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Passive control of Permanent Magnet Synchronous Motor chaotic system based on state observer*

QI Dong-lian†, WANG Qiao
(*School of Electrical Engineering, Zhejiang University, Hangzhou 310027, China*)
†E-mail: qidl@zju.edu.cn
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Abstract: Passive system theory was applied to propose a new passive control method with nonlinear observer of the Permanent Magnet Synchronous Motor chaotic system. Through constructing a Lyapunov function, the subsystem of the Permanent Magnet Synchronous Motor chaotic system could be proved to be globally stable at the equilibrium point. Then a controller with smooth state feedback is designed so that the Permanent Magnet Synchronous Motor chaotic system can be equivalent to a passive system. To get the state variables of the controller, the nonlinear observer is also studied. It is found that the outputs of the nonlinear observer can approximate the state variables of the Permanent Magnet Synchronous Motor chaotic system if the system's nonlinear function is a globally Lipschitz function. Simulation results showed that the equivalent passive system of Permanent Magnet Synchronous Motor chaotic system could be globally asymptotically stabilized by smooth state feedback in the observed parameter convergence condition area.

Key words: Permanent Magnet Synchronous Motor, Passive system, State observer
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INTRODUCTION

Many people paid attention to passive theory and used it to control chaotic dynamical systems (Byrnes and Isidori, 1991; Yu, 1999). Yu (1999) studied the passive equivalence of the Lorenz chaotic system and presented the method of passive control of chaotic system. Using passive theory, Qi and Zhao (2005) researched on the control of Permanent Magnet Synchronous Motor chaotic system, and stabilized the system at different equilibrium points. However, in order to design the controller, it is important to detect the state signal, especially for passive controller. In most situations, it is difficult to detect the state variables or the sensor is expensive, and the state observer is very essential for controlled system (Zhao and Qi, 2001; Femat and Solis-Perales, 1999; Femat and Alvarez-Ramirez, 1997).

In this paper, a new chaos control method is proposed for avoiding chaos in Permanent Magnet Synchronous Motor chaotic system based on nonlinear observer. The remainder of the paper is organized as follows. In Section II, we use the properties of passive system to study the essential conditions, under which Permanent Magnet Synchronous Motor chaotic system could be equivalent to a passive system. In Section III, the state observer of the Permanent Magnet Synchronous Motor chaotic system is given, and the observed parameter convergence condition is also studied. Section IV gives the conclusion, which is that the equivalent passive system transformed by the Permanent Magnet Synchronous Motor chaotic system could be globally asymptotically stabilized with nonlinear observer.

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PASSIVE CONTROL OF PERMANENT MAGNET SYNCHRONOUS MOTOR CHAOTIC SYSTEM

Consider a continuous chaotic system described by the differential equation (Byrnes and Isidori, 1991):

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) \cdot \boldsymbol{u}, \quad \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \tag{1}$$

where, $\boldsymbol{x}\in\mathbb{R}^n$ is state variable, $\boldsymbol{u}\in\mathbb{R}^m$ is the input, $\boldsymbol{y}\in\mathbb{R}^p$ is the output, $\boldsymbol{f}$ and $\boldsymbol{g}$ are smooth vector fields. Function $\boldsymbol{h}$ is smooth mapping. We suppose that the vector field $\boldsymbol{f}$ has at least one equilibrium point, without loss of generality, we can assume that the equilibrium point $\boldsymbol{x}$=0.

A system of Eq.(1) is said to be passive if function $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ exist and there is a real-valued constant β, such that for $\forall t\geq 0$

$$\int_0^t \boldsymbol{u}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau \geq \beta, \tag{2}$$

or if there is a constant $\rho>0$ and real-valued constant β, such that for $\forall t\geq 0$

$$\int_0^t \boldsymbol{u}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho \cdot \boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau. \tag{3}$$

A system of Eq.(1) is said to be passive if there is a nonnegative function $\boldsymbol{V}(\boldsymbol{x})$: $\boldsymbol{X}\rightarrow\mathbb{R}$, $\boldsymbol{V}(0)=0$, named storage function, such that for $\forall \boldsymbol{x}\in\boldsymbol{X}$, $\forall t\geq 0$

$$\boldsymbol{V}(\boldsymbol{x}) - \boldsymbol{V}(\boldsymbol{x}_0) \leq \int_0^t \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{u}(s)\mathrm{d}s, \tag{4}$$

where $\boldsymbol{x}_0$ is the initial value of the state variable.

Given the chaotic dynamical system of Permanent Magnet Synchronous Motor (Zhang *et al*., 2002)

$$\begin{cases} \mathrm{d}\tilde{i}_d / \mathrm{d}t = -\tilde{i}_d + \tilde{\omega}\tilde{i}_q + \tilde{u}_d, \\ \mathrm{d}\tilde{i}_q / \mathrm{d}t = -\tilde{i}_q - \tilde{\omega}\tilde{i}_d + \tilde{u}_q + \gamma\tilde{\omega}, \\ \mathrm{d}\tilde{\omega} / \mathrm{d}t = \sigma(\tilde{i}_q - \tilde{\omega}) - \tilde{T}_L, \end{cases} \tag{5}$$

where $\tilde{u}_d$ and $\tilde{u}_q$ is the voltage of d axes and q axes respectively, $\tilde{T}_L$ is external input torque, γ and σ are parameters.

Suppose $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = 0$, γ=20, σ=5.46 and $x_1 = \tilde{i}_d$, $x_2 = \tilde{\omega}$, $x_3 = \tilde{i}_q$, Eq.(5) can be expressed by differential equation:

$$\begin{cases} \dot{x}_1 = -x_1 + x_2 x_3, \\ \dot{x}_2 = 5.46(x_3 - x_2), \\ \dot{x}_3 = -x_3 - x_1 x_2 + 20x_2. \end{cases} \tag{6}$$

We use Eq.(6) to study the passive control method and design the controller (Qi and Zhao, 2005). The essential method will be given in this paper.

Add the controller u into the third equation of Eq.(6). Suppose that state variable x_3 is the output of the system and suppose $z_1=x_1$, $z_2=x_2$, $y=x_3$, then the system can be expressed by normal form:

$$\begin{cases} \dot{z}_1 = -z_1 + z_2 y, \\ \dot{z}_2 = 5.46(y - z_2), \\ \dot{y} = -y - z_1 z_2 + 20z_2 + u, \end{cases} \tag{7}$$

we have

$$\begin{cases} \dot{\boldsymbol{z}} = \boldsymbol{f}_0(\boldsymbol{z}) + \boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{y}, \\ \dot{\boldsymbol{y}} = \boldsymbol{b}(\boldsymbol{z},\boldsymbol{y}) + \boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{u}, \end{cases} \tag{8}$$

where $\boldsymbol{f}_0(\boldsymbol{z})=[-z_1,\ -5.46z_2]^{\mathrm{T}}$, $\boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})=[z_2,\ 5.46]^{\mathrm{T}}$, $\boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})=1$, $\boldsymbol{b}(\boldsymbol{z},\boldsymbol{y})=-y-z_1z_2+20z_2$.

Choose storage function $\boldsymbol{V}(\boldsymbol{z},\boldsymbol{y})=\boldsymbol{W}(\boldsymbol{z})+0.5\boldsymbol{y}^2$, where $\boldsymbol{W}(\boldsymbol{z})$ is the Lyapunov function of $\boldsymbol{f}_0(\boldsymbol{z})$, and $\boldsymbol{W}(0)=0$,

$$\boldsymbol{W}(\boldsymbol{z}) = 0.5z_1^2 + 0.5z_2^2. \tag{9}$$

Because $\boldsymbol{W}(\boldsymbol{z})$ is Lyapunov function of $\boldsymbol{f}_0(\boldsymbol{z})$, so

$$\frac{\mathrm{d}\boldsymbol{W}}{\mathrm{d}t} = -z_1^2 - 5.46z_2^2 \leq 0. \tag{10}$$

We know that $\boldsymbol{f}_0(\boldsymbol{z})$ should be globally stable if Eq.(6) is equivalent to passive system, where the eigenvalues of $\boldsymbol{f}_0(\boldsymbol{z})$ are $\lambda_1=-1$, $\lambda_1=-5.46$.

The zero dynamical character of Permanent Magnet Synchronous Motor system is Lyapunov stable, so the Permanent Magnet Synchronous Motor chaotic system is minimum phase. From Eq.(10)

$$\begin{aligned}\dot{V} &= \frac{\partial \boldsymbol{W}}{\partial \boldsymbol{z}}\dot{\boldsymbol{z}} + \boldsymbol{y}\dot{\boldsymbol{y}} \\ &= \frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{W}(\boldsymbol{z})\cdot \boldsymbol{f}_0(\boldsymbol{z}) + \frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{W}(\boldsymbol{z})\cdot \boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})\cdot \boldsymbol{y} \\ &\quad + [\boldsymbol{b}(\boldsymbol{z},\boldsymbol{y}) + \boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{u}]\boldsymbol{y}.\end{aligned} \tag{11}$$

Because the equivalent system is minimum phase and have KYP property, we have

$$\frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{W}(\boldsymbol{z})\cdot \boldsymbol{f}_0(\boldsymbol{z}) \le 0.$$

From Eq.(11)

$$\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} \le \frac{\partial}{\partial \boldsymbol{z}}\boldsymbol{W}(\boldsymbol{z})\cdot \boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})\cdot \boldsymbol{y} + [\boldsymbol{b}(\boldsymbol{z},\boldsymbol{y}) + \boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{u}]\boldsymbol{y}. \tag{12}$$

Let

$$\boldsymbol{u}=(1-\alpha)y+v-25.46z_2, \tag{13}$$

where α is positive real constant, v is external input signal.

According to Eq.(6) and Eq.(13), we have

$$\begin{cases}\dot{x}_1 = -x_1 + x_2x_3, \\ \dot{x}_2 = 5.46(x_3 - x_2), \\ \dot{x}_3 = -\alpha x_3 - x_1x_2 - 5.46x_2 + v.\end{cases} \tag{14}$$

In general, if the state variables can be detected, the Permanent Magnet Synchronous Motor chaotic system can be passive equivalence. The design of the nonlinear observer is essential for Permanent Magnet Synchronous Motor system.

NONLINEAR OBSERVER

The observer of the Permanent Magnet Synchronous Motor chaotic system can be described as

$$\begin{cases}\dot{\hat{x}}_1 = -\hat{x}_1 + \hat{x}_2\hat{x}_3 + L_1(\theta)(y-\hat{y}), \\ \dot{\hat{x}}_2 = 5.46\hat{x}_3 - 5.46\hat{x}_2 + L_2(\theta)(y-\hat{y}), \\ \dot{\hat{x}}_3 = \hat{x}_1\hat{x}_2 - \hat{x}_3 + 20\hat{x}_2 + L_3(\theta)(y-\hat{y}), \\ \hat{y} = \hat{x}_1.\end{cases} \tag{15}$$

Suppose that x_1 can be detected, and let $\hat{y} = \hat{x}_1$ in observer. The observer also can be described as:

$$\begin{cases}\dot{\hat{x}} = \boldsymbol{A}\hat{x} + \boldsymbol{B}\boldsymbol{q}(\hat{x}) + \boldsymbol{L}(k)(\boldsymbol{y} - \hat{\boldsymbol{y}}), \\ \hat{\boldsymbol{y}} = \boldsymbol{C}^{\mathrm{T}}\hat{x}.\end{cases} \tag{16}$$

The parameters of the nonlinear observer can be described as

$$\boldsymbol{A} = \begin{bmatrix}-1 & 0 & 0 \\ 0 & -5.46 & 5.46 \\ 0 & 20 & -1\end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix}1 & 0 & 1\end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{C} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^{\mathrm{T}},\ \boldsymbol{q}(\hat{\boldsymbol{x}}) = \begin{bmatrix}\hat{x}_2\hat{x}_3 & 0 & \hat{x}_1\hat{x}_2\end{bmatrix}^{\mathrm{T}}.$$

The method of choosing parameters $\boldsymbol{L}(k)$ will be discussed below. Choose the transforming matrix $\boldsymbol{P}=\boldsymbol{I}$. Consider the linear transformation of coordinates $\boldsymbol{w}=\boldsymbol{P}\boldsymbol{z}$, system Eq.(16) can be described as

$$\begin{cases}\dot{\boldsymbol{w}} = \boldsymbol{A}_0\boldsymbol{w} + \boldsymbol{B}_0\boldsymbol{f}(\boldsymbol{w}), \\ \boldsymbol{y} = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{w},\end{cases} \tag{17}$$

where $\boldsymbol{A}_0=\boldsymbol{P}\boldsymbol{A}\boldsymbol{P}^{-1}$, $\boldsymbol{B}_0=\boldsymbol{P}\boldsymbol{B}$, $\boldsymbol{C}_0^{\mathrm{T}} = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{P}^{-1}$, $\boldsymbol{f}(\boldsymbol{w})=\boldsymbol{q}(\boldsymbol{P}^{-1}\boldsymbol{w})$. Let $\boldsymbol{P}\boldsymbol{L}(k)=\boldsymbol{L}_0(k)$, we have

$$\begin{cases}\dot{\hat{\boldsymbol{w}}} = \boldsymbol{A}_0\hat{\boldsymbol{w}} + \boldsymbol{B}_0\boldsymbol{f}(\hat{\boldsymbol{w}}) + \boldsymbol{L}_0(k)(\boldsymbol{y} - \hat{\boldsymbol{y}}), \\ \hat{\boldsymbol{y}} = \boldsymbol{C}_0^{\mathrm{T}}\hat{\boldsymbol{w}},\end{cases} \tag{18}$$

where $k\ge1$.

Define error $\boldsymbol{e} = \boldsymbol{w} - \hat{\boldsymbol{w}}$. Since matrix $\boldsymbol{P}$ is an invertible matrix, $\boldsymbol{e}(t)\to0$ implies that $\boldsymbol{x}(t) \to \hat{\boldsymbol{x}}(t)$. Hence, it suffices to prove that $\boldsymbol{e}(t)\to0$. There is

$$\dot{\boldsymbol{e}} = \boldsymbol{M}_0\boldsymbol{e} + \boldsymbol{B}_0[\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})], \tag{19}$$

where $\boldsymbol{M}_0 = \boldsymbol{A}_0 - \boldsymbol{L}_0(k)\boldsymbol{C}_0^{\mathrm{T}}$, whose eigen-function is

$$\boldsymbol{p}_n(s)=s^n+c_1s^{n-1}+\dots+c_n. \tag{20}$$

Through choosing proper parameter a_i, all roots of Eq.(20) can be contained in the left-hand side of the complex plane. Lyapunov equation $\boldsymbol{P}_0\boldsymbol{M}_0 + \boldsymbol{M}_0^{\mathrm{T}}\boldsymbol{P}_0 = -\boldsymbol{I}_n$ has $\boldsymbol{P}_0>0$. Choosing Lyapunov function $\boldsymbol{V}=\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_0\boldsymbol{e}$, we have

$$\begin{aligned}\dot{\boldsymbol{V}} &= \dot{\boldsymbol{e}}^{\mathrm{T}}\boldsymbol{P}_0\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_0\dot{\boldsymbol{e}} \\ &\le -\|\boldsymbol{e}\|^2 + 2\left\|\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_0\{\boldsymbol{B}_0[\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})]\}\right\| \\ &\le -\|\boldsymbol{e}\|^2 + 2\left\|\boldsymbol{e}^{\mathrm{T}}\right\|\cdot\|\boldsymbol{P}_0\|\cdot\|\boldsymbol{B}_0\|\cdot|\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})|.\end{aligned} \tag{21}$$

Since $\boldsymbol{q}(\boldsymbol{w})$ is a globally Lipschitz function on $\mathbb{R}^n$, $\boldsymbol{f}(\boldsymbol{w})$ is also a globally Lipschitz function. There exists $\gamma>0$, such that

$$\|\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})\| \le \gamma\|\boldsymbol{w} - \hat{\boldsymbol{w}}\|. \tag{22}$$

Based on Eqs.(21) and (22), we have

$$\begin{aligned}\dot{\boldsymbol{V}} &\le -\|\boldsymbol{e}\|^2 + 2\gamma\boldsymbol{\lambda}_{\max}(\boldsymbol{P}_0)\cdot\|\boldsymbol{B}_0\|\cdot\|\boldsymbol{e}\|^2 \\ &\le -\left(1 - k_f\boldsymbol{\lambda}_{\max}(\boldsymbol{P}_0)\|\boldsymbol{B}_0\|\right)\cdot\|\boldsymbol{e}\|^2,\end{aligned} \tag{23}$$

where $k_f=2\gamma$. If $k_f\boldsymbol{\lambda}_{\max}(\boldsymbol{P}_0)\ \|\boldsymbol{B}_0\|<1$, there should be $\dot{\boldsymbol{V}} < 0$. The state variables of the observer can approximate the state variables of the chaotic system.

Let the initial value of observability parameter be $\boldsymbol{L}_0(\theta)=(3\theta\ \ 3\theta^2\ \ \theta^3)^{\mathrm{T}}$, so that the eigenvalues of $\boldsymbol{p}_n(s)$ are all -1, then $\boldsymbol{L}(\theta)=(3\theta\ \ 3\theta^2\ \ \theta^3)^{\mathrm{T}}$. The observer of the Permanent Magnet Synchronous Motor system has the following form

$$\begin{cases}\dot{\hat{x}}_1 = -\hat{x}_1 + \hat{x}_2\hat{x}_3 + 3\theta(y - \hat{y}), \\ \dot{\hat{x}}_2 = 5.46\hat{x}_3 - 5.46\hat{x}_2 + 3\theta^2(y - \hat{y}), \\ \dot{\hat{x}}_3 = \hat{x}_1\hat{x}_2 - \hat{x}_3 + 20\hat{x}_2 + \theta^3(y - \hat{y}),\end{cases}$$
$$\hat{y} = \hat{x}_1.$$

Let the initial point of Permanent Magnet Synchronous Motor system be (1,1,1) and the initial point of the nonlinear observer be (5,−2,10). The controller $u = (1-\alpha)\hat{x}_3 + v - 25.46\hat{x}_2$. Fig.1 is the state error output between Permanent Magnet Synchronous Motor system and the observer when k=1. Fig.2 is the Permanent Magnet Synchronous Motor system outputs when α=2, v=0, k=1. Those figures show that it takes only very short time for the system to be rapidly stabilized at the equilibrium point (0,0,0) while the outputs of the nonlinear observer approximate the state variables of Permanent Magnet Synchronous Motor chaotic system.

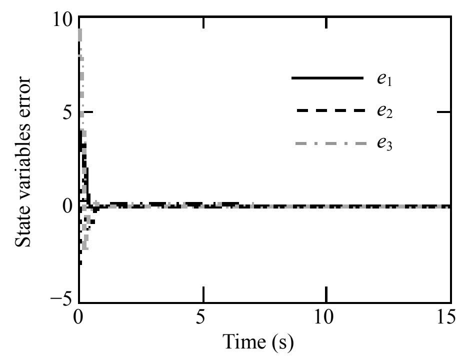

Fig.1 The error between Permanent Magnet Synchronous Motor system and observer when k=1

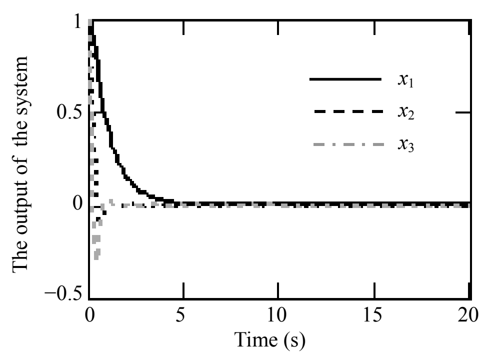

Fig.2 The outputs of Permanent Magnet Synchronous Motor chaotic system when α=2, v=0

Fig.3 is the output of the system when additive noise n=0.2. The system can also be stabilized at the desired point although the additive noise influences the controlled system.

CONCLUSION

The character of passive systems shows a kind of characteristics of dissipative network. Passive net-

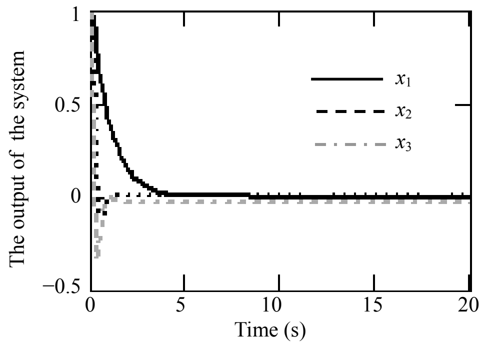

Fig.3 The output of Permanent Magnet Synchronous Motor when additive noise *n*=0.2

work theory can be used to analyze dynamical system characters, such as stabilization and dynamic characteristics. Based on the property of passive system, minimum phase nonlinear system transformed by Permanent Magnet Synchronous Motor system can be globally asymptotically stabilized by state feedback.

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