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Strategic games on a hierarchical network model*

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Abstract: Among complex network models, the hierarchical network model is the one most close to such real networks as world trade web, metabolic network, WWW, actor network, and so on. It has not only the property of power-law degree distribution, but also the scaling clustering coefficient property which Barabási-Albert (BA) model does not have. BA model is a model of network growth based on growth and preferential attachment, showing the scale-free degree distribution property. In this paper, we study the evolution of cooperation on a hierarchical network model, adopting the prisoner's dilemma (PD) game and snowdrift game (SG) as metaphors of the interplay between connected nodes. BA model provides a unifying framework for the emergence of cooperation. But interestingly, we found that on hierarchical model, there is no sign of cooperation for PD game, while the frequency of cooperation decreases as the common benefit decreases for SG. By comparing the scaling clustering coefficient properties of the hierarchical network model with that of BA model, we found that the former amplifies the effect of hubs. Considering different performances of PD game and SG on complex network, we also found that common benefit leads to cooperation in the evolution. Thus our study may shed light on the emergence of cooperation in both natural and social environments.

Key words: Complex network, Hierarchical network model, Barabási-Albert (BA) model, Prisoner's dilemma (PD) game, Snowdrift game (SG)

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INTRODUCTION

In biological, social or economic systems, individuals interplay. One individual takes various strategies against another to maximize its benefits. Two simple strategies have been abstracted as cooperation and defection. A defector can exploit a cooperator's benefit, so individuals should take the defection strategy. But many natural species show altruism, with individuals bearing costs to the benefit of others: vampire bats share blood (Wilkinson, 1984), monkeys groom each other (Seyfarth and Cheney, 1984), alarm calls warn from predators (Clutton-Brock *et al*., 1999), and so on. Understanding why the cooperation emerges among selfish individuals is an important task. When studying the phenomena of cooperation, natural and social scientists often resort to Evolutionary Game Theory as a common framework and strategic games as a model of interacting decision-makers (Maynard Smith, 1982; Gintis, 2000). Two simple games have attracted much attention in both theoretical and experimental studies: the prisoner's dilemma (PD) and the snowdrift game (SG). In the games, two players meet repeatedly. At all the iterations, each player has two options: to cooperate or to defect. Each one gets different payoffs according to its own and its opponent's options. In well-mixed populations under replicator dynamics (Hofbauer and Sigmund, 1998), PD makes cooperators unable to resist the invasion by defectors, while SG leads to an equilibrium frequency for cooperators given by $1-r$, with $0<r\leq 1$ being the cost to benefit ratio of mutual cooperation.

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In the past few years, complex networks have redefined our knowledge about the true nature of real networks. It studies networks whose structure is irregular, complex and dynamically evolving. Many literature (Dorogovtsev and Mendes, 2002; Albert and Barabási, 2002; Newman, 2003; Pastor-Satorras and Vespignani, 2004; Boccaletti *et al*., 2006) have surveyed complex networks' structure and dynamics. The main focus is studying the dynamics on networks. Naturally, we want to see what kind of new behaviors the interplay between complex topologies and game models would eventually produce.

Nowak and May (1992) first introduced network structure as the framework on which strategic games take place. On regular lattice, N individuals play PD game, which they call the spatial PD. Individuals are constrained to play only with their immediate neighbors. They found out that among cooperators they are now able to resist invasion of defectors. In the last years, PD game has been considered a lot on complex networks (Abramson and Kuperman, 2001; Ebel and Bornholdt, 2002a; 2002b; Kim *et al*., 2002; Holme *et al*., 2003; Lieberman *et al*., 2005). Kim *et al*.(2002) have studied numerically the PD game model on a 2D small-world network, considering the case in which there is an influential node with a finite density of directed random links to the other nodes in the network. They have shown that the degree of cooperation does not remain at a steady state level; rather it shows a punctuated equilibrium-type behavior manifested by the existence of sudden breakdowns of cooperation. Lieberman *et al*.(2005) showed that the outcome of evolutionary games could entirely depend on the structure of the underlying graph. Hauert and Doebeli (2004) pointed out that spatial structure often inhibited the evolution of cooperation in the snowdrift game.

Santos and Pacheco (2005) have studied both PD and SG on scale-free networks. They showed that cooperation becomes the dominant trait throughout the entire range of parameters of both games, providing a unifying framework for the emergence of cooperation.

Vukov and Szabó (2005) have studied PD game on hierarchical lattices. Chen *et al*.(2007) have studied PD on community networks. Fu *et al*.(2007) have studied both PD and SG on an online friendship network obtained from a social networking site.

In this paper, we discuss the hierarchy property of real networks and their statistical properties comparing to BA scale-free networks. We adopt the hierarchical network model observed in some real networks (Ravasz and Barabási, 2003). Then we study the evolution of cooperation on the model, adopting the PD and SG as metaphors of the interplay between connected nodes. Simulation results show that although both BA model and hierarchical networks are scale-free networks, games on them are quite different. On hierarchical model, there is no sign of cooperation for PD game, while the frequency of cooperation decreases with the decrease of the common benefit for SG. By comparing the scaling clustering coefficient properties of the hierarchical network model with that of BA model, we found that hierarchical model amplifies the effect of hubs. Considering different performances of PD game and SG on complex network, we also found that common benefit leads to cooperation in the evolution.

STRATEGIC GAMES

A strategic game is a model of interacting decision-makers, consisting of the following ingredients: (1) at least two decision-makers (also known as players or agents) are involved in the game; these players (2) follow the rules of the game, (3) have their own strategies, and (4) play in a way to maximize their gain (Von Neumann and Morgenstern, 1947; Osborne, 2002).

A payoff is a number, also called utility, that reflects the desirability of an outcome to a player, for whatever reason. Payoff matrix of two players is a 2×2 matrix, defining the payoff of one player according to both players' strategies in a single game. See Table 1, they both receive R upon mutual cooperation and P upon mutual defection. A defector exploiting a cooperator gets an amount T and the exploited cooperator receives S.

Table 1 Payoff matrix for a single game of two players

	C	D
C	R	S
D	T	P

C means that the player takes the cooperation strategy; D means that the player takes the defection strategy. The row player gains R, S, T, P according to its and its opponent's strategies

The prisoner's dilemma game (Axelrod and Hamilton, 1981) illustrates that cooperating individuals are prone to exploitation, and that natural selection should favor cheaters. PD takes $T>R>P>S$, which means that it is the best to defect regardless of the co-player's strategy (Table 2a). In our simulation of PD, $T=b>1$, $R=1$, and $P=S=0$, where b represents the advantage of defectors over cooperators, constrained to the interval $1<b\leq 2$.

To illustrate the snowdrift game (also known as the hawk-dove or chicken game) (Sugden, 1986), imagine that two drivers are caught in a blizzard and trapped on either side of a snowdrift. They can either get out and start cooperation or remain in the car (defect). If both players cooperate, they have the benefit β of getting home while sharing the labor c. Notice that $c=1$ in our simulation. Thus, $R=\beta-1/2$. If only one player cooperates, they both get home but the defector avoids the labor cost and gets $T=\beta$, whereas the cooperator gets $S=\beta-1$. The cost-to-benefit ratio of mutual cooperation can be written as $r=1/(2\beta-1)$, where $0<r\leq 1$ is the ratio between the total cost and total benefit when both cooperate, or one cooperates and the other defects. Table 2b gives the payoff matrix of SG expressed with r. SG takes $T>R>S>P$, which means that the best action depends now on the opponent: to defect if the other cooperates, but to cooperate if the other defects. As we can see, in SG even if their strategies are asynchronized, they still have common benefit. This is the major difference between PD and SG.

Table 2 Payoff matrix for strategic games with specified parameters

(a) Payoff matrix for PD with parameter b ($1<b\leq 2$)

	C	D
C	1	0
D	b	0

(b) Payoff matrix for SG with parameter r ($0<r\leq 1$)

	C	D
C	$1/(2r)$	$1/(2r)-1/2$
D	$1/(2r)+1/2$	0

C means that the player takes the cooperation strategy; D means that the player takes the defection strategy

HIERARCHY MEASURE AND HIERARCHICAL NETWORK MODEL

Many networks have the scale-free degree distribution (Albert *et al.*, 1999; Jeong *et al.*, 2000; Wagner, 2001). The Barabási-Albert (BA) model is inspired by WWW and based on growth and preferential attachment (Barabási and Albert, 1999). It has scale-free degree distribution $P(k)\sim k^{-y}$, where y is the degree exponent.

However, recent studies show that real networks are hierarchically organized (Ravasz *et al.*, 2002; Vazquez *et al.*, 2002). These networks are highly modular: one can easily identify groups of nodes that are highly interconnected with each other, but have only a few or no links to nodes outside of the group to which they belong. Usually, models reproducing the scale-free property of real networks (Barabási *et al.*, 2001; Albert and Barabási, 2002; Dorogovtsev and Mendes, 2002) distinguish nodes based only on their degrees, and are blind to node characteristics that could lead to a modular topology. The hierarchy property is usually analyzed by means of the clustering coefficient and the degree-degree correlation.

Clustering, also known as transitivity, is a typical property of acquaintance networks, where two individuals with a common friend are likely to know each other (Wasserman and Faust, 1994). The clustering coefficient C, introduced by Watts and Strogatz (1998), is commonly used as a measure of clustering and defined as follows. A quantity c_i (the local clustering coefficient of node i) is first introduced as

$$c_i = \frac{e_i}{k_i(k_i-1)/2},$$

where k_i is node i's degree, e_i is the number of edges between k_i neighbors. It is defined as the ratio between e_i and $k_i(k_i-1)/2$, the maximum possible number of edges between k_i neighbors. The clustering coefficient C of the graph is then given by the average of c_i over all the nodes:

$$C = \frac{1}{N}\sum_i c_i .$$

By definition, $0\leq c_i\leq 1$ and $0\leq C\leq 1$.

The clustering coefficient of a connectivity class k, i.e., $C(k)$, is defined as the average of c_i taken over all nodes with a given degree k.

The hierarchy property can be characterized in a quantitative manner using the finding that in the deterministic scale-free networks, the clustering coefficient of a node with k links follows the scaling law (Dorogovtsev *et al*., 2002):

$$C(k) \sim k^{-1}.$$

Most hierarchical networks, including metabolic network, WWW, actor network, semantic web, follow this law exactly (Ravasz and Barabási, 2003), while the Internet and world trade web (Serrano and Boguna, 2003) obey $C(k) \sim k^{-w}$, with w=0.75, 0.70, respectively.

Hierarchy is also analyzed by the degree-degree correlation through the conditional probability $P(k|k')$, which measures the probability of a vertex of degree k' to be linked to a vertex of degree k (Pastor-Satorras *et al*., 2001). Because of the difficulty to measure this quantity, it is more often to use the average nearest neighbors degree (ANND), defined as

$$< K_{nn}(K) >= \sum_{k'} k' P(k' \mid k).$$

In this paper, we only use $C(k)$ as the measure of hierarchy.

Ravasz and Barabási (2003) have constructed a hierarchical network model, which has the properties of scale-free degree distribution, scaling clustering coefficient. In some literature, the model is known as the scale-free hierarchical network (Vukov and Szabó, 2005). For the ease of our simulation, we follow this model completely. The model starts with a small cluster of five densely linked nodes. Next, four replicas of this hypothetical module are generated and the four external nodes of the replicated clusters are connected to the central node of the old cluster, obtaining a large 25-node module. Again, four replicas of this 25-node module are generated and the 16 peripheral nodes are connected to the central node of the old module, obtaining a new module of 125 nodes. These replication and connection steps are repeated indefinitely, increasing the numbers of nodes in the network by a factor 5 each step. This process is illustrated by Fig.1. The nodes at the center of the five-node modules have a clustering coefficient C=1. Those at the center of a 25-node module have k=20 and C=3/19, while those at the center of the 125-node modules have k=84 and C=3/83, indicating $C(k) \sim k^{-1}$ (Fig.2b). From the construction process, we can see that hierarchical network model is highly modularized and several submodules combine into larger modules recursively. So it has hierarchy property and shows power-law clustering distribution. Different from BA model, the hubs of parallel modules are not directly connected. In BA model, vertices with highest connectivity (so-called hubs) become directly interconnected thanks to its growth and preferential attachment.

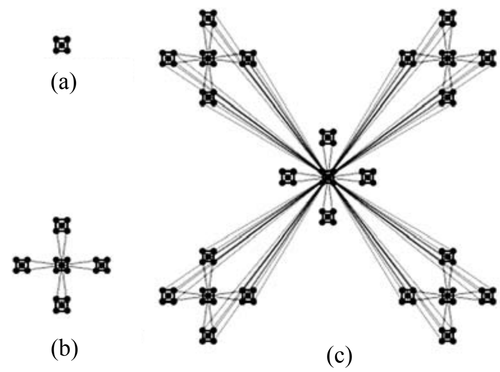

Fig.1 The construction process of hierarchical network model, with the level of the network from *n*=0 to *n*=2. (a) *n*=0, *N*=5; (b) *n*=1, *N*=25; (c) *n*=2, *N*=125

Fig.2 illustrates some properties of the hierarchical model and BA model (Ravasz and Barabási, 2003). We can see that they both have scale-free degree distribution. $C(k)$ of the hierarchical model follows scale-free distribution, but that of BA model converges into one area. Meanwhile, the clustering coefficient $C(N)$ of the hierarchical model keeps constant with the system's size increasing. But $C(N)$ of the BA model decreases when the system size increases.

EVOLUTIONARY DYNAMICS ON HIERARCHICAL NETWORK MODEL

Firstly, we specify our evolutionary dynamics. Whenever a site x is updated, a neighbor y is drawn at random among all N neighbors. The chosen neighbor takes over site with probability $w_y = f(P_y - P_x)$, where the function translates payoff differences into

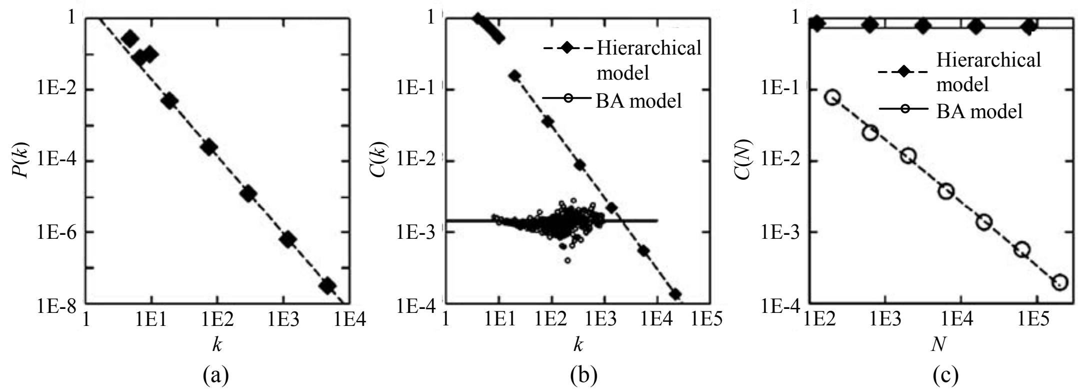

Fig.2 Some properties of the hierarchical model with $N=5^7$ nodes. (a) The numerically determined degree distribution; (b) The $C(k)$ curve of hierarchical model, demonstrating that it follows the scaling law. The open circles show $C(k)$ for BA model of the same size; (c) The dependence of the clustering coefficient C on the network size N

reproductive success. Updating strategy can be either synchronous or asynchronous (Hauert and Doebeli, 2004). For synchronous updates, all individuals interact in their respective neighborhood firstly and then all sites are updated simultaneously through competition with a randomly chosen neighbor. For asynchronous updates, only a single, randomly selected focal site is updated at each simulation step: firstly the payoffs of the focal individual and a random neighbor are determined, afterwards these two individuals compete to re-populate the focal site. Our simulations use the synchronous updating strategy.

Initially, an equal percentage of strategies (cooperators or defectors) were randomly distributed among all individuals of the system. The evolution of the system goes on as follows: in each generation, all pairs of directly connected individuals x and y engage in a single round of a given game, their accumulated payoffs are stored as P_x and P_y, respectively. Whenever a site x is updated, a neighbor y is drawn at random among all k_x neighbors. Whenever $P_y>P_x$, the chosen neighbor takes over site x with probability given by $(P_y-P_x)/(Dk>)$, where $k>$ is the largest value between k_x and k_y, $D=T-S$ for the PD and $D=T-P$ for the SG. Equilibrium frequencies of cooperators and defectors were obtained by averaging over 1000 generations after a transient time of 10000 generations.

Fig.3 gives the simulation results. The networks with 125 nodes and 15625 nodes show the same behavior. In PD, the frequency of cooperators is always near zero for the whole range of b. In SG, the frequency of cooperators decreases from 90% to 10% as r increases. This means that on hierarchical network model, PD game inhibits cooperation while SG game promotes cooperation with small r and inhibits cooperation with large r.

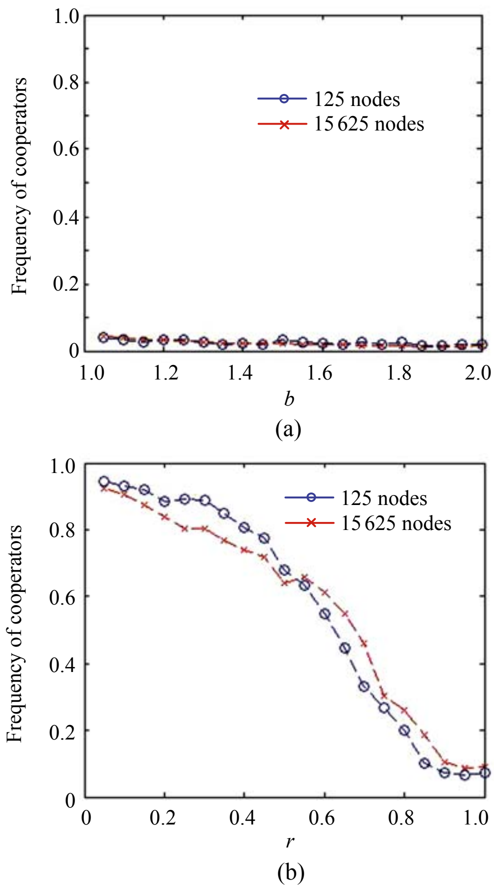

Fig.3 Strategic games on hierarchical network model. Each data point results from averaging over 100 runs. (a) Prisoner's dilemma game; (b) Snowdrift game

DISCUSSION

Santos and Pacheco (2005) pointed out that scale- free networks provide a unifying framework for the emergence of cooperation. For both PD and SG, the frequencies of cooperators on the BA scale-free model are high (above 80% in all the parameters' ranges). On configuration model, the frequencies of cooperators of both PD and SG decrease as parameters increase. And on networks getting from uniform attachment, the dynamics show the same behavior with configuration model. From our work, we can see that although the hierarchical network model shows the same scale-free property like typical BA scale-free model, strategic games on them are quite different. Thus we will make a discussion.

In the evolution, when one's strategy has been changed from cooperation to defection, its total payoffs change differently according to strategic game. We use P_c as one's payoffs before its strategy changes, P_d as one's payoffs after its strategy changes. Then $(P_d-P_c)/P_c$ measures the payoff changing from one's cooperation to defection strategy. Let's denote α as the ratio of defectors of its total neighbors. We have

$$\frac{P_d-P_c}{P_c}=\begin{cases}(b-1)(1-\alpha), & \text{for PD,}\\ r-(1+r)\alpha, & \text{for SG.}\end{cases}$$

We can see from Fig.4a, $(P_d-P_c)/P_c>0$ for PD except the boundary. While for SG, there is a watershed on r-α plane, $(P_d-P_c)/P_c$ is below 0 for more than half of the plane (Fig.4b). Consider the initial of evolution, $\alpha\approx0.5$. For PD, $(P_d-P_c)/P_c>0$, which means that when one's strategy changes from cooperation to defection, its payoff suddenly increases. For SG, $(P_d-P_c)/P_c<0$, which means that when one's strategy changes from cooperation to defection, its payoffs will decrease rather than increase. Notice that, when r increases for SG, the range of α for $(P_d-P_c)/P_c>0$ increases. This means that it behaves close to PD game.

Although the hierarchical network model has the properties of scale-free degree distribution, strategic games on them are quite different from typical scale-free network, such as BA model. In BA model, vertices with the highest connectivity (so-called hubs) become directly interconnected as a result of growth and preferential attachment. Cooperators will tend to occupy the hubs and, since hubs are directly connected, even if a defector occasionally takes over one hub, the probability that it gets reoccupied by a cooperator becomes essentially one. So BA model ensures the prevalence of cooperation in both games.

Hierarchical network model is quite different from BA model due to its power-law clustering distribution. Let us recall the construction process of the hierarchical network model. It is highly modularized, and several submodules combine into the larger modules recursively, making it hierarchical organized, showing the power-law clustering distribution statistically. The hubs of parallel modules are not directly interconnected, making it dominant in the module.

In PD, we can see that when one's strategy changes from cooperation to defection, its payoff

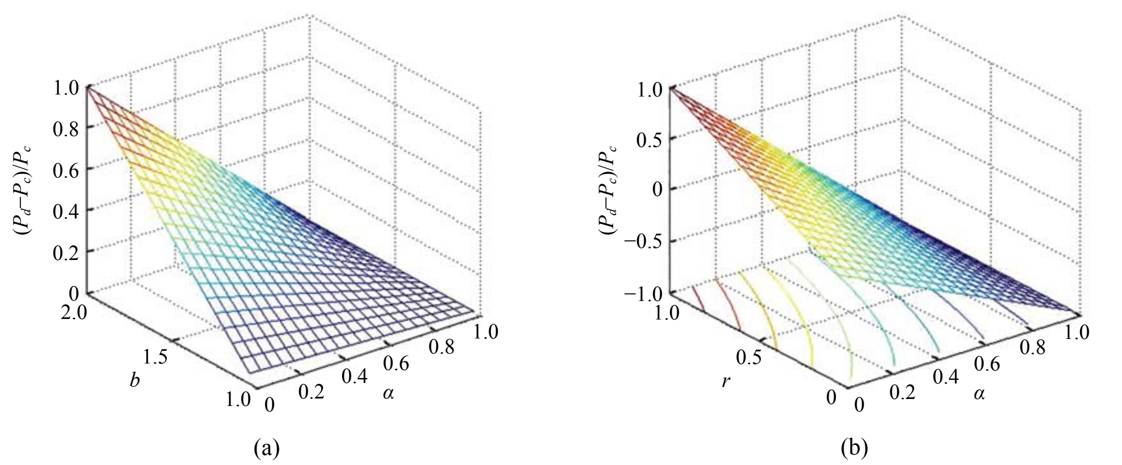

Fig.4 $(P_d-P_c)/P_c$ for strategic games. α is the ratio of defectors of its total neighbors. (a) For PD, $(P_d-P_c)/P_c>0$ except the boundary; (b) For SG, there is a watershed on r-α plane, $(P_d-P_c)/P_c<0$ for more than half of the plane

suddenly increases in whatever situation. So in the evolution, if a defector occasionally takes over a cooperator, the payoffs of the occupied one greatly increases. So the probability that it takes over its neighbors $(P_y-P_x)/(Dk>)$ increases and the probability that it is taken over by its neighbors decreases sharply. Because the parallel nodes are not directly connected, when the hub of the local neighborhood is occupied by a defector, it has little chance to be recovered. Eventually the whole network will be occupied by the defection strategy. In SG, consider the initial of the evolution where $\alpha\approx 0.5$, $(P_d-P_c)/P_c<0$. When one's strategy changes from cooperation to defection, the payoffs will decrease, rather than increase. If a defector takes over the hub, the probability that it takes over its neighbor decreases and the probability that it is taken over by its neighbors increases. So it also could be taken over by a cooperator in the future evolution. With the evolution going on, the number of cooperators increases. When the parameter r increases, SG game behaves close to PD game, and the changes of strategy from cooperation to defection will suddenly increase one's payoffs. It will lead to the low frequency of cooperators.

CONCLUSION

In conclusion, we study the evolution of cooperation on a hierarchical network model, adopting the prisoner's dilemma (PD) game and snowdrift game (SG) as metaphors of the interplay between connected nodes. We found that on hierarchical model, there is no sign of cooperation for PD game, while the frequency of cooperation decreases as the common benefit decreases for SG. This is quite different from BA model which provides a unifying framework for the emergence of cooperation. Comparing the scaling clustering coefficient properties of the hierarchical network model with those of BA model, we found that hierarchical model amplifies the effect of hubs. Considering different performances of PD game and SG in a complex network, we also found that common benefit leads to cooperation in the evolution. The hierarchical network model is more approximate to some real networks observed in nature and the human society. So our work may shed light on the emergence of cooperation in both natural and social environments.

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