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Identification of strategy parameters for particle swarm optimizer through Taguchi method

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Abstract: Particle swarm optimization (PSO), like other evolutionary algorithms is a population-based stochastic algorithm inspired from the metaphor of social interaction in birds, insects, wasps, etc. It has been used for finding promising solutions in complex search space through the interaction of particles in a swarm. It is a well recognized fact that the performance of evolutionary algorithms to a great extent depends on the choice of appropriate strategy/operating parameters like population size, crossover rate, mutation rate, crossover operator, etc. Generally, these parameters are selected through hit and trial process, which is very unsystematic and requires rigorous experimentation. This paper proposes a systematic based on Taguchi method reasoning scheme for rapidly identifying the strategy parameters for the PSO algorithm. The Taguchi method is a robust design approach using fractional factorial design to study a large number of parameters with small number of experiments. Computer simulations have been performed on two benchmark functions—Rosenbrock function and Griewank function—to validate the approach.

Key words: Strategy parameters, Particle swarm optimization (PSO), Taguchi method, ANOVA
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INTRODUCTION

The particle swarm optimization (PSO) method is a member of the broad category of swarm intelligence techniques for finding optimized solutions. The PSO algorithm is based on the social behavior of animals such as flocking of birds and schooling of fish, etc. PSO has its origin in simulation for visualizing the synchronized choreography of bird flock by incorporating concepts such as nearest-neighbor velocity matching and acceleration by distance (Parsopoulos and Vrahatis, 2002; Eberhart and Shi, 2001; Kennedy and Eberhart, 1995; 2001). Later on it was realized that the simulation could be used as an optimizer and resulted in the first simple version of PSO (Kennedy and Eberhart, 1995). Since then, many variants of PSO have been suggested by different researchers (Eberhart and Kennedy, 1995; Shi and Eberhart, 1998; 2001; Xie *et al*., 2002).

In PSO, the particles have an adaptable velocity that determines their movement in the search space. Each particle also has a memory and can remember the best position in the search space ever visited by it. The position corresponding to the best fitness is known as "pbest" and the overall best out of all the particles in the population is called "gbest".

Consider that the search space is d-dimensional and that the ith particle in the swarm can be represented by $\boldsymbol{X}_i=(x_{i1}, x_{i2}, \ldots, x_{id})$ and its velocity can be represented by another d-dimensional vector $\boldsymbol{V}_i=(v_{i1}, v_{i2}, \ldots, v_{id})$. Let the best previously visited position of this particle be denoted by $\boldsymbol{P}_i=(p_{i1}, p_{i2}, \ldots, p_{id})$. If the gth particle is the best particle and the iteration number is denoted by the superscript, then the swarm is modified according to Eqs.(1) and (2) suggested by Shi and Eberhart (1999).

$$v_{id}^{n+1} = wv_{id}^{n} + c_1 r_1^{n}(p_{id}^{n} - x_{id}^{n}) + c_2 r_2^{n}(p_{id}^{n} - x_{id}^{n}), \quad (1)$$

$$x_{id}^{n+1} = x_{id}^{n} + v_{id}^{n+1}, \quad (2)$$

where, w: inertia weight; c_1: cognitive acceleration parameter; c_2: social acceleration parameter; r_1, r_2: random numbers uniformly distributed in the range (0, 1).

These parameters, viz. inertia weight (w), cognitive acceleration (c_1), social acceleration (c_2), and V_{max} (Shi and Eberhart, 1999) are the strategy parameters of PSO algorithm and the performance of PSO to a great extent depends on the selection of these parameters. The parameter V_{max} defined by the user is the maximum velocity along any dimension, which implies that, if the velocity along any dimension exceeds V_{max}, it shall be clamped to this value. The inertia weight governs how much of the previous velocity should be retained from the previous time step. Generally w is not kept fixed and is varied as the algorithm progresses so as to improve performance (Shi and Eberhart, 1999; Parsopoulos and Vrahatis, 2002). This setting allows the PSO to explore a large area at the start of simulation run and to refine the search later by a smaller w. Another parameter has been included which determines the fraction of the total run for which w will be varied and is labelled as *w_vary-for*. The parameters c_1 and c_2 influence the maximum size of the step that a particle can take in a single iteration and random numbers r_1 and r_2 help in maintaining the diversity of the population.

In this paper, Taguchi method has been used for the identification of the strategy parameters of PSO. The Taguchi method, which is a robust design approach, helps in optimization using relatively few experiments.

TAGUCHI METHOD

The fundamental principle of the Taguchi method is to improve the quality of a product by minimizing the effect of the causes of variation without eliminating the causes. In this methodology, the desired design is finalized by selecting the best performance under conditions that produce consistent performance. The conclusions drawn from the small set of experiments are valid over the entire experimental region spanned by the factors and their setting levels. Taguchi approach provides systematic, simple and efficient methodology for the optimization of the near optimum design parameters with only a few well-defined experimental sets (Taguchi *et al*., 2005; Bagchi, 1993; Ross, 1996).

Two major tools used in the Taguchi method are the orthogonal array (OA) and the signal to noise ratio (SNR). OA is a matrix of numbers arranged in rows and columns. Each row represents the level of factors in each run and each column represents a specific level for a factor that can be changed for each run. SNR is indicative of quality and the purpose of the Taguchi experiment is to find the best level for each operating parameter so as to maximize (or minimize) SNR.

Taguchi methodology for optimization can be divided into four phases, viz. planning, conducting, analysis and validation. Each phase has a separate objective and contributes towards the overall optimization process. Taguchi methodology for optimization has been represented in the form of a flowchart as shown in Fig.1.

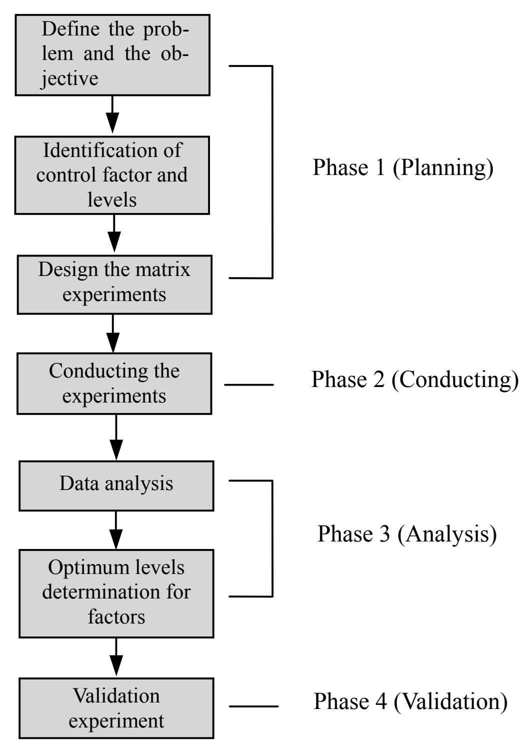

Fig.1 Flowchart representing the Taguchi methodology for optimization

Phase 1—Planning

The first step in Phase 1 is to determine the various factors/parameters and their level that influences the performance characteristic(s). For the problem under consideration, the objective was to identify the optimum parameters of PSO algorithm used for locating the optimum solution for two benchmark functions, viz. Rosenbrock and Griewank. Factors that have significant effect on the performance should be selected. One of the main features of the Taguchi method is its capability to point our insignificant variables, even if they were considered to be significant at the beginning of the optimization process.

For our experiments, we considered five parameters. These five strategy parameters or factors for convenience are represented by the letters *A~E*. The factors (*A~E*) and the corresponding parameters are listed in Table 1. The levels of each operating parameters are listed in Table 2. For our experiments, $L_{16}(4^5)$ OA was selected, which represents 16 experiments with five four-level factors. The selected OA is represented in Table 3. On the other hand, the full factorial design, which represents the traditional or classical approach, considers all possible combinations of input parameters and for the given situation would require 1024 (4^5) experiments to be performed.

Two benchmark functions viz. Rosenbrock and Griewank of 30 dimensions each were used for the experiments. Both these functions are multimodal, having global optimum function values equal to zero. The definitions of these benchmark functions can be found in (Shi and Eberhart, 1999). For both the cases, the fitness value was taken equal to the function value and represents the SNR. Hence, the closer is the value of SNR to zero, the better is the solution. Some of the parameters that remained fixed for the complete run were: swarm size=80, iterations=2000.

Phase 2—Conducting

The experiments were carried out by using the PSO toolbox for Matlab, developed by the authors, which is hosted on SourceForge.net as an open-source initiative and can be downloaded from http://sourceforge.net/projects/psotoolbox. SourceForge.net is the world's largest repository of open-source code and applications.

For each experiment, the mean fitness value of the best particle found for 50 runs has been calculated. The calculated SNRs are summarized in Table 3. The large variations in the values of SNR in the 16 experiments clearly exhibit the dependence of PSO algorithm performance on the choice of strategy parameters and hence validate the importance of careful selection of these parameters.

Table 1 Factors and the corresponding parameters

Factor	Corresponding strategy parameter of PSO
A	$[c_1, c_2]$
B	*w_start*
C	*w_end*
D	*w_vary-for*
E	$V_{\max}$

Table 2 Levels of different parameters (for Rosenbrock function)

Factor	Level			
	1	2	3	4
A	[2, 2]	[2, 1.5]	[1.5, 1]	[0.5, 0.5]
B	1	0.9	2	1.5
C	0.4	0.3	0.2	0.1
D	1	0.9	0.8	0.7
E	90	100	110	120

Table 3 Matrix experiments with L_{16} OA (for Rosenbrock function)

Experiment number	Factor					Response	SNR
	A	*B*	*C*	*D*	*E*		
1	1	1	1	1	1	R_1	227.3041
2	1	2	2	2	2	R_2	101.4337
3	1	3	3	3	3	R_3	123.1993
4	1	4	4	4	4	R_4	151.0734
5	2	1	2	3	4	R_5	138.2774
6	2	2	1	4	3	R_6	146.6493
7	2	3	4	1	2	R_7	262.9157
8	2	4	3	2	1	R_8	144.6070
9	3	1	3	4	2	R_9	322.8132
10	3	2	4	3	1	R_{10}	271.3774
11	3	3	1	2	4	R_{11}	1195.7516
12	3	4	2	1	3	R_{12}	643.4142
13	4	1	4	2	3	R_{13}	5722.1585
14	4	2	3	1	4	R_{14}	600.0516
15	4	3	2	4	1	R_{15}	516635.3345
16	4	4	1	3	2	R_{16}	22573.6059

Phase 3—Analysis

The data generated in Phase 2 is analyzed for identifying optimum level of parameters. The response of each experiment is represented as R_1, R_2, …, R_{16} and the average response of each factor is computed at each level. For example, the average response of factor *A* at Level 3 is calculated as $(R_9+R_{10}+R_{11}+R_{12})/4$. Following this for each factor and for different levels shall generate the entries for the response table (i.e. Table 4) and the response graph. Response table is used for recording the processed data and presents the calculations of effects from the orthogonally designed experiments, whereas the response graph is the graphical representation for the data presented in the response table to quickly identify the effects of different parameters (Taguchi *et al*., 2005). The response graphs corresponding to the different factors are represented in Fig.2. Due to the large difference in the maximum and minimum values of SNRs, these values have been converted into dB for drawing the response graphs.

Table 4 SNR of each operating parameter and level (for Rosenbrock function)

Factor	*SNR* (dB)			
	Level 1	Level 2	Level 3	Level 4
A	150.753 (43.56)	173.112 (44.76)	608.339 (55.68)	136383 (102.69)
B	1602.64 (64.09)	279.878 (48.93)	129554 (102.24)	5878.18 (75.38)
C	6035.83 (75.61)	129380 (102.23)	297.668 (49.47)	1601.88 (64.09)
D	433.421 (52.73)	1790.99 (65.06)	5776.62 (75.23)	129314 (102.23)
E	129320 (102.23)	5815.19 (75.28)	1658.86 (64.39)	521.289 (54.34)

From the response table or the response graph, the optimum level of each factor can be predicted as the level that has the lowest value of SNR. Thus, the optimal configuration for the PSO parameters identified for Rosenbrock function was $A(1)B(2)C(3)D(1)E(4)$. The corresponding parameter values are listed in Table 5.

Table 5 Predicted best strategy parameters (for Rosenbrock function)

Factor (Level)	Value
A (1)	[2, 2]
B (2)	0.9
C (3)	0.2
D (1)	1
E (4)	120

Phase 4—Validation

The confirmatory experiment is run with the optimum control factors obtained in Phase 3. It is basically a validation or invalidation of optimum levels of the control factors. Unsatisfactory confirmatory experiment implies that additional experiments are required to be performed.

To validate this fact, the simulations with predicted parameters were carried out and the mean fitness value of the best particle found for the 50 runs was found to be 95.8358 which is superior to all the SNRs listed in Table 3 and is also better than the simulation results reported in (Shi and Eberhart, 1999), which is 205.5596.

The experiments were repeated for Griewank function also and the fitness value or SNR obtained with the predicted strategy parameters through the Taguchi method was 0.011028 in comparison to the

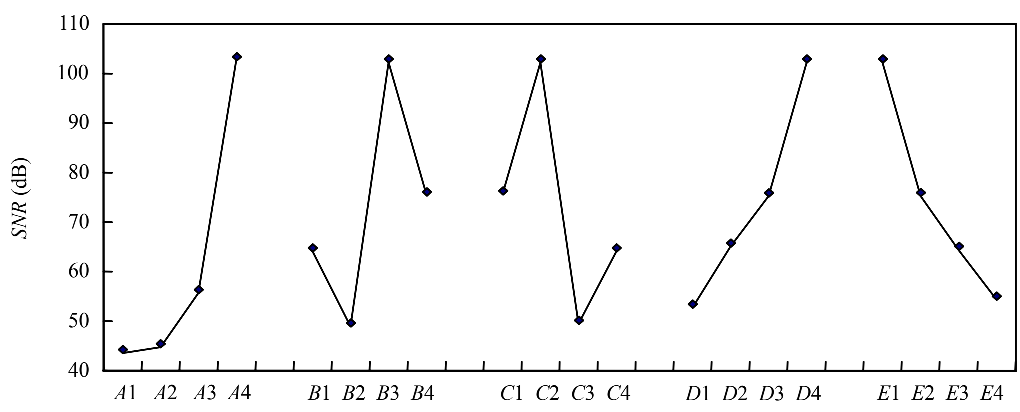

Fig.2 Response graphs

value 0.0128 reported in earlier simulations (Shi and Eberhart, 1999). The results obtained through the Taguchi method approach and those obtained earlier for the two benchmark functions are shown in Fig.3.

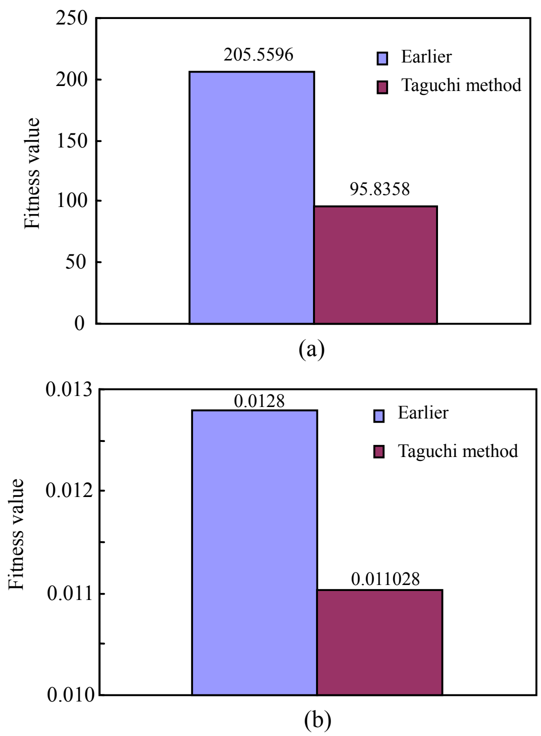

Fig.3 Comparison of fitness values obtained through the Taguchi method approach and those obtained earlier for the two benchmark functions: Rosenbrock function (a) and Griewank function (b)

Analysis of variance (ANOVA)

Statistical methods are powerful tools for extracting useful information contained in data and ANOVA is one of the most frequently used tools. The ANOVA, as commonly understood and practiced today, was developed by Ronald A. Fisher of England (Taguchi *et al*., 2005) in the 1920's. ANOVA is particularly useful in analyzing data from the statistically designed experiments and can decompose variations for any type of data and helps in quickly calculating the magnitude of influence of the cause being considered. Table 6 lists the ANOVA for each operating parameter of PSO algorithm.

The detailed mathematical description of ANOVA can be found in (Scheffe, 1999; Turner and Thayer, 2001). One of the columns of the ANOVA table is percent contribution, which reflects the portion of the total variation observed in an experiment to each significant parameter. It signifies that the parameters with substantial percent contributions are the most important for reducing variation. The contributions of different PSO parameters are also represented in Fig.4. It can be observed from Table 6 that for the problem under consideration, the cognitive acceleration parameter c_1 is the most significant factor for the PSO algorithm used for Rosenbrock function, whereas the factors *w_vary-for* and V_{max} show the least impact amongst the factors considered.

Table 6 ANOVA table

Factor	*DOF*	*SS*	*MS*	Percent contribution (%)
A	3	5.554E+10	1.851E+10	22.34
B	3	4.843E+10	1.614E+10	19.49
C	3	4.825E+10	1.608E+10	19.41
D	3	4.818E+10	1.606E+10	19.38
E	3	4.818E+10	1.606E+10	19.38
Total	15	2.486E+11	8.286E+10	100.00

DOF: degrees of freedom; *SS*: sum of squares; *MS*: mean square; *MS*=*SS*/*DOF*

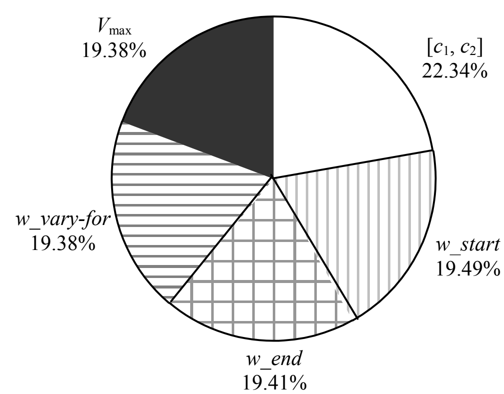

Fig.4 Percent contribution of each parameter

COMPARISON OF COMPUTATIONAL EFFORTS BETWEEN TAGUCHI METHOD AND TRADITIONAL APPROACH

If the process of running experiments is automated and the experiments are being performed 24 h a day and 7 d a week, the computational efforts required for the Taguchi method and for the traditional approach for two benchmark functions viz. Rosenbrock and Griewank are listed in Table 7. The results show that huge savings in simulation time can be achieved by following the Taguchi approach.

Table 7 Comparison of computational efforts between the Taguchi method and the traditional approach for two benchmark functions

Parameter	Full factorial design (Traditional)		Fractional factorial design (Taguchi method)	
	Rosenbrock function	Griewank function	Rosenbrock function	Griewank function
Time for one experiment (s)	92	221	92	221
Total number of experiments (5 factors, each with 4 levels)	1024 (4^5)	1024 (4^5)	16 (with $L_{16}(4^5)$ OA)	16 (with $L_{16}(4^5)$ OA)
Total time for experimentation	1570.1 min (26.17 h)	3771.7 min (62.86 h)	24.53 min (0.41 h)	58.93 min (0.98 h)

CONCLUSION

In this paper, a scheme based on the Taguchi method has been proposed to quickly identify the strategy parameters for the PSO algorithm. The methodology has been used for identifying the strategy parameters for two benchmark functions. Computer simulations are carried out to show that improvements are achieved, when the optimal operating parameters are obtained through the Taguchi method. In this paper, only five parameters of PSO algorithm were considered. Since Taguchi's orthogonal design has a general framework and hence more variables viz. swarm size, number of iterations, etc. can also be considered.

PSO algorithm like other evolutionary algorithms is being used in different engineering applications for optimization of systems, where the PSO operating parameters have to be selected before running the algorithm. On the same lines this approach can be used for finding the best operating parameters of PSO for any other system under consideration with the objective to improve the performance, which can be achieved using a smaller number of experiments and with much less computational effort.

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