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Dynamic response analysis of a moored crane-ship with a flexible boom*

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Abstract: The dynamic response of moored crane-ship is studied. Governing equations for the dynamic response of a crane-ship coupled with the pendulum motion of the payload are derived based on Lagrange's equations. The boom is modeled based on finite element method, while the payload is modeled as a planar pendulum of point mass. The dynamic response was studied using numerical method. The calculation results show that the large-amplitude responses occur at wave periods near the natural period of the payload. Load swing angle is smaller for crane-ship with flexible boom, in comparison with rigid boom. The ship surge motions have large vibrations for crane-ship with flexible boom, which were not observed for a rigid boom. The analysis identifies the significance of key parameters and reveals how the system design can be adjusted to avoid critical conditions.

Key words: Dynamic response, Moored crane-ship, Finite element method, Rigid-flexible coupling dynamic model
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INTRODUCTION

Floating cranes play an important role in the offshore projects. However, the sea wave shakes the crane-ship and may excite its large movement. For heavy duty lifting, the operations of crane-ship in waves, even as the sea is relatively quiet, are often restricted by the excessive motions of the crane load. In some cases, the dynamical behavior of floating cranes is to be considered as critical with respect to the amplitudes of the motion of the ship or the load. A small disturbance of the system can cause the collision between the load and the ship or other objects. In addition, the amplitude of the motion of the ship has to stay small in order to achieve the required precisely positioning and to avoid damages to the mooring system.

The investigation of crane-ship dynamics has been of interest in a large number of recent researches (Dong and Han, 1993; Masoud *et al.*, 2004; Ellermann, 2005). A rigid massless cable and massive point load were used to model the crane load system, and the results of the computer simulation were verified experimentally using a three degree-of-freedom (DOF) ship-motion simulation platform (Henry *et al.*, 2001). The method of multiple scales is used to analyze the dynamics of a cable-suspended load. The results show that a parametric excitation at twice the natural frequency leads to sudden jumps in the response as the cable is unreeled (Chin *et al.*, 2001). Cargo pendulation control of an elastic ship-mounted crane was concerned using the Maryland Rigging system. The dynamics of the crane is described by a multi-model problem depending on the current cable length and boom luff angle. A variable-gain observer and a variable-gain controller are designed. Simulation and experimental results showed that the expressed control strategy has a significant effect on suppressing the vibrations for different operating conditions and payload masses (Al-Sweiti and Söffker, 2007).

As the first approach for evaluating the dynam-

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ics of a crane-ship, a linear theoretical model is used. The linear differential equation of motion is derived under the assumption of small amplitudes, and does not consider occurring restoring forces. A dynamic model was established using multi-body dynamics methods and the dynamics of the crane was analyzed (Chen *et al.*, 2002). The effects of cable reeling and unreeling on cargo pendulations were studied with the boom crane modeled as a planar pendulum and the ship as a rigid body. The results show nonlinear behavior (Kral *et al.*, 1996).

In almost all of these studies, the flexibility of the boom is not taken into consideration. This may be reasonable for short crane booms and small payload-to-ship mass ratios, but for long crane booms and large payload-to-ship mass ratios, the influence of the flexibility of the crane boom cannot be ignored any more.

To the authors' best knowledge, until recently there is no report on the modeling of such system. The study aims to present a rigid-flexible coupling dynamic model for the prediction of the dynamics of a moored crane-ship and its payload. The prediction is the foundation of the dynamical design and an accurate residual working life assessment. It is also a basis for the control of the vibrations of the system.

MODEL DESCRIPTION

The schematic system under investigation is shown in Fig.1. When dimensions and elastic properties of the crane-ship body are considered, it is sufficient to take only the boom as flexible. The following assumptions are used in the analysis of the crane-ship:

(1) The motion is only in the vertical plane. The load is regarded as a point mass, and can have pendulum motion in the plane, but without twisting.

(2) The rope is taken as a rigid rod. This assumption is valid as long as the oscillations of the load in the vertical direction are small and the rope remains in tension.

(3) The structural damping of the system is not taken into account, because cranes are typically lightly damped. Todd *et al.*(1997) reported that the damping of a ship mounted crane is from 0.1% to 0.5% of the critical damping.

As depicted in Fig.1, a coordinate system OXY is fixed to the ground and denoted as "Earth-Fixed", which is taken to be the inertial frame of reference. The other system $O_0X_0Y_0$ is fixed to and, hence, moves with the ship, which is denoted as "Body-Fixed". J is the rotary inertia of the ship, m_{ship} and m_{p} are the masses of the ship and load, respectively. The elastic displacements of the boom tip point B are denoted as u and w. The critical parameters including the boom length L_{b}, the luff angle of the boom β, the length of the rope L, the displacement in surge direction x, the pitch angle θ, the heave motion y and the swing angle of the load α are also indicated in Fig.1.

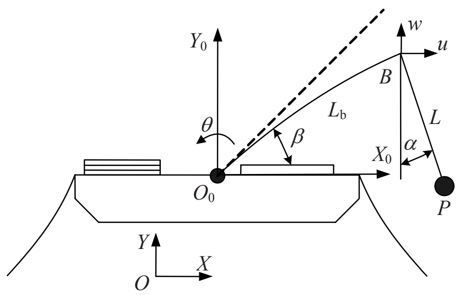

Fig.1 Moored crane-ship model

MATHEMATICAL MODEL

External forces acting on the crane-ship

In the model, different external forces have to be considered (Ellermann *et al.*, 2002), the hydrostatic force

$$\boldsymbol{f}_{\mathrm{b}} = [0,\ -\rho_{\mathrm{w}} g A_{\mathrm{w}} y,\ -(m_{\mathrm{ship}} + m_{\mathrm{p}}) g h_{\mathrm{m}} \theta]^{\mathrm{T}}, \quad (1)$$

with the density of water ρ_{w}, the cross-sectional area of the ship at the still water level A_{w} and the metacentric height h_{m}.

The mooring line forces, which are approximated by a third order polynomial

$$\boldsymbol{f}_{\mathrm{m}} = [-c_1 x - c_2 |x| x - c_3 x^3,\ 0,\ 0]^{\mathrm{T}}, \quad (2)$$

where c_1, c_2, c_3 are the linear, quadratic, and cubic characteristic coefficients of the mooring system.

Forces due to viscous drag

$$\boldsymbol{f}_{\mathrm{d}} = [-\rho_{\mathrm{w}} c_{\mathrm{d}} W T |\dot{x}| \dot{x}/2,\ 0,\ 0]^{\mathrm{T}} \quad (3)$$

are proportional to the density of water ρ_w, the empirical drag coefficient c_d, the width of the ship W and the draught T.

The frequency-dependent wave excitation forces, which can be split into a periodically changing part and the constant drift forces, can be modeled as:

$$\boldsymbol{f}_w = \begin{bmatrix} A(k_{rx}\cos(\omega t) - k_{ix}\sin(\omega t)) + A^2 p_d \\ A(k_{ry}\cos(\omega t) - k_{iy}\sin(\omega t)) \\ A(k_{r\theta}\cos(\omega t) - k_{i\theta}\sin(\omega t)) \end{bmatrix}, \quad (4)$$

with the wave amplitude A, the real and imaginary parts of the frequency dependent coefficient k_{rj} and k_{ij} ($j=x,y,\theta$), and the coefficient of the drift force p_d.

The forces are abbreviated by

$$\boldsymbol{F} = [F_1, F_2, F_3]^T = \boldsymbol{f}_b + \boldsymbol{f}_m + \boldsymbol{f}_d + \boldsymbol{f}_w. \quad (5)$$

Dynamic equation of crane-ship

The position vector of payload P as shown in Fig.1 can be expressed as

$$\boldsymbol{r}_p = (x + L_j\cos(\beta+\theta) + u + L\sin\alpha)\cdot \boldsymbol{i} + (y + L_j\sin(\beta+\theta) + w - L\cos\alpha)\cdot \boldsymbol{j}, \quad (6)$$

where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors along the X- and Y-axes, respectively. It can be seen that, by including the boom tip displacement u and w, the effect of the elastic deformation of the boom is included in the position equation of the payload.

It is assumed that the luff angle of the boom and the length of the payload pendulum are constant. Based on Eq.(6), the velocity vector of the payload P can be obtained by the time derivative of $\boldsymbol{r}_p$ as

$$\boldsymbol{V}_p = (\dot{x} + \dot{u} - \dot{\theta}L_b\sin(\beta+\theta) + \dot{\alpha}L\cos\alpha)\cdot \boldsymbol{i} + (\dot{y} + \dot{w} + \dot{\theta}L_b\cos(\beta+\theta) + \dot{\alpha}L\sin\alpha)\cdot \boldsymbol{j}. \quad (7)$$

So the kinetic energy of the payload can then be derived as

$$\begin{aligned} T_p &= m_p\boldsymbol{V}_p\cdot\boldsymbol{V}_p/2 \\ &= m_p[\dot{x}^2 + \dot{u}^2 + \dot{y}^2 + \dot{w}^2 + \dot{\alpha}^2L^2 + 2\dot{x}\dot{u} + 2\dot{y}\dot{w} \\ &+2\dot{x}\dot{\alpha}L\cos\alpha + 2\dot{y}\dot{\alpha}L\sin\alpha + 2\dot{u}\dot{\alpha}L\cos\alpha \\ &+2\dot{w}\dot{\alpha}L\sin\alpha + \dot{\theta}^2L_b^2 - 2\dot{\theta}\dot{x}L_b\sin(\beta+\theta) \\ &-2\dot{\theta}\dot{u}L_b\sin(\beta+\theta) - 2\dot{\theta}\dot{\alpha}L_bL\sin(\beta+\theta)\cos\alpha \\ &+2\dot{\theta}\dot{y}L_b\cos(\beta+\theta) + 2\dot{\theta}\dot{w}L_b\cos(\beta+\theta) \\ &+2\dot{\theta}\dot{\alpha}L_bL\cos(\beta+\theta)\sin\alpha]/2. \end{aligned} \quad (8)$$

The potential energy of the payload

$$U_p = m_pg(y + L_b\sin(\beta+\theta) + w - L\cos\alpha). \quad (9)$$

The kinetic energy and the potential energy of the ship can be respectively expressed as

$$T_{ship} = [J\dot{\theta}^2 + m_{ship}(\dot{x}^2 + \dot{y}^2)]/2, \quad (10)$$

$$U_{ship} = m_{ship}gy. \quad (11)$$

Based on the finite element discretization, the kinetic and potential energy of boom can be respectively expressed as

$$T_b = \frac{1}{2}\dot{\boldsymbol{U}}^T\boldsymbol{M}\dot{\boldsymbol{U}} = \frac{1}{2}\begin{Bmatrix}\dot{\boldsymbol{U}}_r \\ \dot{u} \\ \dot{w}\end{Bmatrix}^T \begin{bmatrix}\boldsymbol{M}_{rr} & \boldsymbol{M}_{ru} & \boldsymbol{M}_{rw} \\ \boldsymbol{M}_{ur} & m_{uu} & m_{uw} \\ \boldsymbol{M}_{wr} & m_{wu} & m_{ww}\end{bmatrix} \begin{Bmatrix}\dot{\boldsymbol{U}}_r \\ \dot{u} \\ \dot{w}\end{Bmatrix}, \quad (12)$$

$$U_b = \frac{1}{2}\boldsymbol{U}^T\boldsymbol{K}\boldsymbol{U} = \frac{1}{2}\begin{Bmatrix}\boldsymbol{U}_r \\ u \\ w\end{Bmatrix}^T \begin{bmatrix}\boldsymbol{K}_{rr} & \boldsymbol{K}_{ru} & \boldsymbol{K}_{rw} \\ \boldsymbol{K}_{ur} & k_{uu} & k_{uw} \\ \boldsymbol{K}_{wr} & k_{wu} & k_{ww}\end{bmatrix} \begin{Bmatrix}\boldsymbol{U}_r \\ u \\ w\end{Bmatrix}, \quad (13)$$

where $\boldsymbol{M}$ and $\boldsymbol{K}$ are global mass and stiffness matrices of the boom, $\boldsymbol{U}$ and $\dot{\boldsymbol{U}}$ are displacement and velocity vectors of the flexible boom; (u, w) and $(\dot{u}, \dot{w})$ are the nodal displacement and velocities of the boom tip point B. $\boldsymbol{U}_r$ and $\dot{\boldsymbol{U}}_r$ are vectors of displacements and velocities for the rest degrees of freedom of the boom structure.

The Lagrangian function of the system can be expressed as

$$\begin{aligned} L = T - U &= \frac{1}{2}\begin{Bmatrix}\dot{\boldsymbol{U}}_r \\ \dot{u} \\ \dot{w}\end{Bmatrix}^T \begin{bmatrix}\boldsymbol{M}_{rr} & \boldsymbol{M}_{ru} & \boldsymbol{M}_{rw} \\ \boldsymbol{M}_{ur} & m_{uu} & m_{uw} \\ \boldsymbol{M}_{wr} & m_{wu} & m_{ww}\end{bmatrix} \begin{Bmatrix}\dot{\boldsymbol{U}}_r \\ \dot{u} \\ \dot{w}\end{Bmatrix} + \frac{1}{2}J\dot{\theta}^2 \\ &+m_{ship}(\dot{x}^2 + \dot{y}^2)/2 + m_p[\dot{x}^2 + \dot{u}^2 + \dot{y}^2 + \dot{w}^2 + \dot{\alpha}^2L^2 + 2\dot{x}\dot{u} \\ &+2\dot{y}\dot{w} + 2\dot{x}\dot{\alpha}L\cos\alpha + 2\dot{y}\dot{\alpha}L\sin\alpha + 2\dot{u}\dot{\alpha}L\cos\alpha \\ &+2\dot{w}\dot{\alpha}L\sin\alpha + \dot{\theta}^2L_b^2 - 2\dot{\theta}\dot{x}L_b\sin(\beta+\theta) - 2\dot{\theta}\dot{u}L_b\sin(\beta \end{aligned}$$

$$+\theta) - 2\dot{\theta}\dot{\alpha}L_b L\sin(\beta+\theta)\cos\alpha + 2\dot{\theta}\dot{y}L_b\cos(\beta+\theta)$$
$$+2\dot{\theta}\dot{w}L_b\cos(\beta+\theta) + 2\dot{\theta}\dot{\alpha}L_b L\cos(\beta+\theta)\sin\alpha]/2$$
$$-\frac{1}{2}\begin{Bmatrix} \boldsymbol{U}_r \\ u \\ w \end{Bmatrix}^T \begin{bmatrix} \boldsymbol{K}_{rr} & \boldsymbol{K}_{ru} & \boldsymbol{K}_{rw} \\ \boldsymbol{K}_{ur} & k_{uu} & k_{uw} \\ \boldsymbol{K}_{wr} & k_{wu} & k_{ww} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U}_r \\ u \\ w \end{Bmatrix} - m_{ship} g y$$
$$-m_p g(y + L_b\sin(\beta+\theta) + w - L\cos\alpha). \quad (14)$$

The Lagrange's equation is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j, \quad (15)$$

where q_j and $\dot{q}_j$ are general coordinates and general velocities of the system. Q_j is general force of the system.

Substituting Eqs.(5) and (14) into Eq.(15) gives

$$\left(\boldsymbol{M} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & m_p & 0 \\ \boldsymbol{0} & 0 & m_p \end{bmatrix}\right)\begin{bmatrix} \ddot{\boldsymbol{U}}_r \\ \ddot{u} \\ \ddot{w} \end{bmatrix} + \boldsymbol{K}\begin{bmatrix} \boldsymbol{U}_r \\ u \\ w \end{bmatrix}$$
$$= \begin{bmatrix} \boldsymbol{0} \\ m_p(\dot{\alpha}^2 L\sin\alpha + \ddot{\theta}L_b\sin(\beta+\theta) + \\ \dot{\theta}^2 L_b\cos(\beta+\theta) - \ddot{x} - \ddot{\alpha}L\cos\alpha) \\ m_p[-g - \ddot{y} - \ddot{\alpha}L\sin\alpha - \dot{\alpha}^2 L\cos\alpha - \\ \ddot{\theta}L_b\cos(\beta+\theta) + \dot{\theta}^2 L_b\sin(\beta+\theta)] \end{bmatrix}, \quad (16)$$

$$(m_{ship} + m_p)\ddot{x} + m_p\ddot{u} + m_p\ddot{\alpha}L\cos\alpha - m_p\dot{\alpha}^2 L\sin\alpha$$
$$-\ddot{\theta}L_b\sin(\beta+\theta) - \dot{\theta}^2 L_b\cos(\beta+\theta) = F_1, \quad (17)$$

$$(m_{ship} + m_p)\ddot{y} + m_p\ddot{w} + m_p\ddot{\alpha}L\sin\alpha + m_p\dot{\alpha}^2 L\cos\alpha +$$
$$(m_{ship} + m_p)g + \ddot{\theta}L_b\cos(\beta+\theta) - \dot{\theta}^2 L_b\sin(\beta+\theta) = F_2, \quad (18)$$

$$J\ddot{\theta} + m_p L_b(\ddot{\theta}L_b - \ddot{x}\sin(\beta+\theta) - \ddot{u}\sin(\beta+\theta)$$
$$-\ddot{\alpha}L\sin(\beta+\theta)\cos\alpha + \dot{\alpha}^2 L\sin(\beta+\theta)\sin\alpha$$
$$+\ddot{y}\cos(\beta+\theta) + \ddot{w}\cos(\beta+\theta) + \ddot{\alpha}L\cos(\beta+\theta)\sin\alpha$$
$$+\dot{\alpha}^2 L\cos(\beta+\theta)\cos\alpha) + m_p L_b g\cos(\beta+\theta) = F_3, \quad (19)$$

$$\ddot{\alpha}L + \ddot{x}\cos\alpha + \ddot{y}\sin\alpha + \ddot{u}\cos\alpha + \ddot{w}\sin\alpha - \ddot{\theta}L_b\sin(\beta$$
$$+\alpha)\cos\alpha - \dot{\theta}^2 L_b\cos(\beta+\alpha)\cos\alpha + \ddot{\theta}L_b\cos(\beta+$$
$$\alpha)\sin\alpha - \dot{\theta}^2 L_b\sin(\beta+\alpha)\sin\alpha + g\sin\alpha = 0. \quad (20)$$

As in several other investigations, the attention is focused on the horizontal surge motion. In this special case, equations of motion of the system can be reduced to

$$\left(\boldsymbol{M} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & m_p & 0 \\ \boldsymbol{0} & 0 & m_p \end{bmatrix}\right)\begin{bmatrix} \ddot{\boldsymbol{U}}_r \\ \ddot{u} \\ \ddot{w} \end{bmatrix} + \boldsymbol{K}\begin{bmatrix} \boldsymbol{U}_r \\ u \\ w \end{bmatrix}$$
$$= \begin{bmatrix} \boldsymbol{0} \\ m_p(\dot{\alpha}^2 L\sin\alpha - \ddot{\alpha}L\cos\alpha - \ddot{x}) \\ m_p(-g - \ddot{\alpha}L\sin\alpha - \dot{\alpha}^2 L\cos\alpha) \end{bmatrix}, \quad (21)$$

$$(m_{ship} + m_p)\ddot{x} + m_p\ddot{u} + m_p\ddot{\alpha}L\cos\alpha - m_p\dot{\alpha}^2 L\sin\alpha = F_1, \quad (22)$$

$$\ddot{\alpha}L + \ddot{x}\cos\alpha + \ddot{u}\cos\alpha + \ddot{w}\sin\alpha + g\sin\alpha = 0. \quad (23)$$

If the boom structure is assumed to be rigid, the deformation of the boom vanishes in Eqs.(21)~(23). In this case, the equations of motion of the system can be reduced to

$$(m_{ship} + m_p)\ddot{x} + m_p\ddot{\alpha}L\cos\alpha - m_p\dot{\alpha}^2 L\sin\alpha = F_1, \quad (24)$$

$$\ddot{\alpha}L + \ddot{x}\cos\alpha + g\sin\alpha = 0. \quad (25)$$

SIMULATION RESULTS AND DISCUSSION

The dynamic response of the crane-ship is investigated in time domain based on a Newmark method and an iterative approach (Ju *et al.*, 2006). The type of finite element used in modeling the crane boom is space frame element. The basic parameter values of the ship used for the analysis are given as follows: c_1=21200 N/m, c_2=9440 N/m^2, c_3=13.82 N/m^3, k_r=5540 N/m, k_i=426000 N/m, c_d=0.2, p_d=15800 N·m^2, m_p=200000 kg, m_{ship}=1920000 kg, B=25 m, L_b=60 m, T=1.69 m, ρ=1000 kg/m^3, β=60°, A=1.2 m.

Influence of wave excitation frequency

Consider the effect of different wave excitation frequencies on the load-swing angles. The cable length is 30 m, which corresponds to a load natural frequency of 0.626 Hz. The wave frequencies are 0.313 Hz, 0.626 Hz, and 0.8138 Hz. The dynamic responses of load-swing angles α are shown in Fig.2.

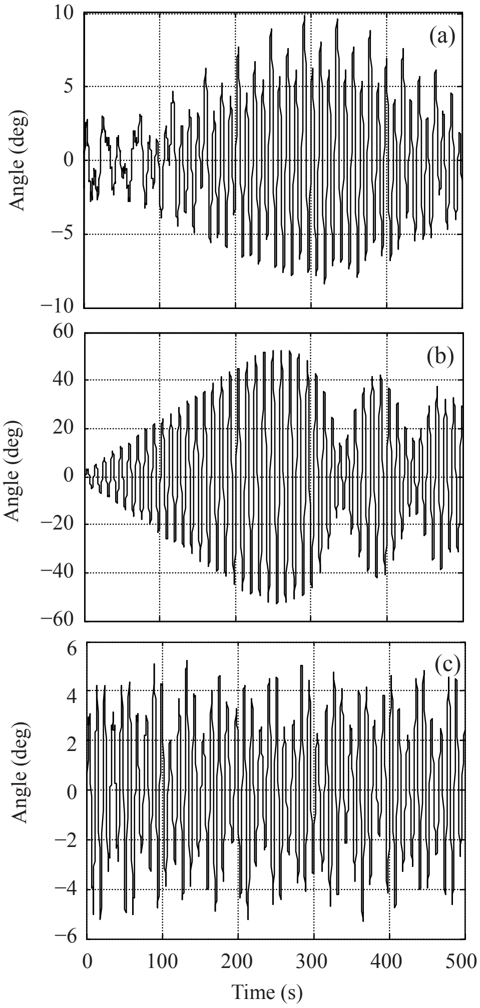

Fig.2 Load swing angle for wave excitation with different frequency
(a) 0.313 Hz; (b) 0.626 Hz; (c) 0.8138 Hz

The results show that the load-swing amplitudes depend on the wave excitation frequency. As the excitation frequency approaches the natural frequency of the load, the amplitudes of the swing angles increase, and the amplitude of the load oscillation was α_{max}=50°.

Influence of flexibility of the boom

Consider the effect of a rigid boom and a flexible boom on the dynamic response of crane-ship. The cable length is 45 m, the excitation wave amplitude is 1.2 m and the wave frequency is 0.626 Hz. The dynamic responses of load-swing angles of crane-ship with the rigid and flexible boom are demonstrated in Fig.3a. The load-swing angle is smaller for the crane-ship with flexible boom as compared with the

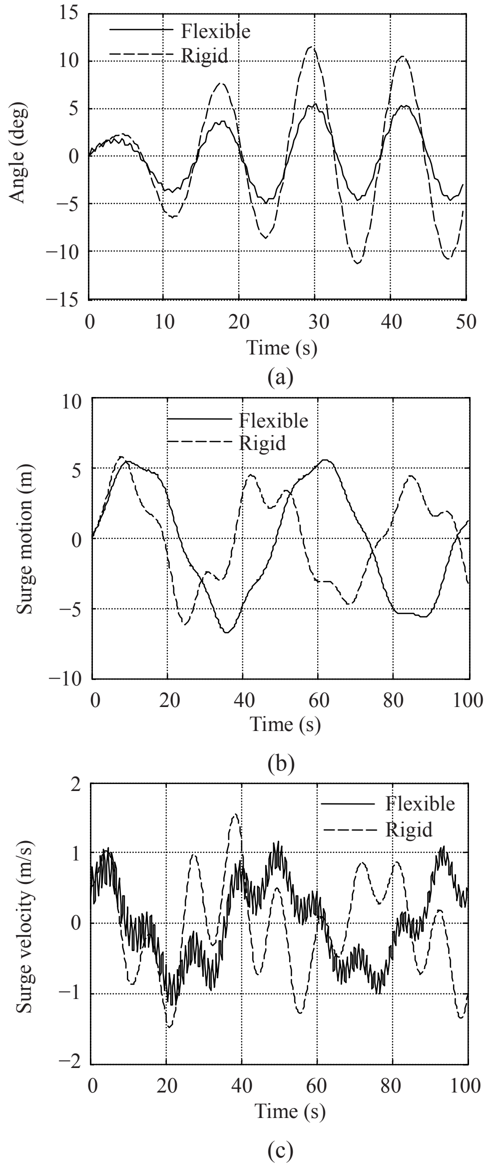

Fig.3 Swing angle (a), surge motion (b) and surge velocity (c) for a crane-ship with the rigid and flexible boom

crane-ship with a rigid boom. The load vibrations during the swing process can be seen from the load swing angle curves in Fig.3a.

The surge motions of a crane-ship with a rigid and a flexible boom are demonstrated in Fig.3b. It can be seen that there is small change in the amplitude of ship surge motion.

The surge velocity of a crane-ship with a rigid and a flexible boom is shown in Fig.3c. It can be seen that there is a high frequency vibration during the ship surge motion for the crane-ship with a flexible boom, but this is not observed for the crane-ship with a rigid boom.

Fig.4 shows the elastic displacement of the boom tip-point *B*. It is seen that at the beginning of the process, the vibration amplitude is about 300 mm and it decreases to about 50 mm at the end of the process.

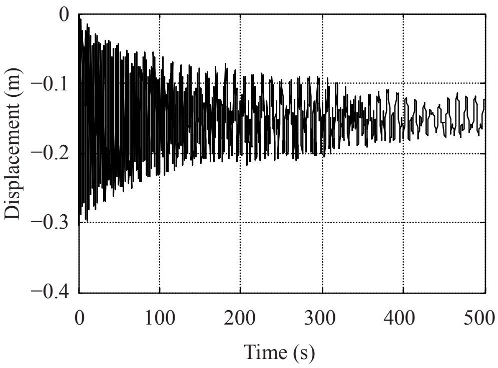

Fig.4 Displacement of flexible boom tip point

Large vibrations during the ship surge motion are only observed for the crane-ship with flexible boom. Crane-ship with flexible boom has a longer period of ship surge motion than that with rigid boom. But the ship surge motion amplitudes have smaller changes for the crane-ship with flexible boom in comparison with rigid boom.

CONCLUSION

The governing equations for the dynamic response of a crane-ship coupled with the pendulum motion of the payload are derived based on Lagrange's equations. The boom is modeled based on a finite element method, while the payload is modeled as a planar pendulum of point mass. The cable is assumed to be massless and inextensible. If the crane structure is assumed to be rigid, the derived equations correctly degenerate to nonlinear differential equations, which exactly correspond to Newton's Law of Motion for the planar pendulum motions. Numerical studies are then carried out for a real crane-ship with planar pendulum motions of the payload. Simulations show that the load-swing amplitude depends on the wave excitation frequency. As the excitation frequency approaches the natural frequency of the payload, the amplitudes of the swing angles increase. The influence of boom flexibility on the ship surge amplitude and load swing angle is also investigated.

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