

Journal of Zhejiang University SCIENCE A  
 ISSN 1009-3095 (Print); ISSN 1862-1775 (Online)  
 www.zju.edu.cn/jzus; www.springerlink.com  
 E-mail: jzus@zju.edu.cn



**Science Letters:**

## Dual structure of composite right/left-handed transmission line\*

HU Xin<sup>†1,2</sup>, ZHANG Pu<sup>1</sup>, HE Sailing<sup>†‡1,2</sup>

<sup>(1)</sup>Center for Optical and Electromagnetic Research, Zhejiang University, Hangzhou 310058, China)

<sup>(2)</sup>Division of Electromagnetic Engineering, School of Electrical Engineering, Royal Institute of Technology, S-100 44 Stockholm, Sweden)

<sup>†</sup>E-mail: huxin@coer.zju.edu.cn; sailing@kth.se

Received Aug. 22, 2006; revision accepted Sept. 1, 2006

**Abstract:** A dual structure of composite right/left handed (CRLH) transmission line (TL) is analyzed in which an inductance  $L_R$  is in parallel with a capacitance  $C_L$  and a shunt capacitance  $C_R$  is in series with an inductance  $L_L$ . Both the distributed and lumped cases are considered. The dispersion diagram and transmission properties of the dual CRLH TL are given and compared with those of a standard CRLH TL. Contrary to the frequency response of a standard CRLH TL, a dual CRLH TL has a left-handed (negative phase shift) band at higher frequencies and a right-handed (positive phase shift) band at lower frequencies. A novel dual-band balun is presented as an application.

**Key words:** Dual structure, Composite right/left handed transmission line (CRLH TL), Dual band balun

doi:10.1631/jzus.2006.A1777

Document code: A

CLC number: O44

### INTRODUCTION

An artificial dielectric medium that exhibits simultaneously negative electric permittivity and magnetic permeability, also known as a left-handed (LH) material, was first envisaged by Veselago, who theoretically predicted that such a medium would exhibit a negative refractive index (NRI) (Veselago, 1968). An artificial NRI medium exhibiting backward-wave propagation characteristics (and therefore a negative refractive index) was first realized experimentally in (Shelby *et al.*, 2001) using a volumetric structure with thin wire strips and split-ring resonators. A planar NRI medium was later realized by periodically loading a conventional transmission line (TL) with lumped series capacitors ( $C_L$ ) and shunt inductors ( $L_R$ ) in a dual-TL (high-pass) configuration (Iyer and Eleftheriades, 2002; Eleftheriades *et al.*, 2002; Caloz and Itoh, 2004). A more

general configuration—composite right/left handed (CRLH) TL meta-structure, which includes both right-handed (RH) and LH effects, was proposed and discussed in (Lai *et al.*, 2004).

In this letter, we study a dual structure (hereafter called dual-CRLH or D-CRLH for simplicity) of the standard CRLH TL. In such a D-CRLH TL, an inductance  $L_R$  is in parallel with a capacitance  $C_L$ , and a shunt capacitance  $C_R$  is in series with an inductance  $L_L$ . Contrary to the properties of a standard CRLH, a D-CRLH has an RH frequency response at lower frequencies and an LH frequency response at higher frequencies. Based on the opposite phase shifting property of a CRLH and a D-CRLH, a novel dual-band balun is designed in this letter.

### THEORY

#### Distributed LC network of D-CRLH

Uniform CRLH and D-CRLH TLs (lossless) are equivalent to the distributed LC networks shown in Figs. 1a and 1b, respectively. The frequencies for the series and shunt resonances of the D-CRLH TL are:

<sup>‡</sup> Corresponding author

\* Project supported by the National Basic Research Program (973) of China (No. 2004CB719802), and the Science and Technology Department of Zhejiang Province (No. 2005C31004), China

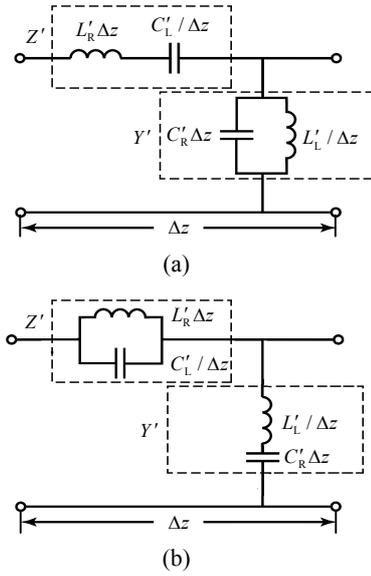


Fig.1 Equivalent circuit model in a distributed LC network. (a) Standard CRLH TL; (b) Dual CRLH TL

$$\omega_{se} = \frac{1}{\sqrt{L'_R C'_L}}, \quad \omega_{sh} = \frac{1}{\sqrt{L'_L C'_R}} \quad (1)$$

In the balanced case (Lai et al., 2004), the series and shunt resonance frequencies are equal, i.e.,  $L'_R C'_L = L'_L C'_R$ .

The propagation constant of a TL is given by

$$\gamma = \alpha + j\beta = \sqrt{Z'Y'}, \quad (2)$$

where  $Z'$  and  $Y'$  are the per-unit length impedance and admittance, respectively. In D-CRLH TL,  $Z'$  and  $Y'$  are defined as

$$Z' = \frac{1}{j\omega C'_L - j(\omega L'_R)^{-1}}, \quad Y' = \frac{1}{j\omega L'_L - j(\omega C'_R)^{-1}} \quad (3)$$

Therefore, the dispersion relation for a distributed D-CRLH TL is

$$\beta = S(\omega) \left[ \left( \omega C'_L - \frac{1}{\omega L'_R} \right) \left( \omega L'_L - \frac{1}{\omega C'_R} \right) \right]^{-1/2}, \quad (4)$$

where

$$S(\omega) = \begin{cases} -1, & \omega > \omega_2, \\ +1, & 0 < \omega < \omega_1, \end{cases} \quad (5)$$

$$\omega_1 = \min(\omega_{se}, \omega_{sh}), \quad \omega_2 = \max(\omega_{se}, \omega_{sh}).$$

Note that when  $\omega \in (\omega_1, \omega_2)$  one has  $S(\omega)=0$  (i.e.,  $\beta=0$ ; however,  $\alpha$  is not zero, and consequently impossible for propagation). The dispersion diagram for D-CRLH TLs is given in Fig.2 showing that in the balanced case the stop band vanishes and the transition from LH to RH occurs at the transition frequency

$$\omega_0 = \sqrt{\omega_{se} \omega_{sh}} \quad (6)$$

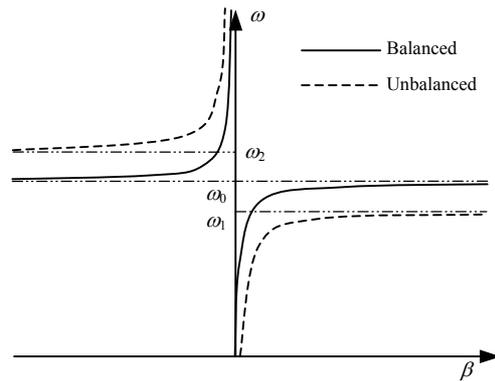


Fig.2 Dispersion diagram of the distributed D-CRLH TL

For a D-CRLH TL, the characteristic impedance is

$$Z_c = \sqrt{\frac{Z'}{Y'}} = \sqrt{\frac{L'_R (\omega^2 L'_L C'_R - 1)}{C'_R (\omega^2 L'_R C'_L - 1)}} \quad (7)$$

In the balanced case, the characteristic impedance ( $Z_c = \sqrt{L'_L / C'_L} = \sqrt{L'_R / C'_R}$ ) is frequency independent, and thus can be matched over a wide band [as the case of CRLH TL in (Lai et al., 2004)].

### Lumped LC network of D-CRLH

An effectively uniform D-CRLH TL can be constructed by periodically cascading the LC unit cell, as shown in Fig.3.

The dispersion relation for a lumped D-CRLH TL is

$$\beta(\omega) = \frac{1}{p} \arccos(1 + ZY/2), \quad (8)$$

where  $p$  is the physical length of the unit cell, and the series impedance ( $Z$ ) and shunt admittance ( $Y$ ) of the LC unit cell are given by

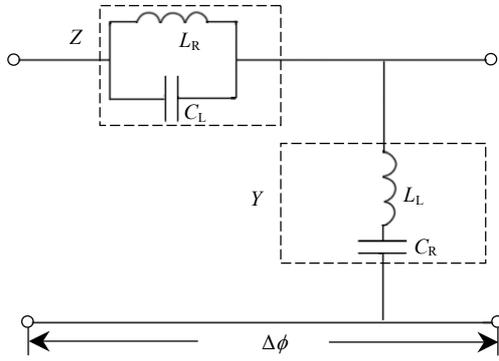


Fig.3 The lumped LC unit cell for a dual CRLH TL

$$Z = \frac{1}{j\omega C_L - j(\omega L_R)^{-1}}, \quad Y = \frac{1}{j\omega L_L - j(\omega C_R)^{-1}}. \quad (9)$$

For the balanced case, the dispersion diagrams of a lumped CRLH TL (dashed line) and a lumped D-CRLH TL (solid line) are given in Fig.4 showing that in a lumped D-CRLH TL a stopband always exists even when the two resonant frequencies are balanced (this is different from the balanced case of distributed D-CRLH TL discussed above). This stopband was utilized to achieve a notch filter in our previous work (Li *et al.*, 2006). Furthermore, D-CRLH has a phase shifting property opposite to that of a standard CRLH, with an LH (negative phase shift) region at higher frequencies and an RH (positive phase shift) region at lower frequencies (below the stopband).

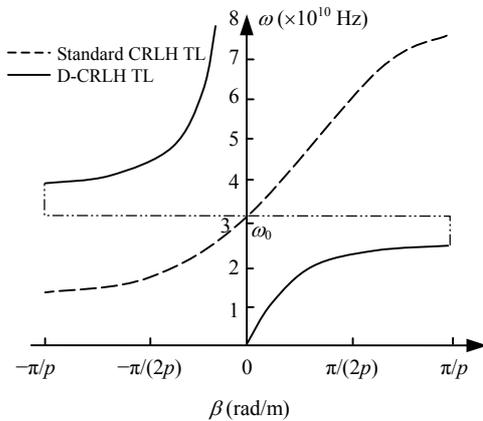


Fig.4 Dispersion diagram for the balanced case of a D-CRLH TL and a standard CRLH TL ( $L_R=L_L=0.01$  nH,  $C_R=C_L=100$  pF)

AN APPLICATION: A DUAL-BAND BALUN

Utilizing the opposite phase shifting property of a D-CRLH and a CRLH, we design here a dual-band balun as shown in Fig.5.

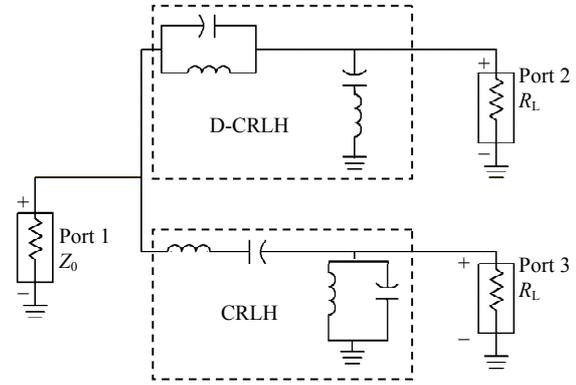


Fig.5 The proposed dual-band balun

The two central frequencies of the dual-band balun are  $\omega_1$  and  $\omega_2$  (e.g. 2.4 GHz and 5 GHz). At  $\omega_1$  and  $\omega_2$ , the top branch is designed to have a phase shift of  $+90^\circ$  ( $+\lambda/4$ ) and  $-90^\circ$  ( $-\lambda/4$ ), respectively, and the bottom branch a phase shift of  $-90^\circ$  and  $+90^\circ$ , respectively. To match the input and output impedances with a  $\lambda/4$  TL, the characteristic impedance of the D-CRLH [see Eq.(7)] and CRLH should satisfy

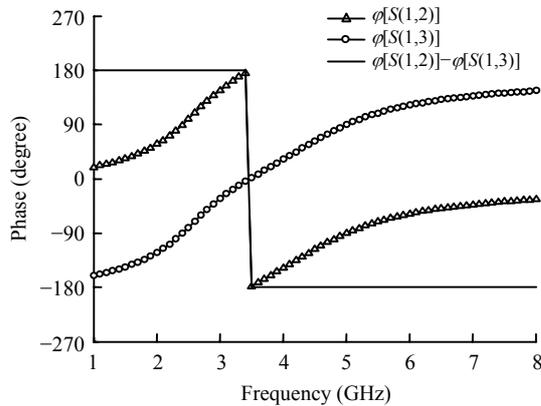
$$Z_c = \sqrt{2Z_0 R_L}, \quad (10)$$

where  $Z_0$  and  $R_L$  are the impedances of the input and output ports (both are  $50 \Omega$  in our case), respectively. The specific values of  $L_L$ ,  $L_R$ ,  $C_L$  and  $C_R$  for the balanced D-CRLH and the balanced CRLH are given in Table 1.

Table 1 Parameters used in the proposed dual-band balun

	$L_R$ (nH)	$L_L$ (nH)	$C_R$ (pF)	$C_L$ (pF)
D-CRLH	2.43	4.33	0.49	0.87
CRLH	4.33	2.43	0.87	0.49

Fig.6 shows the simulated  $S$  parameters of the proposed balun. The solid curve of Fig.6 shows that the difference between phases  $\phi[S(1,3)]$  and  $\phi[S(1,2)]$  remains  $+180^\circ$  (below the transition frequency) or  $-180^\circ$  (above the transition frequency) over a wide band (over 3 GHz).



**Fig.6** Phase responses at the two outputs of the proposed dual-band balun

## CONCLUSION

A dual structure of CRLH transmission line, which consists of an inductance  $L_R$  in parallel with a capacitance  $C_L$  and a shunt capacitance  $C_R$  in series with an inductance  $L_L$ , has been analyzed. Unlike the standard CRLH transmission line, the D-CRLH transmission line has an RH frequency response at lower frequencies and an LH frequency response at higher frequencies. The lumped D-CRLH has a stopband (non-zero even for the balanced case) around the RH-LH transition frequency. As an application of this D-CRLH TL, a novel dual-band balun is presented.

## References

- Caloz, C., Itoh, T., 2004. Transmission line approach of left-handed (LH) materials and microstrip implementation of an artificial LH transmission line. *IEEE Trans. Antennas Propag.*, **52**(5):1159-1166. [doi:10.1109/TAP.2004.827249]
- Eleftheriades, G.V., Iyer, A.K., Kremer, P.C., 2002. Planar negative refractive index media using periodically L-C loaded transmission lines. *IEEE Trans. Micro. Theory Tech.*, **50**(12):2702-2712. [doi:10.1109/TMTT.2002.805197]
- Iyer, A.K., Eleftheriades, G.V., 2002. Negative refractive index metamaterials supporting 2D waves. *Proc. IEEE Int. Symp. Microwave Theory and Tech.*, **2**:1067-1070. [doi:10.1109/MWSYM.2002.1011823]
- Lai, A., Caloz, C., Itoh, T., 2004. Composite right/left-handed transmission line metamaterials. *IEEE Microwave Magazine*, **5**(3):34-50. [doi:10.1109/MMW.2004.1337766]
- Li, J.S., Chen, B., Yao, J., Zou, Y.Z., 2006. A novel notch filter utilizing a composite right/left-handed transmission line. *Microwave and Optical Technology Letters*, **48**(3):626-628. [doi:10.1002/mop.21427]
- Shelby, R.A., Smith, D.R., Schultz, S., 2001. Experimental verification of a negative index of refraction. *Science*, **292**(5514):77-79. [doi:10.1126/science.1058847]
- Veselago, V.G., 1968. The electrodynamics of materials with simultaneously negative values of  $\epsilon$  and  $\mu$ . *Sov. Phys. Usp.*, **10**(4):509-514. [doi:10.1070/PU1968v010n04ABEH003699]

Welcome visiting our journal website: <http://www.zju.edu.cn/jzus>

Welcome contributions & subscription from all over the world

The editor would welcome your view or comments on any item in the journal, or related matters

Please write to: Helen Zhang, Managing Editor of JZUS

E-mail: [jzus@zju.edu.cn](mailto:jzus@zju.edu.cn) Tel/Fax: 86-571-87952276/87952331