Journal of Zhejiang University SCIENCE A
ISSN 1009-3095 (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Fatigue reliability analysis of fixed offshore structures: A first passage problem approach

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Received Mar. 8, 2006; revision accepted July 12, 2006

Abstract: This paper describes a methodology for computation of reliability of members of fixed offshore platform structures, with respect to fatigue. Failure criteria were formulated using fracture mechanics principle. The problem is coined as a "first passage problem". The method was illustrated through application to a typical plane frame structure. The fatigue reliability degradation curve established can be used for planning in-service inspection of offshore platforms. A very limited parametric study was carried out to obtain insight into the effect of important variables on the fatigue reliability.

Key words: Reliability, Fatigue, Fracture mechanics, Offshore structures
doi:10.1631/jzus.2006.A1839 **Document code:** A **CLC number:** TU528.57

INTRODUCTION

Offshore production and drilling platforms are, in general, large and complex structural systems, usually fabricated using steel tubular members interconnected through welded joints. These structures are predominantly subjected to oscillatory environmental loads and fatigue characterizes a primary mode of failure of their components. The fatigue damage at any point in the structure depends on the complete stress history during the structure's service life. The calculation of this stress history and its effects on the material is a complex task. The irregular nature of the sea, size of structure, evaluation of stress concentration factors in welded joints and possible dynamic effects, etc. contribute to the complexity of the fatigue life assessment. Due to the inherent random nature of various input parameters affecting the response of these structures, reliability analysis assumes greater importance in the design of offshore structures. Computation of fatigue reliability is also useful for planning in-service inspection of offshore structures and for checking the design and certification.

Any reliability problem can be formulated as probability of limit state violation. In the case of failure due to fatigue this limit state can be defined either as: (1) $a_c-a_N\leq 0$ representing the serviceability limit state or (2) $K_{IC}-K\leq 0$ representing the ultimate limit state (Madsen *et al*., 1986; Kirkemo, 1988). Madhavan Pillai and Meher Prasad (2000) formulated a procedure for fatigue reliability analysis of fixed offshore structures using the serviceability limit state criteria. In this work fracture mechanics principle is used to formulate failure criteria. Relevant literature on computation of reliability analysis applicable to offshore structures are also discussed. In a similar work Rajasankar *et al*.(2003) formulated the failure equation in terms of number of cycles required for failure to occur. The work has been extended to assess the integrity of tubular joints in offshore structure.

In the present work, fatigue reliability of fixed offshore structures is computed using ultimate limit state criteria. The problem is formulated as a 'first passage problem'. The resistance to fatigue failure is represented as a time-variant barrier and failure is assumed to occur when the stress response first crosses this barrier. The probability of this crossing is computed using relevant stochastic theory (Nigam, 1983).

The stress response in offshore structures is a wide band random process, although it can also be modelled as a stationary, narrow band Gaussian process, after applying suitable wide band correction factors (Wirsching and Light, 1980). For a narrow band stress process, the stress ranges follow Rayleigh distribution (Lin, 1976; Nigam, 1983). The long term fatigue stress process in an offshore structure is non-stationary but is modelled as a sequence of several discrete stationary processes. The wave load due to storms acting on the structure during its entire service life is divided into a set of stationary sea states each being described by wave spectra. The probability of occurrence of each sea state is available from sea scatter diagram and is used to account for the long term distribution (Vughts and Kinra, 1976; Chakrabarti, 1987). The statistical properties of structural response due to each sea state is weighed according to its occurrence probability, to account for long term distribution of sea states for the total life of the structure. Finally, an example problem is solved to demonstrate the procedure. Sensitivity of a few important parameters on the reliability is also highlighted through parametric study.

The three major aspects involved in the reliability analysis of offshore jacket structures, namely structural modelling, hydrodynamic force modelling, and uncertainty modelling are briefly presented in the subsequent sections.

STRUCTURAL MODELLING

As the reliability analysis involves repetition of structural analysis using different sea states, the simplified structural model and analysis suggested by Madhavan Pillai and Meher Prasad (2000) is used in the present work.

HYDRODYNAMIC FORCE MODELLING

Sea state model

The sea state is represented by directional wave spectrum given by:

$$S_\eta(\omega,\theta)=S_\eta(\omega)\,D(\omega,\theta), \tag{1}$$

where $D(\omega,\theta)$ is a spreading function to account for wave energy being continuously distributed both with respect to frequency ω and direction θ, with $S_\eta(\omega)$ being unidirectional wave spectrum. In the present work Pierson-Moskowitz (P-M) spectrum and cosine power spreading function (Sarpkaya and Isaacson, 1981) are used to represent a fully developed wind generated sea state.

The P-M spectrum is given by:

$$S_\eta(\omega)=\pi^3\left(\frac{2H_s}{T_z^2}\right)^2\frac{1}{\omega^5}\exp\left[-\pi^3\left(\frac{2}{T_z\omega}\right)^4\right],\ 0<\omega<\infty, \tag{2}$$

where H_s is the significant wave height and T_z is the mean zero crossing period.

The surface displacement and wave kinematics are generated from the directional wave spectrum of Eq.(1) using linear Airy's wave theory and simulation (Borgman, 1969). In order to account for the variation of water level near the surface, the still water level is stretched up to the instantaneous sea surface using Wheeler (1969)'s stretching approach.

Wave force model

For the type of structure considered, the member dimensions being small compared to wavelengths, the presence of structure does not alter the wave field. Hence Morison's equation is adequate for the computation of wave force. Considering the current velocity, the wave force is given by

$$p_h=C_m\rho V_p\ddot{v}+\frac{1}{2}C_d\rho A_p(\dot{v}+\dot{v}_c)\left|\dot{v}+\dot{v}_c\right|, \tag{3}$$

where C_m is inertia coefficient, C_d is drag coefficient, ρ is density of water, V_p and A_p are volume and projected area of member respectively, $\dot{v}$ and $\ddot{v}$ are water particle velocity and acceleration respectively and $\dot{v}_c$ is current velocity.

UNCERTAINTY MODELLING

The limit state equation is formulated using fracture mechanics principle as it gives a more fundamental view of fatigue crack growth than empirical

Palmgren-Miner rule (S-N curve) approach. Flaws or cracks are inherent in any components owing to the manufacturing or fabrication process. The magnitude of stress at the crack tip region depends on stress intensity factor and geometry of crack. They are related as follows:

$$\Delta K = YS\sqrt{\pi a}, \tag{4}$$

where ΔK is the stress intensity factor range, S is the nominal stress range, a is crack size and Y is a geometry function.

The stress intensity factor range ΔK is related to the rate of crack growth per load cycle, using Paris-Ergodan equation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m,\ \Delta K > 0, \tag{5}$$

where C and m are material constants.

Failure is assumed to occur when the stress intensity factor K, at the leading edge of the crack exceeds the fracture toughness K_{IC} of the material. i.e. the failure function is

$$M = K_{\mathrm{IC}} - K = K_{\mathrm{IC}} - YS\sqrt{\pi a}, \tag{6}$$

or failure occurs if

$$S > \frac{K_{\mathrm{IC}}}{Y\sqrt{\pi a}}, \tag{7}$$

where $S=S(t)$, the stress at time t is the far-field stress.

Thus time varying threshold or barrier can be written as:

$$\xi(t) = \frac{K_{\mathrm{IC}}}{Y\sqrt{\pi a(t)}}, \tag{8}$$

where $a(t)$ is the crack size at time t. This is illustrated in Fig.1.

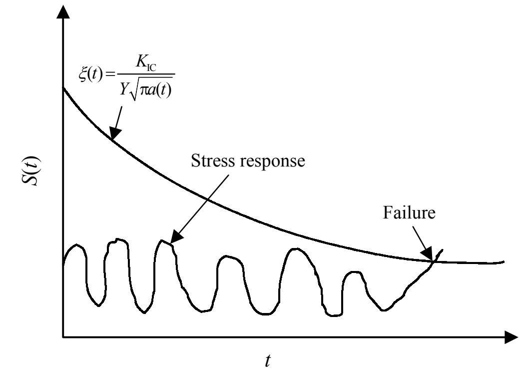

Fig.1 Failure under variable amplitude loading as a first-passage problem

Failure occurs during the time period $[0,T]$ if the stress process $S(t)$ crosses the time varying (Eq.8) in the time interval $[0,T]$ where T can be the service life of the structure.

Combining Eqs.(4) and (5) and using the simplest non-interaction models,

$$\int_{a_0}^{a_N} \frac{\mathrm{d}x}{Y^m(\sqrt{\pi x})^m} = C\sum_{i=1}^{N} S_i^m, \tag{9}$$

where S_i is the stress range in the ith sea state, a_0 is initial crack size and a_N is crack size after N stress cycles.

Accounting for long term stress process, Eq.(9) becomes:

$$\int_{a_0}^{a_c} \frac{\mathrm{d}x}{Y^m(\sqrt{\pi x})^m} = Cf_{0i}Tq_iE[S_i^m], \tag{10}$$

where f_{0i} is zero up-crossing frequency of the stress process, given by $\sqrt{m_{2i}/m_{0i}}$, m_{0i}, m_{2i} being the zeroth and second order moments of the stress spectrum in the ith sea state and q_i is a factor accounting for the fraction of time occupied in the ith sea state, given by γ_i/T_s, γ_i being the probability of occurrence of the ith sea state and T_s is the storm duration (taken as 3 h), a_c is critical crack size after N stress cycles and $E[\cdot]$ is expectation operator.

For Rayleigh distribution with $E[S_i^m]$ being given as:

$$E[S_i^m] = (2\sqrt{2})^m \sigma_i^m \Gamma\left(1+\frac{m}{2}\right), \tag{11}$$

where $\Gamma(\cdot)$ is the Gamma function and σ_i is the RMS value of stress process in the ith sea state which is calculated as $\sigma_i=\sqrt{m_{0i}}$.

Integrating Eq.(10), substituting Eq.(11) for $E[S_i^m]$ and applying Wirsching's wide band correction factor λ_i and stress concentration factor SCF:

$$a(t)=\left[a_0^{\frac{2-m}{2}}+\frac{2-m}{2}C\pi^{\frac{m}{2}}TY^m q_i f_{0i}(2\sqrt{2})^m \cdot\sigma_i^m\Gamma\left(1+\frac{m}{2}\right)\lambda_i(SCF)^m\right]^{\frac{2}{2-m}}. \tag{12}$$

Substituting Eq.(12) for $a(t)$ in Eq.(8), the time varying barrier is given by:

$$\xi(t)=\frac{K_{\text{IC}}}{Y(a)\sqrt{\pi}}\left[a_0^{\frac{2-m}{2}}+\frac{2-m}{2}C\pi^{\frac{m}{2}}TY^m q_i f_{0i}(2\sqrt{2})^m \cdot\sigma_i^m\Gamma\left(1+\frac{m}{2}\right)\lambda_i(SCF)^m\right]^{\frac{1}{m-2}}. \tag{13}$$

Eq.(13) gives the expression for the strength characteristic curve.

The steps involved in the computation of reliability are given below:

Step 1: For a given service life $t=T$, find $\alpha=\xi(t)$, the value of the time varying threshold, using Eq.(13).

Step 2: Find $\upsilon=\frac{1}{2\pi}\frac{\sigma_{\dot{x}}}{\sigma_x}\exp\left(-\frac{\alpha^2}{2\sigma_x^2}\right)$, where σ_x and $\sigma_{\dot{x}}$ are obtained from the spectral analysis of the stress process and υ is rate of crossing the barrier α.

Step 3: The probability of failure $P(T_f\le t)$ is given by $F_{T_f}(t)=1-\exp(-\upsilon t)$ where T_f is the first passage time.

Step 4: Steps 1~3 give the failure probability corresponding to a particular sea state. The total probability of failure is found by using total probability theorem, i.e.,

$$P_F=\sum_{i=1}^{n}P(E_i)P(F\,|\,E_i), \tag{14}$$

where n is total number of sea states, $P(E_i)$ is probability of occurrence of the ith sea state, $P(F|E_i)$ is probability of failure as obtained from Step 3 and P_F is total probability of failure.

Step 5: The reliability index, β can be calculated as

$$\beta=-\Phi^{-1}(P_F), \tag{15}$$

where $\Phi^{-1}(\cdot)$ is inverse cumulative distribution function of a normal variable.

NUMERICAL EXAMPLE

The plane frame tower considered in (Karsan and Kumar, 1990; Madhavan Pillai and Meher Prasad, 2000) is selected for the analysis due to its simplicity. A schematic diagram of the structure is shown in Fig.2. For the reliability analysis the structure is analysed for sample sea-scatter data shown in Table 1. The nominal values of the deterministic variables used are C_d=0.70, C_m=2.0, structural damping ratio ζ=0.05, current velocity $\dot{v}_c$=0.0 m/s and material constant m=3.0.

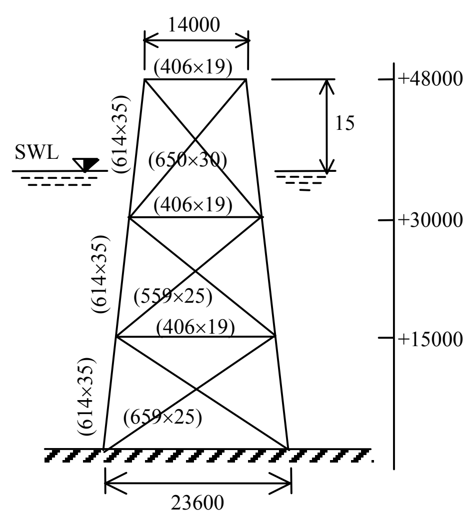

Fig.2 Schematic diagram of plane frame (unit: mm)

Fig.3 shows the degradation of fatigue reliability indices with time for the most critical leg member and diagonal bracing member. It can be seen that the fatigue reliability indices decrease as the time increases. The leg member is more critical compared to diagonal bracing member. This is against the results reported in (Madhavan Pillai and Meher Prasad, 2000), where the

Table 1 Sea-scatter data (Vughts and Kinra, 1976)

H_s (m)	Occurrences No.										
	T_Z=2.0 s	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
0.3048	2	23	17	3	–	–	–	–	–	–	–
0.6096	–	17	53	52	17	32	14	3	–	–	–
0.9144	–	–	16	127	74	11	6	1	–	–	–
1.2192	–	–	2	35	61	20	14	6	2	1	–
1.5240	–	–	1	4	17	16	7	10	2		–
1.8288	–	–	–	4	4	9	3	2	1	1	–
2.1333	–	–	–	–	1	4	14	3	–	–	–
2.4384	–	–	–	–	–	1	7	8	2	2	–
2.7432	–	–	–	–	–	–	2	4	1	–	–
3.0484	–	–	–	–	–	–	–	2	4	–	–
3.3528	–	–	–	–	–	–	–	1	2	2	–
3.9624	–	–	–	–	–	–	–	–	–	1	–
4.2672	–	–	–	–	–	–	–	–	–	–	1
4.5720	–	–	–	–	–	–	–	–	–	1	–

Total number of occurrences is 753

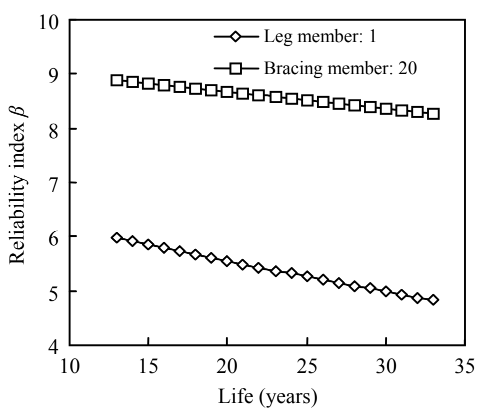

Fig.3 Variation of fatigue reliability with time

diagonal bracing member is more critical when compared to leg member. Thus it is observed that the leg members are more critical under ultimate limit state criteria whereas diagonal bracing members are more critical under serviceability limit state criteria. The fatigue reliability degradation curve is useful for deciding when the next in-service inspection/repair has to be carried out by planning the inspection when the reliability index falls below a predefined target value.

SENSITIVITY OF RELIABILITY INDEX

To provide insight into the effect of the different deterministic variables on the fatigue reliability index, a limited parametric study was carried out. The variables considered are the drag coefficient C_d, inertia coefficient C_m, fracture toughness K_{IC} and initial crack size a_0. The values of these variables are changed one at a time keeping the remaining variables at their nominal values. The results of this parametric study are presented below.

Effect of inertia coefficient

Fig.4 gives the variation of fatigue reliability index with respect to inertia coefficient C_m. The fatigue reliability index decreases with increase in inertia coefficient C_m.

Increase in C_m causes corresponding increase in the inertia force component in wave force which results in the decrease in the reliability index. Because

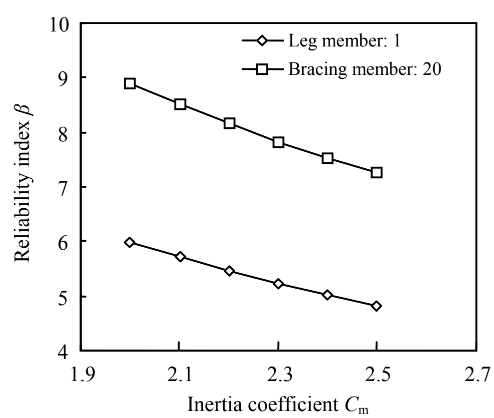

Fig.4 Effect of inertia coefficient on fatigue reliability

reliability index is sensitive to the inertia coefficient, proper value of C_m should be used to get an accurate measure of reliability index.

Effect of drag coefficient

The variation of fatigue reliability index with respect to drag coefficient C_d is depicted in Fig.5. It is clear that there is no considerable variation in reliability index with C_d. However, reliability index tends to decrease with increase in C_d.

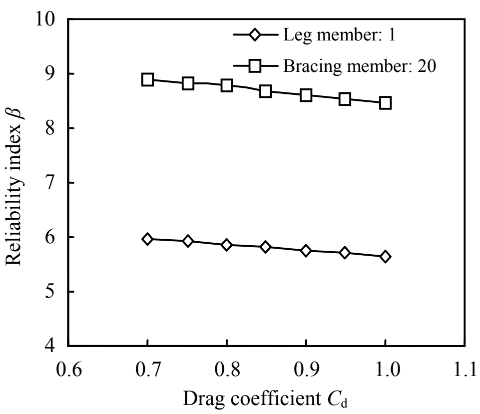

Fig.5 Effect of drag coefficient on fatigue reliability

In the Morison's equation for wave force it is evident that the drag force component is a function of square of the water particle velocity. Thus compared to inertia force, the periodicity of drag force will not cause significant change in the stress cycles. This does not affect the fatigue behaviour. Hence in a rigorous reliability analysis the drag coefficient may be treated as a deterministic variable.

Fracture toughness

Fig.6 shows the variation of reliability index with fracture toughness. As the fracture toughness increases, the reliability index also increases. Fracture toughness K_{IC} is an indication of the resistance to crack growth. The higher the resistance to crack growth, the lesser is the probability of failure.

Initial crack size

The variation of reliability index with the initial crack size is shown in Fig.7. The initial crack size is an indication of weld quality. The larger the initial crack size, the poorer is the quality of weld and the rate of barrier degradation will be faster. This causes the decrease of the reliability index.

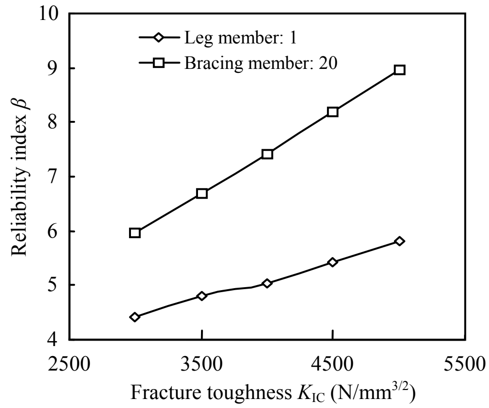

Fig.6 Variation of reliability index with K_{IC}

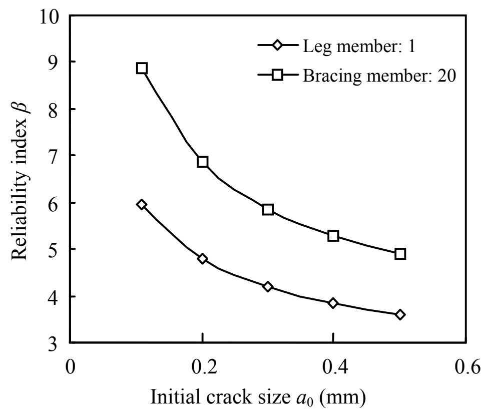

Fig.7 Variation of reliability index with a_0

CONCLUSION

An efficient and simple methodology to compute the time dependent reliability indices with respect to fatigue, of members of fixed offshore structures has been presented. The main thrust of the work is the computation of the fatigue reliability treating the problem as a first passage problem. Ultimate strength criteria are used to formulate the failure function. Fracture mechanics principle is used to develop the necessary equations. A plane frame is considered for numerical study and fatigue reliability degradation curve is established. This fatigue reliability degradation curve can be used to plan in-service inspection of the structure, inspecting the platform when the reliability index falls below a target level. A limited parametric study has also been conducted providing insight into the effect of various parameters on the fatigue reliability indices.

From the study conducted the following specific conclusions can be drawn:

(1) The leg members are more critical compared to diagonal bracing members under ultimate limit state criteria. This is contrary to the results reported by Madhavan Pillai and Meher Prasad (2000) wherein diagonal bracing members are more critical under serviceability limit state criteria.

(2) The fatigue reliability index was found to be sensitive to the inertia coefficient C_m, necessitating the use of proper value of C_m in the analysis.

(3) The reliability indices are insensitive to drag coefficient, C_d and in a rigorous reliability analysis the drag coefficient C_d, can be treated as deterministic variable.

(4) The fatigue reliability index is greatly influenced by weld quality (a_0) and fracture toughness K_{IC}.

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