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Optimal state feedback control of brushless direct-current motor drive systems based on Lyapunov stability criterion

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Abstract: This paper develops a unified methodology for a real-time speed control of brushless direct-current motor drive systems in the presence of measurement noise and load torque disturbance. First, the mathematical model and hardware structure of system is established. Next, an optimal state feed back controller using the Kalman filter state estimation technique is derived. This is followed by an adaptive control algorithm to compensate for the effects of noise and disturbance. Those two algorithms working together can provide a very-high-speed regulation and dynamic response over a wide range of operating conditions. Simulated responses are presented to highlight the effectiveness of the proposed control strategy.

Key words: Brushless DC motor, Adaptive control, Optimal state feedback
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INTRODUCTION

The brushless DC motor drive system is a new type speed regulation system. The main advantages of this drive system are: simple structure, reliable operation, easy to be safeguarded like an AC motor drive system (Sangha and Coles, 1995; Dorrell *et al*., 2006). Also it has high performance of speed regulation, high operation effectiveness, no field energy consumption (Desai, 2004; Desai and Emadi, 2005). Brushless DC motor is widely used in computer, instrument and instrumentation, chemical engineering, textile industries, especially in the computer disk drive systems. So, study on brushless DC motor drive system is very cost effective.

The main contributions of the paper may be divided into three parts:

(1) Development of an algorithm for real-time estimation of the system state variables (including speed) by using the Kalman filter theory.

(2) Development of an optimal state feedback control using the estimated state variable by the Lyapunov stability criterion to get the highlighted effectiveness.

(3) Development of an adaptive control algorithm to compensate the effects of noise and disturbance.

MATHEMATICAL MODEL

Function block diagram of brushless DC motor drive systems

Fig.1 shows the interrelationship among the three parts of this paper. Brushless DC motor is controlled by a DSP, where CAP1, CAP2, CAP3 are the captured sensor signals H1, H2, H3 from rotor position of motor. PWM1~PWM6 are the output of PWM signal, through the gate circuits to control the motor speed and phase commutation. The output signal of the controller $u(k)$ compares with the given signal as an input signal of PWM1~PWM6.

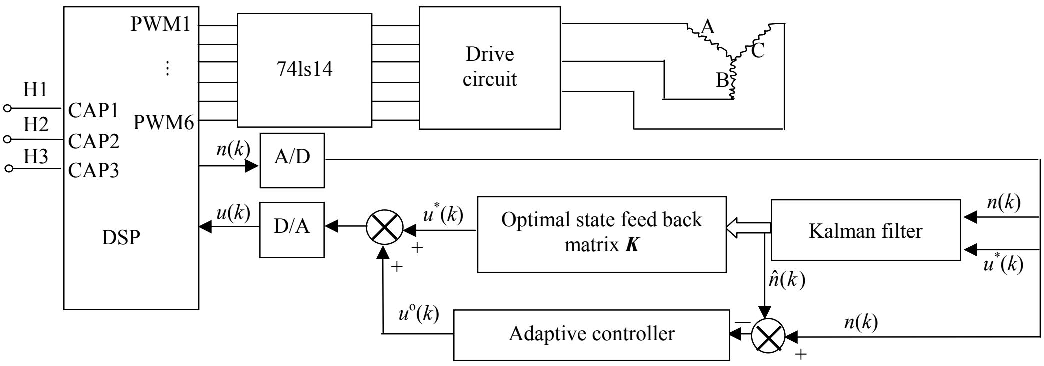

Fig.1 Function block diagram of brushless DC motor drive system

Mathematical model

The dynamic equations of a brushless DC motor may be expressed as

$$\begin{cases} u-\Delta u = E_\mathrm{a} + iR + L\dfrac{\mathrm{d}i}{\mathrm{d}t}, \\ T_\mathrm{a} - T_\mathrm{L} = \dfrac{GD^2}{375}\dfrac{\mathrm{d}n}{\mathrm{d}t}, \\ E_\mathrm{a} = K_\mathrm{e} n, \end{cases} \tag{1}$$

where R is the inner resistance of motor, T_L the load torque of motor, GD^2 the moment of inertia of motor rotor (in N·m^2), Δu the voltage drop of the thyristor, K_e the back EMF coefficient. Torque T_a and back electromotive force (EMF) E_a, using the concept of average torque and average back electromotive force. Due to the bigger ripple (Li *et al*., 2004; Wang *et al*., 2005), the back EMF and torque have

$$T_\mathrm{a} = K_\mathrm{T} I, \tag{2}$$

where I is the current of every phase, K_T the torque coefficient of motor.

Taking Laplace transform of Eqs.(1) and (2) yields the following relationships

$$\begin{cases} U(s)-\Delta U(s) = E_\mathrm{a}(s) + RI(s) + LsI(s), \\ T_\mathrm{a}(s) = K_\mathrm{T} I(s), \\ T_\mathrm{a}(s) - T_\mathrm{L}(s) = \dfrac{GD^2}{375} sN(s), \\ E_\mathrm{a}(s) = K_\mathrm{e} N(s), \end{cases} \tag{3}$$

where ΔU(s) is small to be neglected, then open loop transfer function of the system may be expressed as

$$\frac{E_\mathrm{a}(s)}{U(s)} = \frac{1/(T_\mathrm{l} T_\mathrm{m})}{s(s+1/T_\mathrm{l})} = \frac{T_\mathrm{l}}{T_\mathrm{m}} \cdot \frac{a^2}{s(s+a)}, \tag{4}$$

where $T_\mathrm{l}=L/R$ is the effective electrical time constant, $T_\mathrm{m}=RGD^2/(375K_\mathrm{e}K_\mathrm{T})$ is the mechanical time constant, $a=1/T_\mathrm{l}$.

Allowing for a zero-order hold (sample-hold) in the discrete-time domain representation, the Z transform of the above input-output representation may be expressed as (Kwo, 1992)

$$\begin{aligned} \frac{N(z)}{U(z)} &= \frac{T_\mathrm{l}}{T_\mathrm{m}K_\mathrm{e}}(1-z^{-1})Z\left[\frac{a^2}{s^2(s+a)}\right] \\ &= \frac{T_\mathrm{l}}{T_\mathrm{m}K_\mathrm{e}} \frac{[(aT-1+\mathrm{e}^{-aT}) + (1-\mathrm{e}^{-aT} - aT\mathrm{e}^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})(1-\mathrm{e}^{-aT}z^{-1})} \\ &= \frac{Az+B}{z^2+Fz+D}, \end{aligned} \tag{5}$$

where T is the sample period.

State and output equations at the kth sampled interval are

$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -D & -F \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} A \\ B-AF \end{bmatrix} u(k) \\ \qquad = \boldsymbol{Gx}(k) + \boldsymbol{H}u(k), \\ n(k) = [1 \ \ 0] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \boldsymbol{Cx}(k), \end{cases} \tag{6}$$

where

$$\begin{cases} x_1(k) = n(k), \\ x_2(k) = x_1(k+1) - Au(k), \end{cases} \tag{7}$$

$\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{C}$ are all constant matrices for brushless DC motor. From Eq.(5), the input-output difference equation may be obtained:

$$\begin{aligned} &n(k)+F\cdot n(k-1)+D\cdot n(k-2) \\ &=A\cdot u(k-1)+B\cdot u(k-2). \end{aligned} \tag{8}$$

STATE ESTIMATION USING KALMAN FILTER

In order to realize the optimal feedback control using state feedback, the state vector $\boldsymbol{x}(k)$ needs to be estimated (Rodriguez and Emadi, 2006). Using Kalman filter theory, the algorithm for the derivation $\hat{\boldsymbol{x}}(k)$ can be obtained as

$$\begin{aligned} \hat{\boldsymbol{x}}(k)=\boldsymbol{G}\hat{\boldsymbol{x}}(k-1)+\boldsymbol{H}u(k-1)+\boldsymbol{K}(k)\cdot \\ \left\{n(k)-\boldsymbol{C}\left[\boldsymbol{G}\hat{\boldsymbol{x}}(k-1)+\boldsymbol{H}u(k-1)\right]\right\}, \end{aligned} \tag{9}$$

where $\hat{\boldsymbol{x}}(k)$ is the estimated state vector, and $\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{C}$ are constant matrices known for a brushless DC motor drive system. Choose $\boldsymbol{Q}$, $\boldsymbol{R}$ as the weight matrices, and $\boldsymbol{p}(0)$ the initial matrix, the recurrence algorithms of Kalman filter gain matrix $\boldsymbol{K}(k)$ can be calculated off-line with

$$\begin{cases} \boldsymbol{P}(k)=\boldsymbol{G}\boldsymbol{P}(k-1)\boldsymbol{G}^{\mathrm{T}}+\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\mathrm{T}}, \\ \boldsymbol{K}(k)=\tilde{\boldsymbol{P}}(k)\boldsymbol{C}^{\mathrm{T}}[\boldsymbol{C}\boldsymbol{P}(k)\boldsymbol{C}^{\mathrm{T}}+\boldsymbol{R}]^{-1}, \\ \boldsymbol{P}(k)=[\boldsymbol{I}-\boldsymbol{K}(k)\boldsymbol{C}]\tilde{\boldsymbol{P}}(k), \end{cases} \tag{10}$$

where $\boldsymbol{P}(k)$ is an estimation error covariance matrix, and $\tilde{\boldsymbol{P}}(k)$ is a prediction matrix of the error covariance.

OPTIMAL STATE FEEDBACK CONTROL BY LYAPUNOV STABILITY CRITERION

The state equation of the system is expressed as

$$\boldsymbol{x}(k+1)=\boldsymbol{G}\boldsymbol{x}(k)+\boldsymbol{H}u(k). \tag{11}$$

The control $u(k)$ is defined as

$$u(k)=-\boldsymbol{K}\boldsymbol{x}(k). \tag{12}$$

The design objective is to find the feedback matrix $\boldsymbol{K}$ that will bring the state $\boldsymbol{x}(k)$ from any initial state $\boldsymbol{x}(0)$ to the equilibrium state $\boldsymbol{x}=\boldsymbol{0}$ in some optimal sense.

The basic assumption is that the system of Eq.(11) is asymptotically stable with $u(k)=0$. This guarantees that given a positive-definite real symmetric matrix $\boldsymbol{Q}$, there exists a positive-definite real symmetric matrix $\boldsymbol{P}$ such that

$$\boldsymbol{G}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}-\boldsymbol{P}=-\boldsymbol{Q}. \tag{13}$$

The Lyapunov function is defined as

$$V[\boldsymbol{x}(k)]=\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}\boldsymbol{x}(k), \tag{14}$$

and

$$\begin{cases} \Delta V[\boldsymbol{x}(k)]=V[\boldsymbol{x}(k+1)], \\ -V[\boldsymbol{x}(k)]=-\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{Q}\boldsymbol{x}(k). \end{cases} \tag{15}$$

As an optimal control problem, we find an optimal control $u^{\mathrm{o}}(k)$ which minimizes the performance index

$$J=\Delta V[\boldsymbol{x}(k)] \tag{16}$$

in the instant k.

Since $\Delta V[\boldsymbol{x}(k)]$ represents the discrete rate of change of $V[\boldsymbol{x}(k)]$, we can define $V[\boldsymbol{x}(k)]$. So the minimization of the performance index of Eq.(16) will carry a physical meaning in optimal control. For instance, $\Delta V[\boldsymbol{x}(k)]$ may represent the rate of change of speed, distance, or energy along the trajectory $\boldsymbol{x}(k)$.

For the general form of $V[\boldsymbol{x}(k)]$ given in Eq.(14), we write

$$\begin{aligned} \Delta V[\boldsymbol{x}(k)]&=\boldsymbol{x}^{\mathrm{T}}(k+1)\boldsymbol{P}\boldsymbol{x}(k+1)-\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}\boldsymbol{x}(k) \\ &=\boldsymbol{x}^{\mathrm{T}}(k)(\boldsymbol{G}-\boldsymbol{H}\boldsymbol{K})^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{G}-\boldsymbol{H}\boldsymbol{K})\boldsymbol{x}(k)-\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}\boldsymbol{x}(k). \end{aligned} \tag{17}$$

By the state equation Eq.(11), the last equation can be written as

$$\begin{aligned} \Delta V[\boldsymbol{x}(k)]&=\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{G}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}\boldsymbol{x}(k)+u^{\mathrm{T}}(k)\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}\boldsymbol{x}(k) \\ &+\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{G}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{H}u(k)+u^{\mathrm{T}}(k)\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{H}u(k)-\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}\boldsymbol{x}(k). \end{aligned} \tag{18}$$

For minimum $\Delta V[\boldsymbol{x}(k)]$ with respect to $u(k)$

$$\frac{\partial \Delta V[\boldsymbol{x}(k)]}{\partial u(k)}=0. \tag{19}$$

Substituting Eq.(18) into Eq.(19) leads to

$$2\boldsymbol{H}^{\mathrm{T}}\boldsymbol{PGx}(k)+2\boldsymbol{H}^{\mathrm{T}}\boldsymbol{PH}u(k)=0. \tag{20}$$

Therefore, the optimal control is

$$u(k)=-(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{PH})^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{PGx}(k). \tag{21}$$

Given $\boldsymbol{Q}=\boldsymbol{I}$ (identity matrix) with $V[\boldsymbol{x}]=\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{Px}(k)=\boldsymbol{0}$ and $\boldsymbol{P}$, the optimal control $u^{\mathrm{o}}(k)$ can be obtained.

ADAPTIVE CONTROL TO COMPENSATE EFFECTS OF NOISE AND DISTURBANCE

If a system is subject to process disturbance and measurement noise, which are from power supple and load disturbance, the equations for the system can be expressed as

$$\begin{cases}\boldsymbol{x}(k)=\boldsymbol{Gx}(k-1)+\boldsymbol{H}u(k-1)+\boldsymbol{\xi}(k-1),\\ \boldsymbol{n}(k)=\boldsymbol{Cx}(k)+\boldsymbol{\varepsilon}(k),\end{cases} \tag{22}$$

where $\boldsymbol{\xi}(k)$, $\boldsymbol{\varepsilon}(k)$ are Gaussian noises. Since the output error due to the above noise and disturbance cannot be eliminated by optimal feedback control, use of an adaptive algorithm is needed.

The output expectation of the system due to an arbitrary input $u(k)$ may be expressed as

$$n(k)=-Fn(k-1)-Dn(k-2)+A[u(k-1)+\Delta u(k)] + B[u(k-2)+\Delta u(k)], \tag{23}$$

where $\Delta u(k)$ is the input error, due to the existence of the noise.

The output error is

$$\Delta n(k)=n(k)-\hat{n}(k)=A\Delta u(k)+B\Delta u(k). \tag{24}$$

So an indirect measurement value of $\Delta u(k)$ is

$$\Delta u(k)=\frac{\Delta n(k)}{A+B}=K_3\Delta n(k). \tag{25}$$

Using an exponential smoothing filter

$$\Delta\hat{n}(k)=(1-\beta)\Delta\hat{n}(k-1)+\beta\Delta n(k), \tag{26}$$

where $\Delta\hat{n}(k)$ is the filtered value of $\Delta n(k)$, and β ($0<\beta<1$) is the filter coefficient. The following equation may be used as an adaptive corrector:

$$u^{\mathrm{o}}(k)=u^{\mathrm{o}}(k-1)-K_3\Delta\hat{n}(k). \tag{27}$$

SIMULATION EXPERIMENTS

Technique data of experimented brushless DC motor

Technique data of the experimented brushless DC motor are: rated power P=1.57 kW, rated speed n=1500 rpm, rated current I=4.5 A, moment of inertia-including load J=16 kg/cm^2.

Algorithm

Step 1: Calculate coefficients A, B, F, D and matrices $\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{C}$.

Step 2: Calculate the optimal feedback matrix $\boldsymbol{K}$ and control signal $u^*(k)$.

Step 3: Calculate the adaptive signal $u^{\mathrm{o}}(k)$.

Simulation results

Simulation results are given in Fig.2.

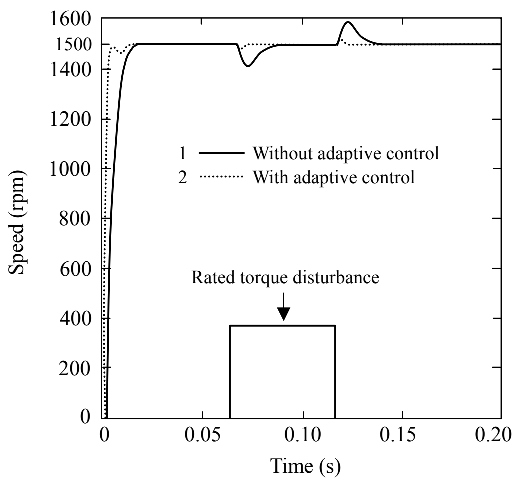

Fig.2 Dynamic response of system underload disturbance

In Fig.2, curve 1 corresponds to the dynamic response of system under the rated load disturbance without adaptive control, while curve 2 corresponds to that with adaptive control. The method of optimal feedback is seen to provide faster step response of output shaft velocity. To further reduce the sensitivity to load disturbances, adaptive torque correct methods were applied in conjunction with optimal feedback control methods.

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