kinds of uncertain fuzzy systems with state time-delay. Simulation results of CSTR process show that the proposed robust predictive control approach is effective and feasible.

Key words: Uncertain Takagi-Sugeno fuzzy model, Time-delay, Model predictive control (MPC), Linear matrix inequalities (LMIs), Robustness
doi:10.1631/jzus.2006.A1723 **Document code:** A **CLC number:** TP273

INTRODUCTION

In recent years, research on nonlinear model predictive control has attracted significant attention because most practical processes are nonlinear. In industry process control, model predictive controllers based on linear models (LMPC) are often used to control nonlinear systems. When the operating conditions undergo significant changes, the performance of LMPC can deteriorate drastically. Under such conditions, predictive control based on nonlinear models (NMPC) should be considered. But the expected control results have not been obtained in practical applications because there still exist two main difficulties: (1) nonlinear modelling problem. Due to the complexity of nonlinear systems, it is impossible to develop nonlinear system identification techniques by straightforward extension of the linear theory; (2) nonlinear optimization problem, which is a certain difficulty in finishing the optimal solution of the prediction control law within a sampling period because the computing time of nonlinear optimization is needed. T-S fuzzy model, presented by Takagi and Sugeno (1985) has wide applications in system modelling and control since it can excellently express the dynamic characteristics of complex nonlinear systems. Fuzzy model predictive control combining T-S model with MPC has also become an attractive strategy in complex systems control (Nounou and Passino, 1999; Matko *et al*., 2000; Sarimveis and Bafas, 2003). A T-S fuzzy model is used as the prediction model (Oblak and Skrjanc, 2005), at each sampling point a local linear model is calculated, and the next incremental control action is solved based on LMPC strategy, which avoids effectively the above two difficulties. Effective optimization approaches were presented in (Mollov *et al*., 2004) that avoid non-

* Project (No. 60421002) supported by the National Natural Science Foundation of China

convex optimization by employing a single local linear model or a set of local linear models to approximate the fuzzy model along the predicted trajectory. The control signal is obtained by solving a constrained quadratic program (QP). To account for errors introduced by the linearization, an iterative optimization scheme is proposed.

However, time delay and parameter uncertainty are frequently encountered in the behavior of many practical processes and are very often the main cause for poor performance and instability of control systems. In view of this, the robustness issue of time-delay and parameter uncertain systems is a topic of great practical importance that has attracted much interest for several decades (Hu and Chen, 2004; Li *et al*., 2000; Wang *et al*., 2004; Zheng *et al*., 2005). A new approach for robust MPC synthesis that allows explicit incorporation of the description of the plant uncertainty in the problem formulation was presented in (Kothare *et al*., 1996). In this approach, a state feedback control law that minimizes a "worst-case" infinite horizon objective function subject to constraints on control input and plant output is reduced to a convex optimization involving linear matrix inequalities (LMIs). An improved state feedback model predictive control approach was given for the systems with delayed state variable, delayed input and measurable disturbance (Yan and Hu, 2004). Casavola *et al*.(2004) presented a novel robust predictive control algorithm for uncertain input-saturated linear systems described by structured norm-bounded model uncertainties.

In this paper, an extended robust predictive control approach using uncertain T-S fuzzy model satisfying sector bound condition is proposed for input constrained discrete nonlinear systems with state time-delay. First, discrete uncertain nonlinear systems with state time-delay are described based on a class of uncertain T-S models satisfying sector bound condition. Then, an extended robust predictive control approach for these kinds of systems is presented. The robust constrained MPC problem with state feedback is formulated as an LMIs problem. Under the existence of the feasible solution, a sufficient condition for robust stability is given. And then, a simulation example of CSTR process is given to show that the proposed approach is effective and feasible. Finally, the conclusion of this paper is given.

TAKAGI-SUGENO MODEL FOR DISCRETE UNCERTAIN SYSTEM WITH TIME-DELAY

The Takagi-Sugeno fuzzy model is described by fuzzy IF-THEN rules that represent local linear input-output relations of a nonlinear system. The ith rule of the uncertain T-S fuzzy model with state time-delay is of the following form

Plant rule i:

IF $z_1(t)$ is M_{i1} and …, and $z_p(t)$ is M_{ip}, THEN

$$\left.\begin{aligned} \boldsymbol{x}(k+1)&=\boldsymbol{A}_i\boldsymbol{x}(k)+\boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(k-d)+\boldsymbol{B}_i\boldsymbol{u}(k)+f_i(\boldsymbol{x},\boldsymbol{u}),\\ \boldsymbol{x}(k)&=\boldsymbol{\theta}(k),\quad k\in[-d,0],\ i=1,\dots,N \end{aligned}\right\} \tag{1}$$

satisfying the following input constraints

$$\|\boldsymbol{u}(k)\|_2 \le u_{\max}, \tag{2}$$

where $\boldsymbol{x}(k)\in\mathbb{R}^{n_x}$ is the state vector, $\boldsymbol{u}(k)\in\mathbb{R}^{n_u}$ is the input vector, $u_{\max}$ is a positive real number, $\boldsymbol{A}_i, \boldsymbol{A}_{\mathrm{d}i}\in\mathbb{R}^{n_x\times n_x}$, $\boldsymbol{B}_i\in\mathbb{R}^{n_x\times n_u}$ are constant matrices, M_{ij} is the fuzzy set, N is the number of IF-THEN rules, $z_1(k)$, …, $z_p(k)$ are the premise variables, d is the state time delay, $\boldsymbol{\theta}(k)$ is the initial condition of the state defined on $-d\le k\le 0$, $f_i(\boldsymbol{x},\boldsymbol{u})\in\mathbb{R}^{n_x}$ includes the parameter uncertainty and modelling error which satisfies the following sector bound condition

$$f_i^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{u})f_i(\boldsymbol{x},\boldsymbol{u})=\mu_i^2\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{u}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{u}\end{bmatrix}. \tag{3}$$

In this paper, the premise variables are assumed to be independent of the input variable $\boldsymbol{u}(k)$. Given a pair of ($\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$), the output of the fuzzy system Eq.(1) is expressed as follows

$$\begin{aligned}\boldsymbol{x}(k+1)=\sum_{i=1}^{N}\{h_i[\boldsymbol{z}(k)][\boldsymbol{A}_i\boldsymbol{x}(k)\\ +\boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(k-d)+\boldsymbol{B}_i\boldsymbol{u}(k)+f_i(\boldsymbol{x},\boldsymbol{u})]\},\end{aligned} \tag{4}$$

where

$$\boldsymbol{z}(k)=[z_1(k), z_2(k), \dots, z_p(k)],$$

$$h_i[\boldsymbol{z}(k)]=w_i[\boldsymbol{z}(k)]\Big/\sum_{i=1}^{N}w_i[\boldsymbol{z}(k)],$$

$$w_i[\boldsymbol{z}(k)]=\prod_{j=1}^{p}M_{ij}[z_j(k)].$$

$\forall k$, $M_{ij}[z_j(k)]$ is the grade of membership of $z_j(k)$ in M_{ij}. Note that

$$\sum_{i=1}^{N} h_i[\boldsymbol{z}(k)]=1, \quad h_i[\boldsymbol{z}(k)]\geq 0, \; i=1, \ldots, N. \tag{5}$$

Let

$$\boldsymbol{A}(k)=\sum_{i=1}^{N} h_i[\boldsymbol{z}(k)]\boldsymbol{A}_i, \;\; \boldsymbol{A}_{\rm d}(k)=\sum_{i=1}^{N} h_i[\boldsymbol{z}(k)]\boldsymbol{A}_{{\rm d}i},$$

$$\boldsymbol{B}(k)=\sum_{i=1}^{N} h_i[\boldsymbol{z}(k)]\boldsymbol{B}_i, \;\; \mu(k)=\sum_{i=1}^{N} h_i[\boldsymbol{z}(k)]\mu_i.$$

The final output of the uncertain fuzzy system with state time-delay is rewritten as follows

$$\boldsymbol{x}(k+1)=\boldsymbol{A}(k)\boldsymbol{x}(k)+\boldsymbol{A}_{\rm d}(k)\boldsymbol{x}(k-d)+\boldsymbol{B}(k)\boldsymbol{u}(k)+f(\boldsymbol{x},\boldsymbol{u}). \tag{6}$$

ROBUST FUZZY PREDICTIVE CONTROL BASED ON DISCRETE UNCERTAIN SYSTEM WITH TIME-DELAY

MPC, also known as moving horizon control or receding horizon control, has become very successful in process industries, especially in the control of processes that are constrained, multivariable, uncertain and time delayed. In general, MPC solves online an open-loop optimal control problem subject to system dynamics and constraints at each time instant and implements only the first element of the control profile. At each sampling time k, plant measurements are obtained and a model of the process is used to predict the future outputs of the process. Using these predictions, control series $\boldsymbol{u}(k+j|k)$ ($j\geq 0$) are computed by minimizing a nominal objective function $J(k)$ over a prediction horizon as follows

$$\min_{\boldsymbol{u}(k+j|k),\, j=0,1,\cdots,\infty} J(k). \tag{7}$$

Kothare *et al*.(1996) presented a min-max MPC design technique in which the minimization of the nominal cost function is modified to a minimization of the "worst-case" objective function. The min-max objective function of robust MPC is described as follows

$$\min_{\substack{\boldsymbol{u}(k+j|k)\\ j=0,1,\cdots,\infty}} \max_{\substack{\boldsymbol{A}(k+j),\,\boldsymbol{A}_{\rm d}(k+j),\\ \boldsymbol{B}(k+j)}} J(k). \tag{8}$$

At sampling time k, consider a quadratic Lyapunov-Krasovskii function for the system Eq.(6)

$$V[\boldsymbol{x}(k)]=\boldsymbol{x}^{\rm T}(k)\boldsymbol{P}\boldsymbol{x}(k)+\sum_{l=1}^{d}\boldsymbol{x}^{\rm T}(k-l)\boldsymbol{P}_{\rm d}\boldsymbol{x}(k-l), \tag{9}$$

where $\boldsymbol{P}$ and $\boldsymbol{P}_{\rm d}$ are symmetric positive definite matrices. Supposing V satisfies the following inequality

$$V[\boldsymbol{x}(k+j+1|k)]-V[\boldsymbol{x}(k+j|k)]\leq$$
$$-\boldsymbol{x}^{\rm T}(k+j|k)\boldsymbol{Q}\boldsymbol{x}(k+j|k)-\boldsymbol{u}^{\rm T}(k+j|k)\boldsymbol{R}\boldsymbol{u}(k+j|k), \tag{10}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are positive definite matrices. With the conditions $V[\boldsymbol{x}(\infty|k)]=0$ and $\boldsymbol{x}(\infty|k)=0$, summing Eq.(10) from $j=0$ to $j=\infty$ gives

$$-V[\boldsymbol{x}(k|k)]\leq -J(k), \tag{11}$$

Then the upper bound on the objective function $J(k)$ is derived as

$$\max_{\boldsymbol{A}(k+j),\boldsymbol{A}_{\rm d}(k+j),\boldsymbol{B}(k+j)} J(k)\leq V[\boldsymbol{x}(k|k)]. \tag{12}$$

The robust MPC problem is reduced to the following minimization problem

$$\min_{\boldsymbol{u}(k+j|k),\, j=0,1,\cdots,\infty} V[\boldsymbol{x}(k|k)] \tag{13}$$

subject to Eqs.(2), (6) and (10).

Theorem 1 Let $\boldsymbol{x}(k|k)$ be the state of the uncertain fuzzy system with time-delay measured at the sampling time k, $\boldsymbol{u}(k+j|k)=\boldsymbol{K}\boldsymbol{x}(k+j|k)$ ($j\geq 0$) be the state feedback predictive control law satisfying the Euclidean norm constraints $\|\boldsymbol{u}(k+j|k)\|_2\leq u_{\max}$, then the state feedback matrix $\boldsymbol{K}$ in the control law that minimizes the upper bound $V[\boldsymbol{x}(k|k)]$ on the robust performance objective function $J(k)$ at the sampling time k is given by

$$\boldsymbol{K}=\boldsymbol{Y}\bar{\boldsymbol{P}}^{-1}, \tag{14}$$

where $\bar{\boldsymbol{P}}>0$ and $\boldsymbol{Y}$ are obtained from the solution (if it exists) of the following linear objective minimization problem

$$\min_{\substack{\gamma,\,\bar{\boldsymbol{P}},\,\boldsymbol{Y},\,\boldsymbol{u}(k+j|k)\\ j=0,\cdots,\infty}} \gamma \tag{15}$$

subject to

$$\begin{bmatrix} 1 & \boldsymbol{x}^{\mathrm{T}}(k|k) & \boldsymbol{x}^{\mathrm{T}}(k-1|k) & \cdots & \boldsymbol{x}^{\mathrm{T}}(k-d|k) \\ \boldsymbol{x}(k|k) & \bar{\boldsymbol{P}} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{x}(k-1|k) & \boldsymbol{0} & \bar{\boldsymbol{P}}_{\mathrm{d}} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{x}(k-d|k) & \boldsymbol{0} & \boldsymbol{0} & \cdots & \bar{\boldsymbol{P}}_{\mathrm{d}} \end{bmatrix} \geq \boldsymbol{0} \tag{16}$$

$$\begin{bmatrix} \bar{\boldsymbol{P}} & \bar{\boldsymbol{P}} & \bar{\boldsymbol{P}}\boldsymbol{Q}^{1/2} & \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{R}^{1/2} & \mu_i\bar{\boldsymbol{P}} & \mu_i\boldsymbol{Y}^{\mathrm{T}} & (\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{Y})^{\mathrm{T}} & \boldsymbol{0} \\ \bar{\boldsymbol{P}} & \bar{\boldsymbol{P}}_{\mathrm{d}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{Q}^{1/2}\bar{\boldsymbol{P}} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{R}^{1/2}\boldsymbol{Y} & \boldsymbol{0} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \mu_i\bar{\boldsymbol{P}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \mu_i\boldsymbol{Y} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ (\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{Y}) & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \bar{\boldsymbol{P}}-\gamma\boldsymbol{I} & \boldsymbol{A}_{\mathrm{d}i}^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_{\mathrm{d}i} & \bar{\boldsymbol{P}}_{\mathrm{d}}^{-1} \end{bmatrix}$$

$$\geq \boldsymbol{0}, \quad i=1, \ldots, N, \tag{17}$$

$$\begin{bmatrix} u_{\max}^2\boldsymbol{I} & \boldsymbol{Y} \\ \boldsymbol{Y}^{\mathrm{T}} & \bar{\boldsymbol{P}} \end{bmatrix} \geq 0. \tag{18}$$

Proof The minimization of $V[\boldsymbol{x}(k|k)]$ is equivalent to the linear objective minimization problem of the following LMIs problem

$$\min_{\substack{\gamma, \bar{\boldsymbol{P}}, \boldsymbol{Y}, \boldsymbol{u}(k+j|k) \\ j=0,\cdots,\infty}} \gamma$$

subject to

$$\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}\boldsymbol{x}(k)+\sum_{l=1}^{d}\boldsymbol{x}^{\mathrm{T}}(k-l)\boldsymbol{P}_{\mathrm{d}}\boldsymbol{x}(k-l) \leq \gamma. \tag{19}$$

After substituting $\bar{\boldsymbol{P}}=\gamma\boldsymbol{P}^{-1}$, $\bar{\boldsymbol{P}}_{\mathrm{d}}=\gamma\boldsymbol{P}_{\mathrm{d}}^{-1}$ into Eq.(19), and using Schur complements (Boyd *et al.*, 1994), we can obtain Eq.(16).

To obtain Eq.(17), substitute $\boldsymbol{x}(k+j|k)$, $\boldsymbol{u}(k+j|k)=\boldsymbol{K}\boldsymbol{x}(k+j|k)$ ($j\geq 0$) into Eq.(10) to get

$$\boldsymbol{M}^{\mathrm{T}}\begin{bmatrix} \bar{\boldsymbol{A}}_{11} & \bar{\boldsymbol{A}}_k^{\mathrm{T}}(j)\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j) & \bar{\boldsymbol{A}}_k^{\mathrm{T}}(j)\boldsymbol{P} \\ \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P}\bar{\boldsymbol{A}}_k(j) & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j)-\boldsymbol{P}_{\mathrm{d}} & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P} \\ \boldsymbol{P}\bar{\boldsymbol{A}}_k(j) & \boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j) & \boldsymbol{P} \end{bmatrix}\boldsymbol{M}$$

$$\leq 0, \tag{20}$$

where $\boldsymbol{M}=\begin{bmatrix} \boldsymbol{x}_k(j) \\ \boldsymbol{x}_{k-d}(j) \\ f[\boldsymbol{x}_k(j)] \end{bmatrix}$. $\bar{\boldsymbol{A}}_{11}=\bar{\boldsymbol{A}}_k^{\mathrm{T}}(j)\boldsymbol{P}\bar{\boldsymbol{A}}_k(j)+\boldsymbol{P}_{\mathrm{d}}-\boldsymbol{P}+$ $+\boldsymbol{Q}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{K}$, $\bar{\boldsymbol{A}}_k(j)=\boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{K}$. $\boldsymbol{x}_k(j)$, $\boldsymbol{x}_{k-d}(j)$, $\boldsymbol{A}_k(j)$, $\boldsymbol{A}_{\mathrm{d}k}(j)$, $\boldsymbol{B}_k(j)$ are the simplified expressions of $\boldsymbol{x}(k+j|k)$, $\boldsymbol{x}(k-d+j|k)$, $\boldsymbol{A}(k+j)$, $\boldsymbol{A}_{\mathrm{d}}(k+j)$, $\boldsymbol{B}(k+j)$ respectively.

From Eq.(3), considering $\boldsymbol{u}(k+j|k)=\boldsymbol{K}\boldsymbol{x}(k+j|k)$, we can obtain

$$f^{\mathrm{T}}[\boldsymbol{x}_k(j)]f[\boldsymbol{x}_k(j)]-\mu^2\boldsymbol{x}_k^{\mathrm{T}}(j)(\boldsymbol{I}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{x}_k(j)\leq 0, \tag{21}$$

Eq.(21) is equivalent to

$$\boldsymbol{M}^{\mathrm{T}}\begin{bmatrix} -\mu^2(\boldsymbol{I}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}\boldsymbol{M} \leq 0. \tag{22}$$

From S-procedure (Boyd *et al.*, 1994), we can know that if Eqs.(20) and (22) are both satisfied, and that $\forall\varepsilon>0$ exists, Eq.(23) can be obtained as

$$\begin{bmatrix} \bar{\boldsymbol{A}}_{11}-\varepsilon\mu^2(\boldsymbol{I}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}) & \bar{\boldsymbol{A}}_k^{\mathrm{T}}(j)\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j) & \bar{\boldsymbol{A}}_k^{\mathrm{T}}(j)\boldsymbol{P} \\ \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P}\bar{\boldsymbol{A}}_k(j) & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j)-\boldsymbol{P}_{\mathrm{d}} & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P} \\ \boldsymbol{P}\bar{\boldsymbol{A}}_k(j) & \boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j) & \boldsymbol{P}-\varepsilon\boldsymbol{I} \end{bmatrix} \leq \boldsymbol{0}, \tag{23}$$

with ε^{-1}, $\varepsilon^{-1}\boldsymbol{P}_{\mathrm{d}}$, $\varepsilon^{-1}\boldsymbol{Q}$, $\varepsilon^{-1}\boldsymbol{R}$ being replaced with $\boldsymbol{P}$, $\boldsymbol{P}_{\mathrm{d}}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ respectively to eliminate variable ε, Eq.(23) further becomes

$$\begin{bmatrix} \bar{\boldsymbol{A}}_{11}-\mu^2(\boldsymbol{I}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}) & \bar{\boldsymbol{A}}_k^{\mathrm{T}}(j)\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j) & \bar{\boldsymbol{A}}_k^{\mathrm{T}}(j)\boldsymbol{P} \\ \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P}\bar{\boldsymbol{A}}_k(j) & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j)-\boldsymbol{P}_{\mathrm{d}} & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\boldsymbol{P} \\ \boldsymbol{P}\bar{\boldsymbol{A}}_k(j) & \boldsymbol{P}\boldsymbol{A}_{\mathrm{d}k}(j) & \boldsymbol{P}-\boldsymbol{I} \end{bmatrix} \leq \boldsymbol{0}. \tag{24}$$

Considering $\bar{\boldsymbol{A}}_{11}$ and $\bar{\boldsymbol{A}}_k(j)$, using Schur complements, Eq.(24) is equivalent to

$$\begin{bmatrix} \boldsymbol{P}-\boldsymbol{P}_{\mathrm{d}}-\boldsymbol{Q}-\boldsymbol{K}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{K}-\mu^2(\boldsymbol{I}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_{\mathrm{d}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} -$$

$$\begin{bmatrix} [\boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{K}]^{\mathrm{T}} \\ \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j) \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{P}[\boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{K}\boldsymbol{A}_{\mathrm{d}k}(j)\boldsymbol{I}] \geq \boldsymbol{0}. \tag{25}$$

Further using Schur complements shows that Eq.(25) is equivalent to

$$\begin{bmatrix} \boldsymbol{N} & \boldsymbol{0} & \boldsymbol{0} & [\boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{K}]^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{P}_{\mathrm{d}} & \boldsymbol{0} & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j) \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{I} \\ \boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{K} & \boldsymbol{A}_{\mathrm{d}k}(j) & \boldsymbol{I} & \boldsymbol{P}^{-1} \end{bmatrix} \geq \boldsymbol{0} \tag{26}$$

where

$$\boldsymbol{N} = \boldsymbol{P} - \boldsymbol{P}_{\mathrm{d}} - \boldsymbol{Q} - \boldsymbol{K}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{K} - \mu^2(\boldsymbol{I} + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}).$$

Eq.(26) is further rewritten to

$$\begin{bmatrix} \boldsymbol{N} & [\boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{K}]^{\mathrm{T}} \\ \boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{K} & \overline{\boldsymbol{P}}^{-1} - \boldsymbol{I} - \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j)\overline{\boldsymbol{P}}_{\mathrm{d}}^{-1}\boldsymbol{A}_{\mathrm{d}k}(j) \end{bmatrix} \geq \boldsymbol{0}. \tag{27}$$

Substitute $\overline{\boldsymbol{P}} = \gamma\boldsymbol{P}^{-1}$, $\overline{\boldsymbol{P}}_{\mathrm{d}} = \gamma\boldsymbol{P}_{\mathrm{d}}^{-1}$, $\boldsymbol{Y} = \boldsymbol{K}\overline{\boldsymbol{P}}$ into Eq.(27), and after pre- and post-multiplying by $\begin{bmatrix} \overline{\boldsymbol{P}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$, using Schur complements, we can obtain Eq.(28) (see below).

Since Eq.(28) is affine on $\boldsymbol{A}_i$, $\boldsymbol{A}_{\mathrm{d}i}$, $\boldsymbol{B}_i$, μ_i (i=1, …, N) and in prediction horizon the strategy of single-step linearization on T-S model is used, then Eq.(28) is equivalent to Eq.(17).

To obtain Eq.(18), define an invariant set of the state of the uncertain system

$$\boldsymbol{x}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}^{-1}\boldsymbol{x}(k) + \sum_{l=1}^{d}\boldsymbol{x}^{\mathrm{T}}(k-l)\overline{\boldsymbol{P}}_{\mathrm{d}}^{-1}\boldsymbol{x}(k-l) \leq 1. \tag{29}$$

Considering $\overline{\boldsymbol{P}} > 0$, $\overline{\boldsymbol{P}}_{\mathrm{d}} > 0$, from Eq.(29), we can obtain

$$\boldsymbol{x}^{\mathrm{T}}(k\,|\,k)\overline{\boldsymbol{P}}^{-1}\boldsymbol{x}(k\,|\,k) \leq 1. \tag{30}$$

For the predictive control input series $\boldsymbol{u}(k+j|k)$ ($j\geq0$) satisfying constraints condition over the prediction horizon, we have

$$\max_{j\geq0} \|\boldsymbol{u}(k+j\,|\,k)\|_2^2 = \max_{j\geq0} \|\boldsymbol{K}\boldsymbol{x}(k+j\,|\,k)\|_2^2 \leq \lambda_{\max}(\overline{\boldsymbol{P}}^{-1/2}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\overline{\boldsymbol{P}}^{-1/2}) \leq u_{\max}^2. \tag{31}$$

Using Schur complements, we obtain

$$\begin{bmatrix} u_{\max}^2\boldsymbol{I} & \boldsymbol{K}\overline{\boldsymbol{P}} \\ \overline{\boldsymbol{P}}\boldsymbol{K}^{\mathrm{T}} & \overline{\boldsymbol{P}} \end{bmatrix} \geq \boldsymbol{0}. \tag{32}$$

Substitute $\boldsymbol{Y} = \boldsymbol{K}\overline{\boldsymbol{P}}$, Eq.(32) is equivalent to Eq.(18).

So by solving the linear objective minimization problem (15) with LMIs constraints (16), (17), (18), the state feedback predictive control law is obtained, and only the first computed control move $\boldsymbol{u}(k|k)$ is implemented. At the next sampling time, the optimization problem is solved again with new measurements from the plant.

Lemma 1 (Feasibility) Any feasible solution of the optimization in Theorem 1 at time k is also feasible for all time $t>k$, thus if the optimization problem in Theorem 1 is feasible at time k then it is feasible for all time $t>k$.

Proof See (Kothare *et al.*, 1996).

$$\begin{bmatrix} \overline{\boldsymbol{P}} & \overline{\boldsymbol{P}} & \overline{\boldsymbol{P}}\boldsymbol{Q}^{1/2} & \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{R}^{1/2} & \mu\overline{\boldsymbol{P}} & \mu\boldsymbol{Y}^{\mathrm{T}} & [\boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{Y}]^{\mathrm{T}} & \boldsymbol{0} \\ \overline{\boldsymbol{P}} & \overline{\boldsymbol{P}}_{\mathrm{d}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{Q}^{1/2}\overline{\boldsymbol{P}} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{R}^{1/2}\boldsymbol{Y} & \boldsymbol{0} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \mu\overline{\boldsymbol{P}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \mu\boldsymbol{Y} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \gamma\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{A}_k(j)+\boldsymbol{B}_k(j)\boldsymbol{Y} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{P}}-\gamma\boldsymbol{I} & \boldsymbol{A}_{\mathrm{d}k}^{\mathrm{T}}(j) \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_{\mathrm{d}k}(j) & \overline{\boldsymbol{P}}_{\mathrm{d}}^{-1} \end{bmatrix} \geq \boldsymbol{0}. \tag{28}$$

Theorem 2 (Robust stability) The feasible receding horizon state feedback control law obtained from Theorem 1 robustly asymptotically stabilizes the closed-loop system.

Proof From the existence of the feasible solution of the optimization problem in Theorem 1 and feasibility in Lemma 1, we can know the feasible state feedback predictive control law $\boldsymbol{u}(k+j|k)=\boldsymbol{Kx}(k+j|k)$ ($j\geq 0$) guaranteeing that the Lyapunov function $V[\boldsymbol{x}(k|k)]$ is strictly decreasing in the closed-loop system, so we can conclude that the closed-loop system is asymptotically stable. That is $\boldsymbol{x}(\infty|k)\rightarrow 0$.

SIMULATION STUDY

Consider the first order irreversible, exothermic reaction $A\rightarrow B$ carried out in a well-mixed continuously stirred tank reactor (CSTR). The material and energy balance equations of CSTR are described as follows (Cao and Frank, 2000)

$$V\frac{\mathrm{d}A}{\mathrm{d}t}=\lambda_q A_0+q(1-\lambda)A(t-\alpha)-qA(t)$$
$$-VK_0A(t)\exp\left(\frac{-E}{RT(t)}\right), \tag{33}$$

$$VC_\rho\frac{\mathrm{d}T}{\mathrm{d}t}=qC_\rho[\lambda T_0+(1-\lambda)T(t-\alpha)-T(t)]$$
$$+V(-\Delta H)K_0A(t)\exp\left(\frac{-E}{RT(t)}\right), \tag{34}$$

where $A(t)=\phi_1(t)$ and $T(t)=\phi_2(t)$ for $t\in[-\alpha, 0]$, $A(t)$ is the concentration of chemical A, $T(t)$ is the reactor temperature. The remaining constants are defined in (Cao and Frank, 2000).

Let

$$x_1(t)=\frac{A_0-A(t)}{A_0},\ x_2(t)=\frac{T(t)-T_0}{T_0}\left(\frac{-E}{RT(t)}\right),$$

$$\theta_1(t)=\frac{A_0-\phi_1(t)}{A_0},\ \theta_2(t)=\frac{\phi_2(t)-T_0}{T_0}\left(\frac{-E}{RT(t)}\right),$$

$$t_{\text{new}}=\frac{t}{v},\ v=\frac{V}{q\lambda},\ \tau=\frac{\alpha}{v},\ \gamma_0=\frac{E}{RT(t)},\ \beta=\frac{Uv}{VC_\rho},$$

$$D_\alpha=K_0v\exp(-\gamma_0),\ \ H=-\Delta HA_0E\big/(C_\rho T_0^2R),$$

$$u(t)=\frac{T_0-T_\text{w}}{T_0}\left(\frac{-E}{RT(t)}\right).$$

Then CSTR process can be described in the following dimensionless forms

$$\dot{x}_1(t)=g_1(\boldsymbol{x})+\left(\frac{1}{\lambda}-1\right)x_1(t-\tau), \tag{35}$$

$$\dot{x}_2(t)=g_2(\boldsymbol{x})+\left(\frac{1}{\lambda}-1\right)x_2(t-\tau)+\beta u(t), \tag{36}$$

where $\boldsymbol{x}(t)=[x_1(t)\ \ x_2(t)]^\mathrm{T}$, τ is the state time delay,

$$g_1(\boldsymbol{x})=D_\alpha[1-x_1(t)]\exp\left[\frac{x_2(t)}{1+x_2(t)/\gamma_0}\right]-\frac{1}{\lambda}x_1(t),$$

$$g_2(\boldsymbol{x})=HD_\alpha[1-x_1(t)]\exp\left[\frac{x_2(t)}{1+x_2(t)/\gamma_0}\right]-\left(\frac{1}{\lambda}+\beta\right)x_2(t),$$

$$x_i(t)=\theta_i(t),\ \ i=1, 2;\ \ t\in[-\tau, 0].$$

The parameters are given as $\gamma_0=20$, $H=8$, $\beta=0.3$, $D_\alpha=0.072$, $\lambda=0.8$, $\tau=1$.

It is easy to find three steady states when $u=0$, they are $\boldsymbol{x}_{\text{d1}}=(0.1440, 0.8862)$, $\boldsymbol{x}_{\text{d2}}=(0.4472, 2.7520)$, $\boldsymbol{x}_{\text{d3}}=(0.7646, 4.7052)$.

In (Cao and Frank, 2000), the continuous T-S fuzzy model of the above CSTR process is given for any expected operating point ($\boldsymbol{x}_\text{d}$, $\boldsymbol{u}_\text{d}$), which is a stationary point of the nonlinear system. We assume that the system is perturbed by a bounded nonlinear perturbation (Wang *et al.*, 2004) because the modelling and linearization of the system bring about uncertainties. Let the sampling time $T_\text{s}=0.2$ s, here the continuous fuzzy models are discrete, we obtain:

Rule 1 IF the temperature is low [i.e. $x_2(k)$ is about 0.8862], THEN

$$\hat{\boldsymbol{x}}(k+1)=\boldsymbol{A}_1\hat{\boldsymbol{x}}(k)+\boldsymbol{A}_{\text{d1}}\hat{\boldsymbol{x}}(k-d)+\boldsymbol{B}_1\hat{\boldsymbol{u}}(k)+f_1[\hat{\boldsymbol{x}}(k)].$$

Rule 2 IF the temperature is medium [i.e. $x_2(k)$ is about 2.752], THEN

$$\hat{\boldsymbol{x}}(k+1)=\boldsymbol{A}_2\hat{\boldsymbol{x}}(k)+\boldsymbol{A}_{\text{d2}}\hat{\boldsymbol{x}}(k-d)+\boldsymbol{B}_2\hat{\boldsymbol{u}}(k)+f_2[\hat{\boldsymbol{x}}(k)].$$

Rule 3 IF the temperature is high [i.e., $x_2(k)$ is about 4.7052], THEN

$$\hat{\boldsymbol{x}}(k+1)=\boldsymbol{A}_3\hat{\boldsymbol{x}}(k)+\boldsymbol{A}_{d3}\hat{\boldsymbol{x}}(k-d)+\boldsymbol{B}_3\hat{\boldsymbol{u}}(k)+f_3[\hat{\boldsymbol{x}}(k)].$$

The model parameters $\boldsymbol{A}_i$, $\boldsymbol{A}_{di}$, $\boldsymbol{B}_i$ (i=1, 2, 3) can be calculated by the following formulas

$$\boldsymbol{A}_i=\boldsymbol{I}+\tilde{\boldsymbol{A}}_iT_s, \boldsymbol{A}_{di}=(e^{\tilde{A}_iT_s}-\boldsymbol{I})\tilde{\boldsymbol{A}}_i^{-1}\tilde{\boldsymbol{A}}_{di}, \boldsymbol{B}_i=(e^{\tilde{A}_iT_s}-\boldsymbol{I})\tilde{\boldsymbol{A}}_i^{-1}\tilde{\boldsymbol{B}}_i, \tag{37}$$

where $\hat{\boldsymbol{x}}(k)=\boldsymbol{x}(k)-\boldsymbol{x}_d$, $\hat{\boldsymbol{x}}(k-d)=\boldsymbol{x}(k-d)-\boldsymbol{x}_d$, $\hat{\boldsymbol{u}}(k)=\boldsymbol{u}(k)-\boldsymbol{u}_d$. $\tilde{\boldsymbol{A}}_i$, $\tilde{\boldsymbol{A}}_{di}$, $\tilde{\boldsymbol{B}}_i$ are parameters of the continuous fuzzy models (Cao and Frank, 2000). Using Eq.(37), we can obtain

$$\boldsymbol{A}_1=\begin{bmatrix}0.7145 & 0.0151\\ 0.2838 & -0.1888\end{bmatrix},\ \boldsymbol{A}_2=\begin{bmatrix}0.5895 & 0.0792\\ -0.2813 & 0.2234\end{bmatrix},$$

$$\boldsymbol{A}_3=\begin{bmatrix}0.0994 & 0.0633\\ -5.2454 & 1.1967\end{bmatrix},\ \boldsymbol{A}_{d1}=\begin{bmatrix}0.05 & 0\\ -0.3358 & 0.2875\end{bmatrix},$$

$$\boldsymbol{A}_{d2}=\begin{bmatrix}0.05 & 0\\ 1.7404 & -0.5461\end{bmatrix},\ \boldsymbol{A}_{d3}=\begin{bmatrix}0.05 & 0\\ 0 & 0.05\end{bmatrix},$$

$$\boldsymbol{B}_1=\begin{bmatrix}0\\ 0.3454\end{bmatrix},\ \boldsymbol{B}_2=\begin{bmatrix}0\\ -0.6554\end{bmatrix},\ \boldsymbol{B}_3=\begin{bmatrix}0\\ 0.06\end{bmatrix}.$$

The time delay $d=\tau/T_s=5$, the perturbation

$$f_i[\hat{\boldsymbol{x}}(k)]=\begin{bmatrix}\mu_i\hat{x}_1(k)\sin(\hat{x}_1(k)+\pi/2)\\ \mu_i\hat{x}_2(k)\sin(\hat{x}_2(k)+\pi/2)\end{bmatrix}$$

with $\mu_1\le0.4352$, $\mu_2\le0.2672$, $\mu_3\le0.0660$.

From the perturbation description, we can know

$$f_i^T(\hat{\boldsymbol{x}})f_i(\hat{\boldsymbol{x}})\le\mu_i^2\|\hat{\boldsymbol{x}}\|_2.$$

The membership functions are shown in Fig.1. The weight matrices are $\boldsymbol{Q}=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ and R=0.02, u_{max}=2. We assume that $\boldsymbol{x}_{d2}$=(0.4472, 2.7520) is the expected operating point. Under different initial condition $\boldsymbol{x}(0)$ and $\boldsymbol{\theta}(t)=\boldsymbol{x}(0)$ for t<0, the simulation results of the open-loop system are shown in Fig.2. From Fig.2, we find that the open-loop state responses may converge to two different steady states. It is shown that the middle steady state of the system is unstable. Figs.3 and 4 give the simulation results of the closed-loop system using robust fuzzy control (RFC) and the proposed robust fuzzy predictive control (RFPC), respectively. The simulation results showed that under different initial conditions the closed-loop state responses can converge to the expected operating point $\boldsymbol{x}_{d2}$=(0.4472, 2.7520) by two control approaches. But the response speed of the closed-loop system based on RFPC is evidently faster than RFC. Fig.5 is the comparison results of errors performance index using RFC (solid line) and RFPC (dashed line).

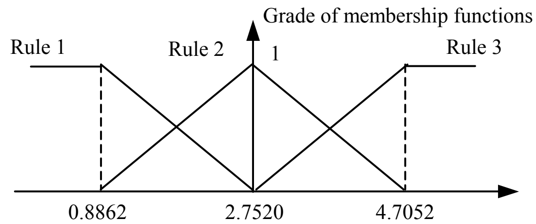

Fig.1 Membership function of CSTR fuzzy model

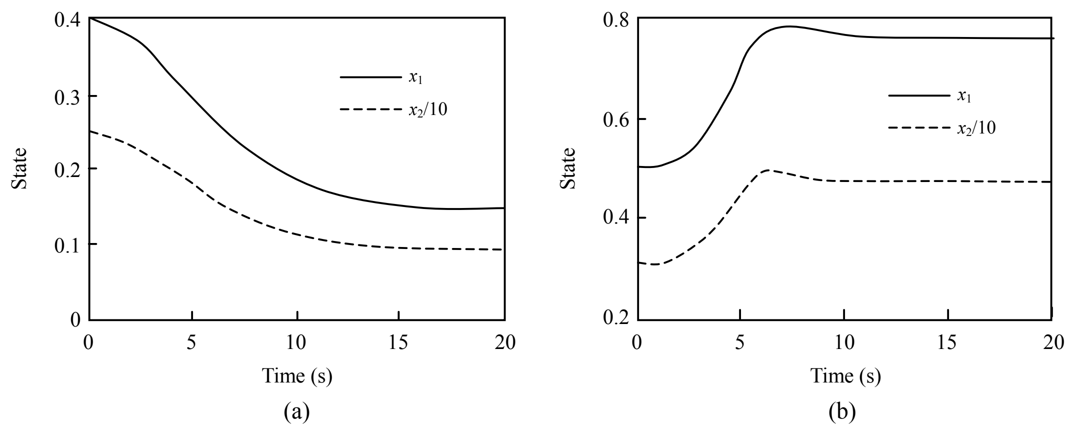

Fig.2 Open-loop responses x_d=(0.4472, 2.7520). (a) Initial state x_0=(0.4, 2.5); (b) Initial state x_0=(0.5, 3.0)

To further show the control performance of the proposed RFPC, we assume that the temperature is the process output, i.e. $y(k)=[0\ 1]\boldsymbol{x}(k)$. Fig.6 gives the control results of tracking the set point changes using classical PID, RFC and RFPC respectively. Obviously a quite satisfactory control result is obtained based on the proposed RFPC.

From these figures, we can see that the control performance of the closed-loop system based on RFPC is obviously better than classical PID and RFC.

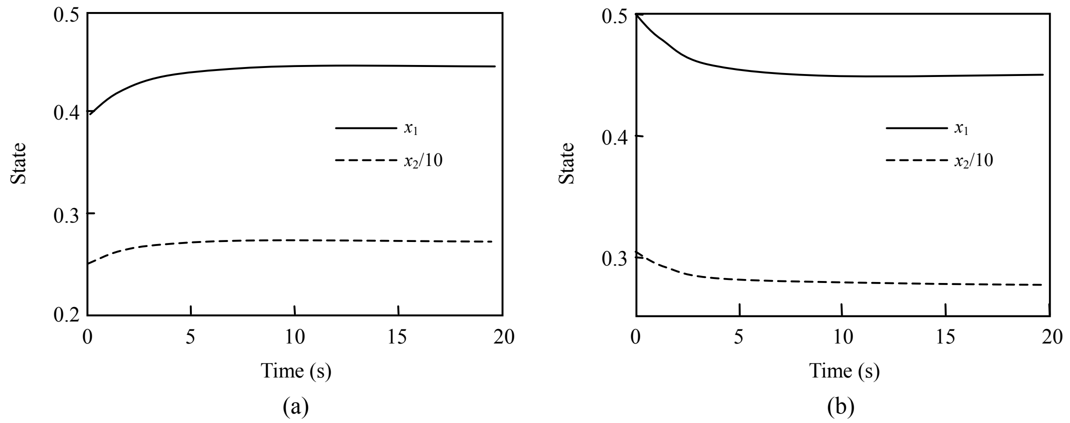

Fig.3 Closed-loop responses via RFC. (a) Initial state x_0=(0.4, 2.5); (b) Initial state x_0=(0.5, 3.0)

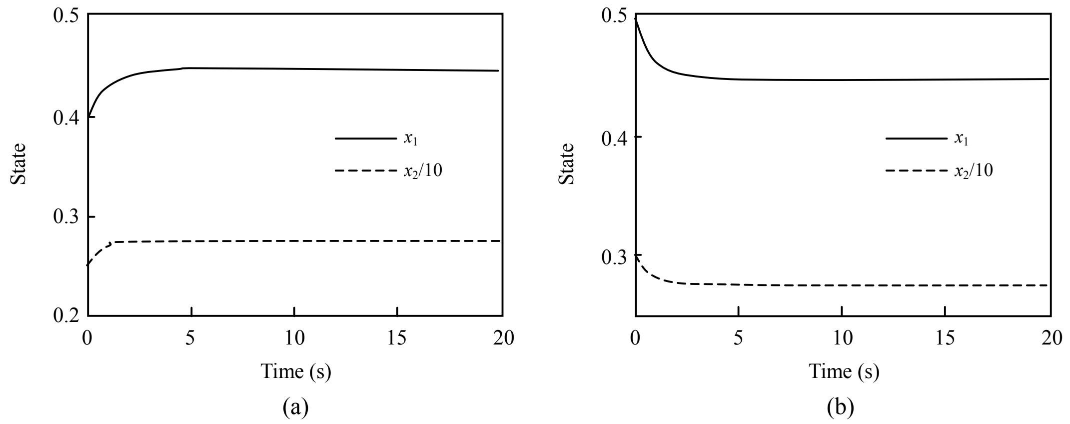

Fig.4 Closed-loop responses via RFPC. (a) Initial state x_0=(0.4, 2.5); (b) Initial state x_0=(0.5, 3.0)

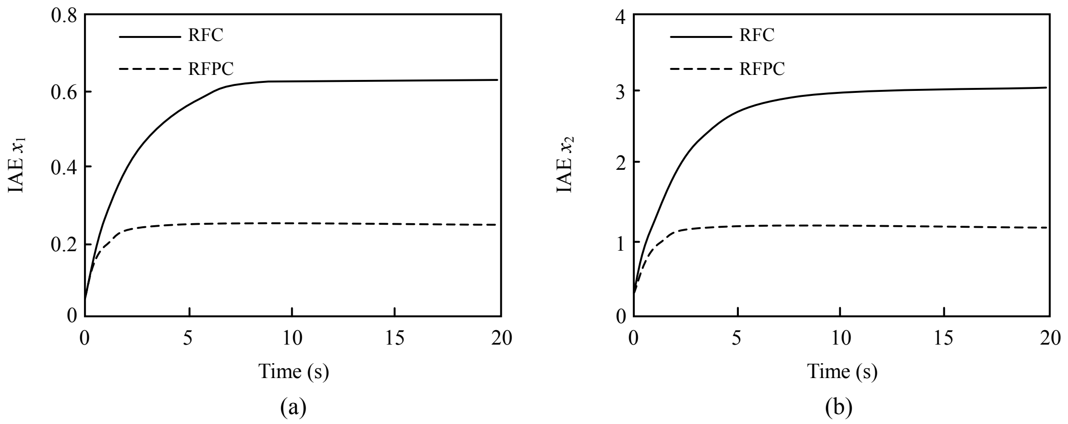

Fig.5 Comparison of error performance index. (a) Initial state x_{10}=0.5; (b) Initial state x_{20}=3.0

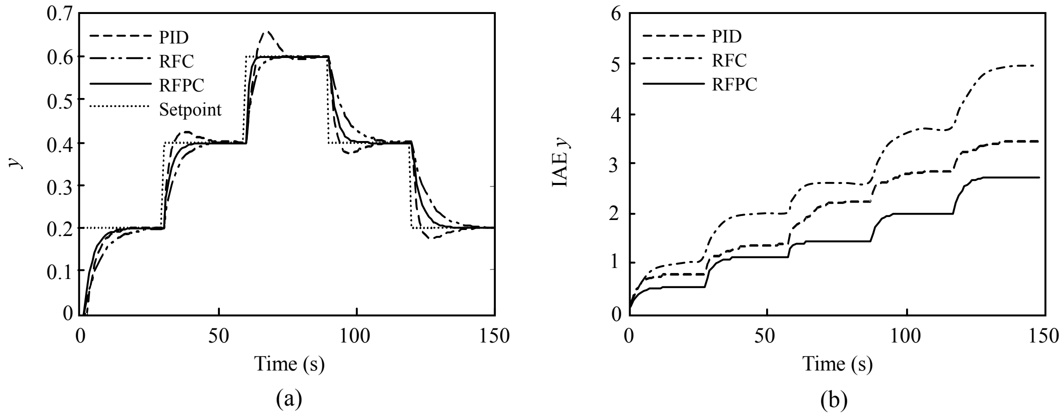

Fig.6 Comparison results of tracking performance. (a) Output responses; (b) Error performance index

CONCLUSION

This paper has presented an extended robust predictive control approach for input constrained discrete uncertain nonlinear systems with time-delay based on a class of uncertain T-S fuzzy models which satisfy sector bound condition. This approach will convert the min-max optimization problem in robust model predictive control into linear objective minimization problem with LMIs constrains. The state feedback control law is obtained by solving convex optimization of a set of LMIs. A sufficient condition for the robust stability of the system is given. CSTR example shows that the proposed approach is an effective control strategy with excellent tracking performance and strong robustness. The results can also easily be extended to the uncertain nonlinear systems with input time-delay and state multiple time-delays. Possible future research work is how to realize robust predictive control when the time delay and uncertain bound of the system are unknown.

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Editors-in-Chief: Pan Yun-he
ISSN 1009-3095 (Print); ISSN 1862-1775 (Online), monthly

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