



Optimal transmission strategy for spatially correlated MIMO systems with channel statistical information

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Abstract: In real multiple-input multiple-output (MIMO) systems, the perfect channel state information (CSI) may be costly or impossible to acquire. But the channel statistical information can be considered relatively stationary during long-term transmission. The statistical information can be obtained at the receiver and fed back to the transmitter and do not require frequent update. By exploiting channel mean and covariance information at the transmitter simultaneously, this paper investigates the optimal transmission strategy for spatially correlated MIMO channels. An upper bound of ergodic capacity is derived and taken as the performance criterion. Simulation results are also given to show the performance improvement of the optimal transmission strategy.

Key words: MIMO, Optimal transmission strategy, Channel statistical information

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INTRODUCTION

MIMO systems have recently attracted considerable attention as they offer substantial capacity improvement over single antenna systems when perfect or partial channel state information (CSI) is available at the transmitter (Foschini and Gans, 1998; Telatar, 1999). For slowly time-varying wireless channels, the instantaneous channel information can be estimated at the receiver and fed back to the transmitter. But when the channel varies rapidly, it is costly or impossible to acquire instantaneous CSI at the transmitter. The channel statistical information can be seen as relatively stationary. Designing optimal transmitters based on channel statistical information has been an active research area.

There are several performance criteria in studying MIMO systems. When system error probability is taken as the performance criterion, a common system includes a space-time block code and a linear precoder which exploits the CSI at the transmitter. Examples of prior works using this criterion include schemes exploiting imperfect channel estimates

(Jongren *et al.*, 2002; Liu and Jafarkhani, 2005), the channel mean (Zhou and Giannakis, 2002), or the channel covariance (Sampath and Paulraj, 2002; Zhou and Giannakis, 2003). Vu and Paulraj (2004) designed a precoder to simultaneously exploit both the channel mean and the transmit correlation to minimize the error probability. Another performance criterion is to maximize ergodic capacity using the CSI at the transmitter. Researches in this category include studies on exploiting the channel mean (Visotsky and Madhow, 2001; Venkatesan *et al.*, 2003), or the channel covariance (Visotsky and Madhow, 2001; Jafar and Goldsmith, 2004; Jorswieck and Boche, 2004). Otherwise, by taking the average MMSE as the performance measure, Jorswieck and Boche (2006) derived the optimal transmit strategy with statistical CSI.

In this paper, taking ergodic capacity as the criterion and exploiting both the channel mean and covariance information at the transmitter, we investigate the optimal transmission strategy for spatially correlated MIMO system. We consider the general case with double-sided correlation and arbitrary rank

channel mean. The optimal input signal covariance matrix that maximizes the channel ergodic capacity is derived and the optimal power allocation algorithm is presented. Some numerical results are given for studying the effects of the system parameters on the optimal transmission strategy.

SYSTEM MODEL

Consider a MIMO wireless system with N_t transmit and N_r receive antennas modeled by

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where $\mathbf{r} \in \mathbb{C}^{N_r \times 1}$ is the received signal vector, $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ is the transmitted signal vector satisfying the power constraint $E[\mathbf{x}^H \mathbf{x}] \leq E_x$, $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is a vector of zero-mean additive complex Gaussian noise with $E[\mathbf{n}\mathbf{n}^H] = \sigma_n^2 \mathbf{I}_{N_r}$, and $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix. Assuming the channel has non-zero mean and spatially correlation at both ends of the MIMO link, then the channel can be written in the form (Vu and Paulraj, 2004)

$$\mathbf{H} = \sqrt{a}\bar{\mathbf{H}} + \sqrt{b}\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w\mathbf{R}_t^{\frac{1}{2}}, \quad (2)$$

where $\bar{\mathbf{H}}$ is the normalized channel mean, \mathbf{H}_w is a complex Gaussian random matrix with independent zero-mean unit-variance entries, a and b are power normalization coefficients, \mathbf{R}_r and \mathbf{R}_t are normalized receive and transmit correlation matrices, such that

$$\text{tr}(\bar{\mathbf{H}}^H \bar{\mathbf{H}}) = N_t N_r; \quad \text{tr}\mathbf{R}_r = N_r; \quad \text{tr}\mathbf{R}_t = N_t. \quad (3)$$

The ergodic capacity of the MIMO system in Eq.(1) can be expressed by (Telatar, 1999)

$$C = \max_{\mathbf{Q} \succeq \mathbf{0}} E \left[\log_2 \det \left(\mathbf{I}_{N_r} + \frac{\gamma}{N_t} \mathbf{H}\mathbf{Q}\mathbf{H}^H \right) \right], \quad (4)$$

where $\gamma = E_x / \sigma_n^2$ is the signal to noise ratio (SNR), $\mathbf{Q} = E[\mathbf{x}\mathbf{x}^H]$ is the covariance of the input signal, \mathbf{I}_{N_r} is an $N_r \times N_r$ identity matrix. Throughout this paper, we

assume that the channel is perfectly known at the receiver and the channel mean ($\bar{\mathbf{H}}$) and fading channel covariance matrices ($\mathbf{R}_r, \mathbf{R}_t$) are known at the transmitter. The problem of maximizing the channel ergodic capacity reduces to finding the optimal covariance of the Gaussian input signal as a function of the channel statistical information obtained at the transmitter.

OPTIMAL TRANSMISSION STRATEGY

The SVD of the channel mean is $\sqrt{a}\bar{\mathbf{H}} = \mathbf{U}\mathbf{A}\mathbf{V}^H$, where \mathbf{U} and \mathbf{V} are unitary matrices. From Eq.(2), we can derive that

$$\begin{aligned} \mathbf{H}\mathbf{Q}\mathbf{H}^H &= (\sqrt{a}\bar{\mathbf{H}} + \sqrt{b}\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w\mathbf{R}_t^{\frac{1}{2}})\mathbf{Q}(\sqrt{a}\bar{\mathbf{H}} + \sqrt{b}\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w\mathbf{R}_t^{\frac{1}{2}})^H \\ &= \mathbf{U}(\mathbf{A} + \sqrt{b}\mathbf{U}^H\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w\mathbf{R}_t^{\frac{1}{2}}\mathbf{V})\mathbf{V}^H\mathbf{Q}\mathbf{V} \\ &\quad \cdot (\mathbf{A}^H + \sqrt{b}\mathbf{V}^H\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w^H\mathbf{R}_t^{\frac{1}{2}}\mathbf{U})\mathbf{U}^H. \end{aligned} \quad (5)$$

Then the channel ergodic capacity can be obtained:

$$\begin{aligned} C &= \max_{\mathbf{Q} \succeq \mathbf{0}} E \left\{ \log_2 \det \left[\mathbf{I}_{N_r} + \frac{\gamma}{N_t} (\mathbf{A} + \sqrt{b}\mathbf{U}^H\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w\mathbf{R}_t^{\frac{1}{2}}\mathbf{V}) \right. \right. \\ &\quad \left. \left. \cdot \mathbf{V}^H\mathbf{Q}\mathbf{V}(\mathbf{A}^H + \sqrt{b}\mathbf{V}^H\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w^H\mathbf{R}_t^{\frac{1}{2}}\mathbf{U}) \right] \right\} \\ &= \max_{\mathbf{Q} \succeq \mathbf{0}} E \left\{ \log_2 \det \left[\mathbf{I}_{N_t} + \frac{\gamma}{N_t} \mathbf{Q}^{\frac{H}{2}}\mathbf{V}(\mathbf{A}^H + \sqrt{b}\mathbf{V}^H\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w^H\mathbf{R}_t^{\frac{1}{2}}\mathbf{U}) \right. \right. \\ &\quad \left. \left. \cdot \mathbf{H}_w^H\mathbf{R}_t^{\frac{1}{2}}\mathbf{U}(\mathbf{A} + \sqrt{b}\mathbf{U}^H\mathbf{R}_r^{\frac{1}{2}}\mathbf{H}_w\mathbf{R}_t^{\frac{1}{2}}\mathbf{V})\mathbf{V}^H\mathbf{Q}^{\frac{1}{2}} \right] \right\}. \end{aligned} \quad (6)$$

Eq.(6) is obtained using the property $|\mathbf{I} + \mathbf{A}\mathbf{B}| = |\mathbf{I} + \mathbf{B}\mathbf{A}|$, for arbitrary \mathbf{A} and \mathbf{B} .

Define $\tilde{\mathbf{H}} = \sqrt{b}\mathbf{U}^H\mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2}\mathbf{V}$ and $\tilde{\mathbf{Q}}^{1/2} = \mathbf{V}^H\mathbf{Q}^{1/2}$, then the power constraint of the input signal covariance changes to be

$$\text{tr}\mathbf{Q} = \text{tr}(\mathbf{V}\tilde{\mathbf{Q}}\mathbf{V}^H) = \text{tr}\tilde{\mathbf{Q}}. \quad (7)$$

And Eq.(6) can be written as

$$C = \max_{\mathbf{Q} \succeq \mathbf{0}} E \left\{ \log_2 \det \left[\mathbf{I}_{N_t} + \frac{\gamma}{N_t} \tilde{\mathbf{Q}}^{\frac{H}{2}} (\mathbf{A}^H + \tilde{\mathbf{H}}^H) (\mathbf{A} + \tilde{\mathbf{H}}) \tilde{\mathbf{Q}}^{\frac{1}{2}} \right] \right\}. \quad (8)$$

As the expression $\tilde{\mathbf{Q}}^{H/2}(\mathbf{A}^H + \tilde{\mathbf{H}}^H)(\mathbf{A} + \tilde{\mathbf{H}})\tilde{\mathbf{Q}}^{1/2}$ is of non-central matrix-variate complex quadratic form, the optimization problem in Eq.(8) is difficult to solve. Therefore, we derive an upper bound of the ergodic capacity and find the optimal input signal covariance matrix to maximize the upper bound satisfying the power constraint.

Noting that the ‘log₂det’ function is a concave function on the set of Hermitian positive-definite matrices (Telatar, 1999) and using Jensen’s inequality, we can get the upper bound of Eq.(8)

$$C \leq C_{\text{bound}} = \max_{\text{tr}\tilde{\mathbf{Q}} \leq 1} \log_2 \det \left\{ \mathbf{I}_{N_t} + \frac{\gamma}{N_t} \tilde{\mathbf{Q}}^H \cdot E[(\mathbf{A}^H + \tilde{\mathbf{H}}^H)(\mathbf{A} + \tilde{\mathbf{H}})]\tilde{\mathbf{Q}} \right\}. \quad (9)$$

From the definition of $\tilde{\mathbf{H}}$, we can see that it is zero mean. Then

$$E[(\mathbf{A}^H + \tilde{\mathbf{H}}^H)(\mathbf{A} + \tilde{\mathbf{H}})] = \mathbf{A}^H \mathbf{A} + E(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}), \quad (10)$$

and

$$E(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}) = bE[(\mathbf{V}^H \mathbf{R}_t^{H/2} \mathbf{H}_w^H \mathbf{R}_t^{H/2} \mathbf{U})(\mathbf{U}^H \mathbf{R}_t^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} \mathbf{V})] = b\mathbf{V}^H \mathbf{R}_t^{H/2} E(\mathbf{H}_w^H \mathbf{R}_t \mathbf{H}_w) \mathbf{R}_t^{1/2} \mathbf{V}. \quad (11)$$

As a complex Gaussian matrix, \mathbf{H}_w can be noted by $\mathbf{H}_w \sim CN_{N_r, N_t}(\mathbf{0}_{N_r \times N_t}, \mathbf{I}_{N_t} \otimes \mathbf{I}_{N_r})$, where \otimes denotes Kronecker product. For the Hermitian positive-definite matrix \mathbf{R}_t , the matrix $\mathbf{H}_w^H \mathbf{R}_t \mathbf{H}_w$ is of non-central matrix-variate complex quadratic form. From Theorem 7.7.1 in (Gupta and Nagar, 2000), we can derive that

$$E(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}) = b\mathbf{V}^H \mathbf{R}_t^{H/2} N_r \mathbf{I}_{N_t} \mathbf{R}_t^{1/2} \mathbf{V} = bN_r \mathbf{V}^H \mathbf{R}_t \mathbf{V}. \quad (12)$$

Substituting Eq.(10) and Eq.(12) into Eq.(9), the upper bound can be obtained as

$$C_{\text{bound}} = \max_{\text{tr}\tilde{\mathbf{Q}} \leq 1} \log_2 \det \left(\mathbf{I}_{N_t} + \frac{\gamma}{N_t} \tilde{\mathbf{Q}}^H \boldsymbol{\Sigma} \tilde{\mathbf{Q}} \right), \quad (13)$$

where $\boldsymbol{\Sigma} = \mathbf{A}^H \mathbf{A} + bN_r \mathbf{V}^H \mathbf{R}_t \mathbf{V}$ is a positive-definite matrix. Denoting the SVD of $\boldsymbol{\Sigma}$ to be $\mathbf{U}_\Sigma \mathbf{A}_\Sigma \mathbf{U}_\Sigma^H$,

Eq.(13) can be written as

$$C_{\text{bound}} = \max_{\text{tr}\tilde{\mathbf{Q}} \leq 1} \log_2 \det \left(\mathbf{I}_{N_t} + \frac{\gamma}{N_t} \tilde{\mathbf{Q}}^H \mathbf{U}_\Sigma \mathbf{A}_\Sigma \mathbf{U}_\Sigma^H \tilde{\mathbf{Q}} \right). \quad (14)$$

Define $\mathbf{Q}_a^{1/2} = \mathbf{U}_\Sigma^H \tilde{\mathbf{Q}}^{1/2}$, then $\text{tr}\mathbf{Q}_a = \text{tr}\tilde{\mathbf{Q}}$,

$$C_{\text{bound}} = \max_{\text{tr}\mathbf{Q}_a \leq 1} \log_2 \det \left(\mathbf{I}_{N_t} + \frac{\gamma}{N_t} \mathbf{A}_\Sigma \mathbf{Q}_a \right). \quad (15)$$

The matrix $\mathbf{I}_{N_t} + \gamma \mathbf{A}_\Sigma \mathbf{Q}_a / N_t$ being positive semi-definite and using Hadamard’s inequality, the determinant in Eq.(15) is maximized when \mathbf{Q}_a is diagonal. The optimal signal covariance matrix that maximizes the upper bound of the ergodic capacity can be determined by $\mathbf{Q} = \mathbf{V} \mathbf{U}_\Sigma \mathbf{Q}_a \mathbf{U}_\Sigma^H \mathbf{V}^H$. Comparing it with the SVD of \mathbf{Q} , we can see that the optimal transmission directions $\mathbf{U}_Q = \mathbf{V} \mathbf{U}_\Sigma$.

From the above analysis, we can see that the optimal input signal covariance matrix boils down to a power allocation strategy when the optimal transmission directions are determined. Denote the i th diagonal element of the diagonal matrices \mathbf{Q}_a and \mathbf{A}_Σ to be λ_i and λ_{Σ_i} respectively, with the elements being in nonincreasing order. The optimization problem is to find the optimal λ_i such that the following summation is maximized:

$$C_{\text{bound}} = \max_{\lambda_i} \sum_{i=1}^{N_t} \log_2 \left(1 + \frac{\gamma}{N_t} \lambda_{\Sigma_i} \lambda_i \right) \quad \text{s.t.} \quad \sum_{i=1}^{N_t} \lambda_i = 1. \quad (16)$$

The optimal λ_i ($i=1, 2, \dots, N_t$) in Eq.(16) can be obtained by water-filling (Vucetic and Yuan, 2004).

NUMERICAL RESULTS

We consider a power normalization model with $a=K/(K+1)$ and $b=1/(K+1)$, where K is the ratio of the power in the fixed (mean) component with respect to the average power in the fading components. Since $a+b=1$, for fixed total transmit power, the receiver SNR remains constant for any value of K .

The correlation matrices are generated using the practical channel model in (Bolcskei et al., 2003). There are uniform linear arrays at the transmitter and

the receiver. The antenna spacing between adjacent antennas (measured in number of wavelength $\lambda=c/f$, where f is the frequency of the narrowband signal) is denoted as d_r at the receiver and d_t at the transmitter. In all the following simulations, we assume $d_r=1/2$ and $d_t=1/2$. The mean angle of arrival (AoA), mean angle of departure (AoD), receive angle spread and transmit angle spread are denoted as $\theta_r, \theta_t, \sigma_r^2, \sigma_t^2$ respectively and let $\underline{\theta}_r = \theta_r + \hat{\theta}_r$ and $\underline{\theta}_t = \theta_t + \hat{\theta}_t$ denote the actual AoA and AoD respectively, with $\hat{\theta}_r \sim N(0, \sigma_r^2)$ and $\hat{\theta}_t \sim N(0, \sigma_t^2)$. With these assumptions, the (p, q) th entry of correlation matrices is given by

$$\begin{cases} \mathbf{R}_r(p, q) = \exp[-j2\pi(q-p)d_r \cos \theta_r] \\ \quad \cdot \exp\left\{-\frac{1}{2}[2\pi(q-p)d_r \sigma_r \sin \theta_r]^2\right\} \\ \mathbf{R}_t(p, q) = \exp[-j2\pi(q-p)d_t \cos \theta_t] \\ \quad \cdot \exp\left\{-\frac{1}{2}[2\pi(q-p)d_t \sigma_t \sin \theta_t]^2\right\} \end{cases} \quad (17)$$

The mean matrix $\bar{\mathbf{H}}$ is generated using

$$\bar{\mathbf{H}} = \sum_{i=1}^P \beta_i \mathbf{a}(\theta_{r,i}) \mathbf{a}^T(\theta_{t,i}), \quad (18)$$

where P is the number of dominant parts, β_i is the complex amplitude of the i th path, and

$$\mathbf{a}(\theta) = [1 \quad e^{j2\pi d \cos \theta} \quad \dots \quad e^{j2\pi(M-1)d \cos \theta}]^T. \quad (19)$$

Fig.1 gives the upper bound of the ergodic capacity for 4x4 MIMO systems vs SNR. High levels of correlation are considered at both ends, with $\theta_r = \pi/2, \theta_t = \pi/2, \sigma_r^2 = \pi/64, \sigma_t^2 = \pi/128$. The mean matrix is rank 1 and $K=20$. The curves denote three different power allocation strategies: optimal, equal and beamforming. We see that the optimal power allocation scheme has better capacity performance than beamforming and equal power allocation schemes. The optimal scheme tends to allocate power equally over the transmitters at high SNR and converges to beamforming when the SNR is small.

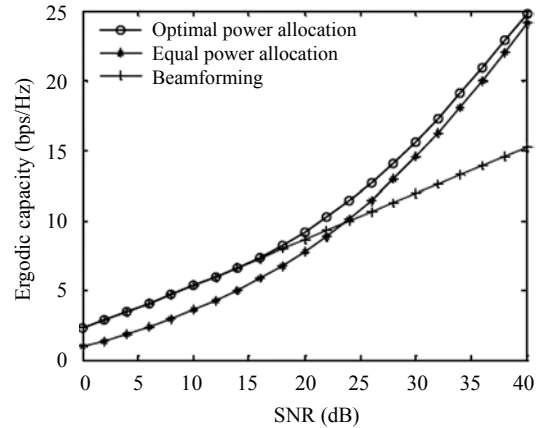


Fig.1 Upper bound of ergodic capacity for 4x4 MIMO system with different power allocation schemes

Fig.2 shows the upper bound of ergodic capacity for 2x2 MIMO system with different K . The correlation and mean matrix parameters are the same as for Fig.1 and the SNR is set to 20 dB. This figure shows that the capacity of beamforming changes little in all the range of K . While the capacity of optimal and equal power allocation schemes decrease monotonically with K . When K is large enough, the optimal power allocation converges to beamforming, because when K is large, the fixed component has more power gain than the fading components, so that the optimal scheme tends to allocate all transmit power to the fixed component.

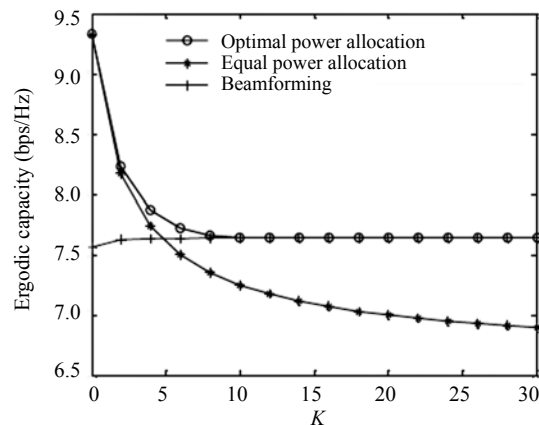


Fig.2 Upper bound of ergodic capacity for 2x2 MIMO system with different K

Fig.3 analyzes the effect of the transmit angle spread on the capacity upper bound for 4x4 MIMO system with optimal power allocation scheme. The system parameters are the same as those for Fig.1

except for the transmit angle spread. Small transmit angle spread means high correlation between the transmitters. We can see that the transmit correlation degrades the ergodic capacity's upper bound especially at high SNR.

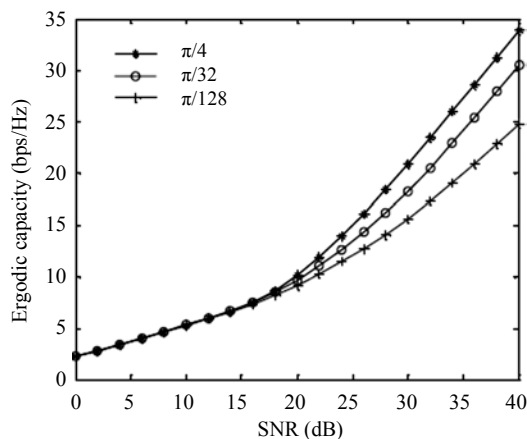


Fig.3 Upper bound of ergodic capacity for 4×4 MIMO system with different transmit angle spreads

CONCLUSION

In this paper, by exploiting channel statistical information at the transmitter, we have investigated the optimal transmission strategy for spatially correlated MIMO system. An upper bound of ergodic capacity is taken as the performance criterion. The optimal transmission directions are derived and the optimal power allocation algorithm is presented. Simulation results show that the optimal power allocation scheme has better performance than beamforming and equal power allocation schemes. The effects of power ratio K and angle spread on the upper bound of ergodic capacity are also analyzed.

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