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An unscented particle filter for ground maneuvering target tracking*

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Abstract: In this study, an unscented particle filtering method based on an interacting multiple model (IMM) frame for a Markovian switching system is presented. The method integrates the multiple model (MM) filter with an unscented particle filter (UPF) by an interaction step at the beginning. The framework (interaction/mixing, filtering, and combination) is similar to that in a standard IMM filter, but an UPF is adopted in each model. Therefore, the filtering performance and degeneracy phenomenon of particles are improved. The filtering method addresses nonlinear and/or non-Gaussian tracking problems. Simulation results show that the method has better tracking performance compared with the standard IMM-type filter and IMM particle filter.

Key words: Interacting multiple model (IMM), Unscented particle filter (UPF), Ground target tracking, Particle filter (PF)

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INTRODUCTION

With the increased demands in recent years of modern warfare, ground target tracking has become the focus of increased investigation. Tracking ground targets is quite different from tracking airborne and sea-surface targets in some aspects such as high target density, complex target kinematics, heavy clutter, low detection rate, and terrain and road constraints (Kirubarajan *et al.*, 1998; Chong *et al.*, 2000; Kreucher and Kastella, 2001). Variable terrain conditions not only limit the maneuverability of targets but also degrade the quality of measurement data. As an alternative, terrain and road information are important sources for data to improve the tracking performance of ground targets.

A single model cannot effectively model the complex variables and maneuvering complexity of a ground target tracking system. One popular technique for tracking maneuvering targets is the multiple model (MM) approach, especially the interacting multiple model (IMM) approach (Bar-Shalom *et al.*, 1989; 2005). The MM approach has proven to be a suboptimal method for handling such tracking problems. The IMM estimator performs much better than MM methods (Bar-Shalom and Li, 1993). The application of IMM for tracking ground target is presented using a ground moving target indicator (GMTI) (Kirubarajan *et al.*, 1998).

Complex, multiple ground target tracking, with non-maneuvering, low maneuvering and high maneuvering targets simultaneously, belongs to the class of linear/nonlinear and Gaussian/non-Gaussian filtering problem. There is no way to obtain more accurate estimates using only Kalman filter (KF) or extended Kalman filter (EKF). Gordon *et al.*(1993) proposed the first working particle filter (PF) or bootstrap filter. By using a large number of random samples (particles), the probability density of state distribution can be directly approximated. Paticle filter based methods, therefore, can deal with any nonlinearity or non-Gaussianity in the dynamical

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state and measurement models. To improve the filtering performance, an unscented particle filter (UPF) (van der Merwe *et al.*, 2000; van der Merwe, 2004), which uses an unscented Kalman filter (UKF) (Julier and Uhlmann, 1997) to generate the importance proposal distribution, is used for updating stage of sequential importance sampling (SIS). The UPF method has two advantages: first, it makes efficient use of the latest available information; secondly, it has more heavy tails. To approximate the true mean and covariance of Gaussian random variable, the EKF method achieves only the first-order. The unscented filter, in contrast, accurately captures up to the third order (Taylor series expansions) for any nonlinearity (Wan and van der Merwe, 2000). Boers and Driessen (2003) and Hong *et al.*(2006) proposed an IMM particle filter which uses a regularized particle filter for the filtering step.

Based on the IMM framework, an interacting multiple model unscented particle filter (IMMUPF) is presented for tracking ground maneuvering targets. The central idea is to integrate an UPF with the IMM estimator. The algorithm uses a multiple model framework with the number of particles fixed in each model. The interaction/mixing between models is accomplished by interaction of the particle mass from each model. In the filtering stage, some particle filters are used for updating. At the same time, the UKF is applied to update the true mean and covariance of the proposal distribution for improving estimate accuracy. Moreover, the residual resampling scheme is adopted to curtail the degeneracy of data by using a fixed number of particles.

The organization of this paper is as follows. The dynamic models for ground target tracking are described in Section 2. The general UPF approach and an UPF algorithm based on the IMM frame for tracking ground maneuvering targets are presented in Section 3 and Section 4, respectively. To validate the IMMUPF algorithm, a ground target-tracking example is given in Section 5. Finally, Section 6 concludes the paper.

DYNAMIC SYSTEM

A general dynamic system for multiple models in discrete time is rendered by

$$\begin{aligned}\boldsymbol{x}(k+1) = g(\boldsymbol{x}(k), t_k, M(k))w(\boldsymbol{x}(k), M(k)) \\ + f(\boldsymbol{x}(k), t_k, M(k)),\end{aligned} \tag{1}$$

$$\boldsymbol{z}(k) = h(\boldsymbol{x}(k), t_k, M(k)) + v(\boldsymbol{x}(k), M(k)), \tag{2}$$

where $f(\cdot)$ and $h(\cdot)$ are the parameterized state transition and measurement functions; $\boldsymbol{x}(k)\in\mathbb{R}^{n[M(k)]}$ is the dynamic state of the system in mode $M(k)$, and $M(k)\in M\subset\mathbb{N}$ is the modal state of the system, and the system itself is a Markov chain with r states; w, v are the process noise and measurement noise with means $\bar{w}$ and $\bar{v}$ and covariances $\boldsymbol{Q}(k)$ and $\boldsymbol{R}(k)$, respectively, and the two noise sequences are white and possibly mode-dependent; $\boldsymbol{z}(k)\in\mathbb{R}^{p[M(k)]}$ is the measurement in mode $M(k)$; $g(\cdot)$ is the input; t_k is the sampling time, in this context a constant variable.

The model transition probability is modeled as a Markov chain with

$$\begin{aligned}\pi_{ij} = Prob\{M_k = j \mid M_{k-1} = i\}, \\ \forall i, j \in M,\ M = \{1, ..., r\},\end{aligned} \tag{3}$$

where r is the number of possible models. In continuous time, $M(k)$, under assumption, is in effect within the semi-closed interval $(t_{k-1},t_k]$.

UNSCENTED PARTICLE FILTER

The PF is a recursive estimation method using Monte Carlo simulation within a Bayesian framework (Farina and Ristic, 2002; Arulampalam *et al.*, 2002). The central idea is to obtain the MMSE (minimum mean-square error) of state from a set of random samples (particles) of state space to approximate the required probability density function (PDF). It is often impossible to directly sample from the posterior probability density. Thus, an importance proposal distribution, $q(\boldsymbol{x}_{0:k}|\boldsymbol{Z}_{1:k})$, with identical distribution to the posterior distribution is introduced, where $\boldsymbol{Z}_{1:k}$ are all measurements from t=1 to t=k. The bootstrap filter (Gordon *et al.*, 1993) simply takes the prior distribution as the proposal distribution by

$$q(\boldsymbol{x}_k^i \,|\, \boldsymbol{x}_{0:(k-1)}^i, \boldsymbol{Z}_{1:k}) = p(\boldsymbol{x}_k^i \,|\, \boldsymbol{x}_{k-1}^i) \approx N(f(\boldsymbol{x}_{k-1}^i), \boldsymbol{Q}_{k-1}) \tag{4}$$

in calculating the importance weights. However, it would cause a larger error if there is little overlap between prior and the likelihood. To obtain more accurate proposal distribution within the particle filter frame, UKF may be used to update the mean and covariance of the Gaussian approximation to the state distribution given by

$$p(\boldsymbol{x}_k^i \left| \boldsymbol{x}_{0:(k-1)}^i, \boldsymbol{Z}_{1:k}\right.) \approx N(\hat{\boldsymbol{x}}_{k-1}^i, \boldsymbol{P}_{k|k}^i),\ \ i=1,\ ...,\ N, \quad (5)$$

where N is the number of sampling particles. So the UPF can be derived (van der Merwe *et al*., 2000). For completeness, one cycle of the UPF algorithm can be described in detail as follows:

1. Initialization

Let $\{\boldsymbol{x}_0^i\}_{i=1}^N$ be a set of particles sampled from the prior $p(\boldsymbol{x}_0)$ at k=0 and set

$$\hat{\boldsymbol{x}}_0^i = E[\boldsymbol{x}_0^i],\ \boldsymbol{P}_0^i = E[(\boldsymbol{x}_0^i - \hat{\boldsymbol{x}}_0^i)(\boldsymbol{x}_0^i - \hat{\boldsymbol{x}}_0^i)^{\mathrm{T}}]. \quad (6)$$

2. Importance sampling

(1) Update each particle with the UKF to obtain mean $\hat{\boldsymbol{x}}_{k|k}^i$ and covariance $\boldsymbol{P}_{k|k}^i$ (Julier and Uhlmann, 1997; Julier, 2002). Let n-dimensional state vector $\boldsymbol{x}_{k-1}$ with mean $\hat{\boldsymbol{x}}_{(k-1)|(k-1)}$ and covariance $\boldsymbol{P}_{(k-1)|(k-1)}$ be approximated by $2n$+1 weighted samples or sigma points. Then one cycle of the UKF is as follows:

(i) Calculate sigma points:

$$\begin{cases} \boldsymbol{\chi}_{(k-1)|(k-1)}^0 = \hat{x}_{(k-1)|(k-1)}, & i=0, \\ \boldsymbol{\chi}_{(k-1)|(k-1)}^i = \hat{\boldsymbol{x}}_{(k-1)|(k-1)} + \left(\gamma\sqrt{\boldsymbol{P}_{(k-1)|(k-1)}}\right)_i, & i=1,\ ...,\ n, \\ \boldsymbol{\chi}_{(k-1)|(k-1)}^{i+n} = \hat{\boldsymbol{x}}_{(k-1)|(k-1)} - \left(\gamma\sqrt{\boldsymbol{P}_{(k-1)|(k-1)}}\right)_i, & i=1,\ ...,\ n, \end{cases} \quad (7)$$

where $\gamma=\sqrt{n+\kappa}$, the corresponding weights are given by

$$\begin{cases} w_0 = \kappa/(n+\kappa), \\ w_i = 1/[2(n+\kappa)],\ i=1,\ 2,\ ...,\ 2n, \end{cases} \quad (8)$$

where κ is a scaling factor and $(\sqrt{(n+\kappa)\boldsymbol{P}_{k|k-1}})_i$ is the ith row or column of the matrix square root of $(n+\kappa)\boldsymbol{P}_{k|(k-1)}$ and w_i is the weight associated with the ith point such that $\sum_{i=0}^{2n} w_i = 1$. Numerically efficient and stable methods such as the Cholesky decomposition (Press *et al.*, 1992) are needed for the matrix square root.

(ii) Propagation (time update): the sigma points are propagated and the estimated mean and covariance of the state are computed as follows:

$$\boldsymbol{\chi}_{k|(k-1)}^i = f(\boldsymbol{\chi}_{(k-1)|(k-1)}^i),\ \hat{\boldsymbol{x}}_{k|(k-1)} = \sum_{i=0}^{2n} w_i \boldsymbol{\chi}_{k|(k-1)}^i, \quad (9)$$

$$P_{k|(k-1)} = \sum_{i=0}^{2n} w_i [\boldsymbol{\chi}_{(k-1)|(k-1)}^i - \hat{\boldsymbol{x}}_{k|(k-1)}][\boldsymbol{\chi}_{(k-1)|(k-1)}^i - \hat{\boldsymbol{x}}_{k|(k-1)}]^{\mathrm{T}} + \boldsymbol{Q}(k-1). \quad (10)$$

(iii) Measurement update: using $h(\cdot)$ to calculate the measurement sigma points $\xi_{k|(k-1)}^i$ and update the mean and covariance by

$$\xi_{k|(k-1)}^i = h(\boldsymbol{\chi}_{k|(k-1)}^i),\ \ \hat{z}_{k|(k-1)} = \sum_{i=0}^{2n} w_i \xi_{k|(k-1)}^i, \quad (11)$$

$$\boldsymbol{P}_z = \sum_{i=0}^{2n} w_i (\xi_{k|(k-1)}^i - \hat{z}_{k|(k-1)})(\xi_{k|(k-1)}^i - \hat{z}_{k|(k-1)})^{\mathrm{T}}. \quad (12)$$

(iv) Calculate filter gain:

$$\begin{cases} \boldsymbol{P}_{xz} = \sum_{i=0}^{2n} w_i (\boldsymbol{\chi}_{k|(k-1)}^i - \hat{\boldsymbol{x}}_{k|(k-1)})(\xi_{k|(k-1)}^i - z_{k|(k-1)})^{\mathrm{T}}, \\ \boldsymbol{K}_k = \boldsymbol{P}_{xz}\boldsymbol{P}_z^{-1}. \end{cases} \quad (13)$$

(v) Output:

$$\begin{cases} \hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|(k-1)} + \boldsymbol{K}_k(z_k - \hat{z}_{k|(k-1)}), \\ \boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|(k-1)} - \boldsymbol{K}_k \boldsymbol{P}_z \boldsymbol{K}_k^{\mathrm{T}}. \end{cases} \quad (14)$$

(2) Sample $\hat{\boldsymbol{x}}_{k|k}^i$ can be drawn from $q(\boldsymbol{x}_k^i \mid \boldsymbol{x}_{0:(k-1)}^i, \boldsymbol{Z}_{1:k}) = N(\bar{\boldsymbol{x}}_{k|k}^i, \boldsymbol{P}_{k|k}^i)$. Set $\tilde{\boldsymbol{x}}_{0:k}^i = \{\boldsymbol{x}_{0:(k-1)}^i, \hat{\boldsymbol{x}}_{k|k}^i\}$ and $\tilde{\boldsymbol{P}}_{0:k}^i = \{\boldsymbol{P}_{0:(k-1)}^i, \boldsymbol{P}_{k|k}^i\}$.

(3) The importance weight can be evaluated by

$$w_k^i \propto \frac{p(z_k \left| \hat{\boldsymbol{x}}_{k|k}^i\right.) p(\boldsymbol{x}_k^i \left| \boldsymbol{x}_{(k-1)}^i\right.)}{q(\hat{\boldsymbol{x}}_{k|k}^i \left| \boldsymbol{x}_{0:(k-1)}^i, \boldsymbol{Z}_{1:k}\right.)},\ \ i=1,\ ...,\ N. \quad (15)$$

The normalized weight is given by

$$\tilde{w}_k^i = w_k^i \Big/ \sum_{i=1}^N w_k^i. \quad (16)$$

3. Selection or resampling

To obtain N random particles $\{\hat{\boldsymbol{x}}_{0:k}^i, \hat{\boldsymbol{P}}_{0:k}^i\}_{i=1}^N$, it is necessary to resample N times from $\{\tilde{\boldsymbol{x}}_{0:k}^i, \tilde{\boldsymbol{P}}_{0:k}^i\}_{i=1}^N$ according to the importance weights. The procedure multiplies the high-weight particles and eliminates the low-weight particles in the state space to match the changes of the PDF over the state transition. Then, set the weights $w_k^i = \tilde{w}_k^i = 1/N$. Note that the above resampling scheme reduces the degeneracy phenomenon at the cost of increasing the computational load and losing the particle diversity. Other efficient resampling schemes, such as regularized particle filter (Musso *et al.*, 2001) and MCMC (Markov chain Monte Carlo) step (Robert and Casella, 1999), may be applied to compensate the drawbacks.

4. Output

Via the output of a set of samples used to approximate the posterior distribution given by

$$p(\boldsymbol{x}_{0:k} \mid \boldsymbol{Z}_{1:k}) \approx \hat{p}(\boldsymbol{x}_{0:k} \mid \boldsymbol{Z}_{1:k}) = \frac{1}{N}\sum_{i=1}^{N} \delta_{(\boldsymbol{x}_{0:k}^i)}(\mathrm{d}\boldsymbol{x}_{0:k}), \quad (17)$$

the state is eventually updated by means of the particles as follows:

$$\hat{\boldsymbol{x}}_{k|k} \approx \frac{1}{N}\sum_{i=1}^{N} \hat{\boldsymbol{x}}_{k|k}^i. \quad (18)$$

INTERACTING MULTIPLE MODEL UNSCENTED PARTICLE FILTER

Integrating the IMM filter with an UPF, an IMMUPF algorithm is derived to address ground target tracking problems. First, the mixed initial condition and the mixing probabilities at k–1 are calculated. Second, a set of particles is randomly sampled in each mode according to the prior probability density $\hat{p}_{0j}((k-1)\mid(k-1))$. The sample is used as the basis for the update. Third, the state and its covariance are updated for each mode with the UPF. The innovations, residual covariance, likelihoods and weights at time step k are then computed simultaneously. The mode probability then may be updated and the new mixing probability can be calculated based on the likelihoods. Last, the state mean and covariance are combined for the next cycle. Starting from k–1, one cycle of the IMMUPF algorithm can be described in detail as follows:

1. Interaction/mixing

Based on the Markov model, the model likelihoods and the posterior probability densities for the different modes at time step k–1, the initial densities $\hat{p}_{0j}(\boldsymbol{x}_{0j}(k-1)\mid \boldsymbol{Z}_{1:(k-1)})$ are computed as Gaussian sum probability densities. At the beginning of the estimation cycle, state estimate and covariance for each filter are initialized as

$$\hat{\boldsymbol{x}}_{0j}((k-1)\mid(k-1)) = \sum_{i=1}^{r} \hat{\boldsymbol{x}}_i((k-1)\mid(k-1))\boldsymbol{\mu}_{i|j}((k-1)\mid(k-1)), \quad (19)$$

$$\hat{\boldsymbol{P}}_{0j}((k-1)\mid(k-1)) = \sum_{i=1}^{r} \boldsymbol{\mu}_{i|j}((k-1)\mid(k-1))\{\boldsymbol{P}_i((k-1)\mid(k-1)) + [\hat{\boldsymbol{x}}_i((k-1)\mid(k-1)) - \hat{\boldsymbol{x}}_{0j}((k-1)\mid(k-1))] \cdot[\hat{\boldsymbol{x}}_i((k-1)\mid(k-1)) - \hat{\boldsymbol{x}}_{0j}((k-1)\mid(k-1))]^{\mathrm{T}}\}, \quad (20)$$

where $\hat{\boldsymbol{x}}_{0j}((k-1)\mid(k-1))$ and $\hat{\boldsymbol{P}}_{0j}((k-1)\mid(k-1))$ are the mixed initial condition for mode-matched filter j at time k–1.

Then the Gaussian mixing probabilities are computed via the equations

$$\begin{cases} \mu_{i|j}((k-1)\big|(k-1)) = \pi_{ij}\mu_i(k-1)/c_j, \\ c_j = \sum_{i=1}^{r}\pi_{ij}\mu_i(k-1), \end{cases} \quad (21)$$

where c_j is a normalization factor.

2. State update/importance sampling

$\{\hat{\boldsymbol{x}}_j^n((k-1)\mid(k-1)), \hat{\boldsymbol{P}}_j^n((k-1)\mid(k-1))\}_{j=1,\dots,r}^{n=1,\dots,N}$ as the sample set is drawn from the state $\hat{\boldsymbol{x}}_{0j}((k-1)\mid(k-1))$ with the probability $\hat{p}_{0j}((k-1)\mid(k-1))$. Then propagate and update the sample set by using the UPF to obtain the posterior samples $\{\hat{\boldsymbol{x}}_j^n(k\mid k), \tilde{w}_j^n(k), \hat{\boldsymbol{P}}_j^n(k\mid k)\}_{j=1,\dots,r}^{n=1,\dots,N}$ at k, where $\tilde{w}_i^n$ is the normalized importance weight. So the innovations, residual covariance and likelihoods can be obtained as follows:

Mean of propagate output over the sample set

$$\bar{h}_j(k) = \sum_{n=1}^{N} h(\hat{\boldsymbol{x}}_j^n(k), k, j). \quad (22)$$

Innovations

$$\boldsymbol{r}_j(k)=\boldsymbol{z}(k)-h(\hat{\boldsymbol{x}}_j^n(k),k,j). \tag{23}$$

Residual covariance over the sample set

$$\hat{\boldsymbol{S}}_j(k)=\sum_{n=1}^{N}\left\{[h(\hat{\boldsymbol{x}}_j^n(k),k,j)-\bar{h}_j(k)]\cdot[h(\hat{\boldsymbol{x}}_j^n(k),k,j)-\bar{h}_j(k)]^{\mathrm{T}}\right\}, \tag{24}$$

Likelihoods

$$\boldsymbol{L}_j^n(k)=N(\boldsymbol{r}_j^l(k);0;\hat{\boldsymbol{S}}_j(k)). \tag{25}$$

3. Residual resampling

Resample the sample set by evaluating the importance weights of all particles for each model to ensure the particles are distinct for an accurate posterior. The procedure, propagating the particles with higher weights and suppressing the particles with lower weights, is to generate a new sample set with identical weights $\{\hat{\boldsymbol{x}}_j^n(k\,|\,k),\ \tilde{w}_j^n(k)=1/N\}_{j=1,\dots,r}^{n=1,\dots,N}$. Here, a residual resampling scheme is adopted for its smaller Monte Carlo variance and favorable computation time (Arulampalam *et al.*, 2002).

4. Update of the mode probabilities:

$$\mu_j(k)=\boldsymbol{L}_j^n(k)c_j/c,\quad c=\sum_{n=1}^{r}\boldsymbol{L}_j^n(k)c_j. \tag{26}$$

5. Combination (output)

Taking into account the mode probabilities, a combined state estimate can be obtained by averaging over the samples in Step 3 as follows:

$$\hat{\boldsymbol{x}}^n(k\,|\,k)=\sum_{j=1}^{r}\hat{\boldsymbol{x}}_j^n(k\,|\,k)\mu_j^n(k), \tag{27}$$

and

$$\hat{\boldsymbol{P}}^n(k\,|\,k)=\sum_{j=1}^{r}\mu_j^n(k)\{\hat{\boldsymbol{P}}_j^n(k\,|\,k)+[\hat{\boldsymbol{x}}_j^n(k\,|\,k)-\hat{\boldsymbol{x}}(k\,|\,k)][\hat{\boldsymbol{x}}_j^n(k\,|\,k)-\hat{\boldsymbol{x}}(k\,|\,k)]^{\mathrm{T}}\}. \tag{28}$$

The mean and covariance of state at k are given by

$$\hat{\boldsymbol{x}}(k\,|\,k)=\frac{1}{N}\sum_{n=1}^{N}\hat{\boldsymbol{x}}^n(k\,|\,k), \tag{29}$$

$$\hat{\boldsymbol{P}}(k\,|\,k)=\frac{1}{N}\sum_{n=1}^{N}\hat{\boldsymbol{P}}^n(k\,|\,k). \tag{30}$$

AN EXAMPLE OF GROUND MANEUVERING TARGET TRACKING

In this section, the IMMUPF algorithm is applied to a ground target tracking scenario, and its performance is compared with that of the IMMPF method (Boers and Driessen, 2003) and a standard IMMEKF filter (Mazor *et al.*, 1998).

Nonlinear models

To accurately model ground target dynamics, two models based on lateral and longitudinal accelerations are utilized to describe the kinematics of both non-maneuvering and maneuvering ground target (McGinnity and Irwin, 2000; Kreucher and Kastella, 2001; Cui *et al.*, 2005). The target state is defined as $\boldsymbol{x}=[x, y, \theta, v]^{\mathrm{T}}$, where (x, y) is the target's Cartesian location in the x-y plane, θ is the target heading, and v is the target velocity.

Model 1: a constant model (CV) is modeled by (Kreucher and Kastella, 2001; Cui *et al.*, 2005)

$$\begin{cases}\dot{x}=v\cos\theta,\ \dot{y}=v\sin\theta,\\ \dot{\theta}=-[\theta-\theta_0(x,y)]/\tau_\theta(x,y)+w(\theta),\\ \dot{v}=-[v-v_0(x,y)]/\tau_v(x,y)+w(v),\end{cases} \tag{31}$$

where $\theta_0(x,y)$ and $\tau_\theta(x,y)$ are the preferred heading and the mean time to take heading correction, respectively; $v_0(x,y)$ and $\tau_v(x,y)$ are the varying preferred speed and the mean time to speed correction, respectively; $w(\theta)$ and $w(v)$ are respectively zero-mean Gaussian white noise processes with variances given by

$$q_\theta(x,y)=2\sigma_\theta^2/\tau_\theta(x,y),\ q_v(x,y)=2\sigma_v^2/\tau_v(x,y), \tag{32}$$

where σ_θ^2 and σ_v^2 are the variances of heading and velocity deviation from their preferred values, respectively.

Model 2: a general constant lateral acceleration model with non-zero mean noise is defined as (McGinnity and Irwin, 2000; Cui *et al.*, 2005)

$$\dot{x}=v\cos\theta,\ \dot{y}=v\sin\theta,\ \ \dot{\theta}=-u_{\text{lat}}/v,\ \ \dot{v}=0, \tag{33}$$

where u_{lat} is a constant lateral acceleration. This model is called a coordinate turn (CT) model.

To meet the research needs, it is necessary to make Models 1 and 2 discrete. The general dynamic discrete system for the multiple models is defined in Eqs.(1) and (2), where $M\in\{1,2\}$; M=1 and M=2 correspond to Model 1 (CV) and Model 2 (CT), respectively.

Measurement model: the measurement function $h(\cdot)$ is mode-independent and time-independent, and the measurement model is given by

$$\begin{aligned} z(k) &= h(\boldsymbol{x}(k), t_k, M(k)) + v(\boldsymbol{x}(k), M(k)) \\ &= \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \arctan(y_k / x_k) \end{bmatrix} + \begin{bmatrix} v_{r_k} \\ v_{\phi_k} \end{bmatrix}, \end{aligned} \tag{34}$$

where v is a zero-mean Gaussian noise with variance $\boldsymbol{R}(k)$=diag(σ_r^2, σ_θ^2), and σ_r, σ_θ are standard deviations for the range and bearing, respectively. The sensor is assumed at the original point.

Scenario and simulation

From the location of x=y=3 km, the target starts to make an almost CV movement for 150 s with an initial velocity of 20 m/s and heading of 30°. Then it turns for 350 s with a positive turn rate of about 0.15°/s. At last, the target resumes a CV motion for 100 s. The true trajectory for a maneuvering target is shown in Fig.1.

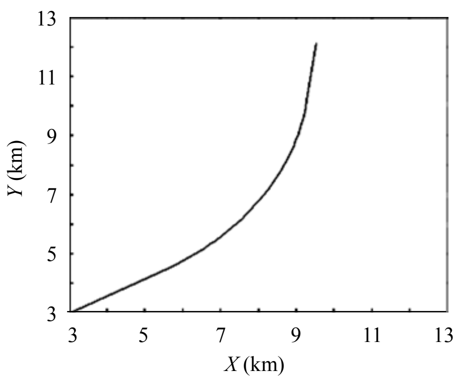

Fig.1 A true trajectory for a maneuvering target

The mode transition matrix is given by

$$\boldsymbol{\pi} = (p_{ij})_{2\times 2} = \begin{bmatrix} 0.97 & 0.03 \\ 0.03 & 0.97 \end{bmatrix}. \tag{35}$$

The model parameters are set as $u_{\rm lat}$=−0.2 rad/s^2 for Model 2. The standard deviations of Model 2 are taken to be: q_x=q_y=3 m, q_θ=0.05°, q_v=0.005 m/s for the process noise and σ_r=4 m, σ_θ=1.15° for the measurement noise. For the IMMPF and IMMUPF algorithms, the total particle number is set as N=1000 with 500 particles for each model at the initiation stage.

Five hundred Monte Carlo simulations with different measurement noises to the same trajectory have been done. The root mean square errors (RMSEs) plots of each state vector for the three filters (IMMEKF, IMMPF and IMMUPF) are shown in Fig.2. It can be seen from Fig.2 and Table 1 that the three filters had almost the same performance in the non-maneuvering phase (k<150 s and k>500 s); but in the maneuvering phase (k=150~500 s), the IMMPF and IMMUPF algorithms perform much better than the standard IMM nonlinear filters in reducing the state estimate errors. The reason for the enhanced performance is that the particle-based multiple model filters use the mixture of multiple Gaussians while the IMM-type filter (IMMEKF) only uses single Gaussian mixing. When target maneuvering occurs, the single Gaussian approximation introduces large errors in the standard IMM filters. In contrast, the two particle filters matched have no limitation to the non-Gaussian condition. They can handle well any non-Gaussianity in the dynamic systems. Comparatively, the filtering accuracy of the IMMUPF algorithm is superior to that of the IMMPF algorithm. The reason lies in that the UKF method is used to generate the importance proposal distribution. As a result, much more accurate mean and covariance can be updated.

Table 1 Comparison of RMSEs for the three filters (100 runs)

Filters	RMSEs-x (m)	RMSEs-y (m)	RMSEs-θ ($\times10^{-3}$ rad)	RMSEs-v (m/s)
IMMEKF	56.81	67.46	7.1631	1.7220
IMMPF	15.31	17.54	4.3912	0.9332
IMMUPF	14.88	16.07	4.1284	0.7932

It is an undeniable fact that the particle-based filters are computationally expensive. However, certain parallel mechanism in implementing the IMM-based filtering approaches (including the particle-based filters and IMM-type filters) can reduce the computational burden. Except for the total time consumed in each filtering approach, the computational

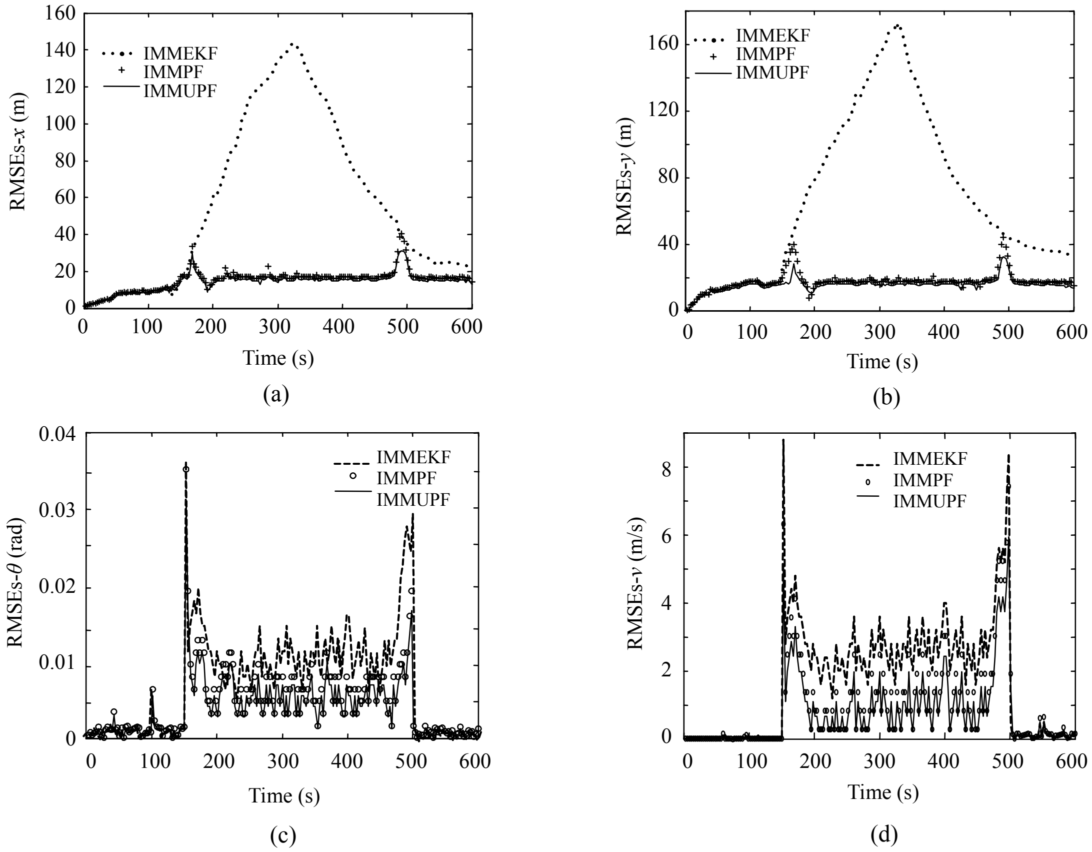

Fig.2 Comparison of the three filters. (a) RMSEs-*x*; (b) RMSEs-*y*; (c) RMSEs-*θ*; (d) RMSEs-*v*

complexity of per particle/model is compared simultaneously. Table 2 shows a comparison of total computational loads as well as the loads per particle/model for the scenario according to the CPU time (system configurations: Celeron (R) CPU 2.0 GHz, memory 512 MB).

Table 2 Comparison of computational complexity for the three filters (1 run, *N*=500, 2 models)

Filters	Total time (s)	Per particle/per model (s)
IMMEKF	0.8930	0.4465
IMMPF	114.85	0.1098
IMMUPF	483.67	0.4726

CONCLUSION

In this research, a particle filter based on the IMM framework using UKF to generate the importance proposal distribution has been presented. Besides the theoretical development of the filtering method, the simulation results demonstrated that the IMM particle filters perform well whereas the standard IMM-type filter behaves poorly when the target is in the maneuvering phase. Furthermore, the filtering method, IMMUPF, performs much better than IMMPF. However, the computational complexity of IMMEKF is far lower than those of IMMPF and IMMUPF. Therefore, for maneuvering targets with weak nonlinearity, IMMEKF is the most efficient filter with acceptable performance; for maneuvering targets with strong nonlinearity, the choice of the filters depends on the user's emphasis. If the parallel processing capacity is sufficient, the IMMPF or IMMUPF should be used to obtain better performance; if not, the IMMEKF should be chosen for computational saving. The priority for future work is to adjust adaptively the number of resampling particles according to the filtering performance.

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