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Time-domain training sequences design for MIMO OFDM channel estimation*

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Abstract: This paper describes a Least Squares (LS) channel estimation scheme for MIMO OFDM systems based on time-domain training sequence. We first compute the minimum mean square error (MSE) of the LS channel estimation, and then derive the optimal criteria of the training sequence with respect to the minimum MSE. It is shown that optimal time-domain training sequence should satisfy two criteria. First, the autocorrelation of the sequence transmitted from the same antenna is an impulse function in a region longer than the channel maximum delay. Second, the cross-correlation between sequences transmitted from different antennas is zero in this region. Simulation results show that the estimator using optimal time-domain training sequences has better performance than that using optimal frequency training sequence at low signal-to-noise ratio (SNR). To reduce the training overhead, a suboptimal training sequence is also proposed. Comparing with optimal training sequence, it has low computation complexity and high transmission efficiency at the expense of little performance degradation.

Key words: MIMO, OFDM, Channel estimation, Time-domain training sequence
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INTRODUCTION

The combination of multiple input multiple output (MIMO) system and orthogonal frequency division multiplexing (OFDM) technologies is currently being considered as a powerful candidate for next generation communication systems. A MIMO system offers high system capacity and provides diversity in a fading environment (Bolcskei *et al*., 2002). An OFDM system is suitable for data transmission over wideband wireless channels since it is robust to severe multipath fading. This system is also less complex in channel equalization, where by using fast Fourier transform (FFT) at the transmitter and the receiver, the multipath channel can be equalized by a single scalar multiplication in the frequency domain. However, such systems rely upon the knowledge of channel state information (CSI) at the receiver.

In a fading channel environment, channel estimation is essential for decoding the received signals. For a MIMO OFDM system where two or more transmitter antennas are used, this channel estimation becomes a challenging task. When estimating the channel state for a transmitter-receiver pair, the signals from other transmitter antennas become interference, which disturbs the accuracy of the estimation process. To separate the transmitted signals from different antennas and to reduce channel estimation complexity for OFDM system with multiple transmit-antennas, Li (2002) and Minn *et al*.(2002) have designed and analyzed orthogonal training sequences for different transmitter antennas in the frequency domain. In (Barhumi *et al*., 2003; Minn and Al-Dhahir, 2006), the optimal frequency-domain training sequence design for frequency selective block fading channel estimation was analyzed based on minimizing the channel estimation mean square error (MSE).

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These training sequences should be equi-powered and equi-spaced. Furthermore, the training sequences from different antennas must be phase-shift orthogonal. However, the time-domain channel estimation method has the advantage of inter-symbol time averaging and can catch significant taps (Minn and Bhargava, 2000). Moreover, time-domain approach has lower complexity than frequency-domain approach (Yeh *et al*., 2000). These motivate us to design time-domain training sequence and estimate MIMO channels in the time domain.

SYSTEM MODEL

For a MIMO OFDM with P transmitter antennas, Q receiver antennas and N subcarriers as shown in Fig.1, the data are coded by space-time code and modulated by inverse fast Fourier transform (IFFT), and then cyclic prefix (CP) is inserted to form OFDM symbol. Training sequence (TS) and its CP form training symbol. An OFDM frame is composed of one training symbol and several OFDM symbols. Since the same channel estimation procedure is performed at each receiver antenna, we only need to consider P transmitter antennas and one of the receiver antennas in designing training sequences. The cyclic prefix G is assumed to be longer than the largest multipath delay. Let $s_p(n)$ be the training symbol of the pth transmitter antenna where $n=-G,\dots,K-1$ and K is the length of training sequence. Define $\boldsymbol{S}_p$ as the training sequence matrix of size $K\times L$ for the pth transmitter antenna whose elements are given by $\boldsymbol{S}_p[i, j]=s_p(i-j)$.

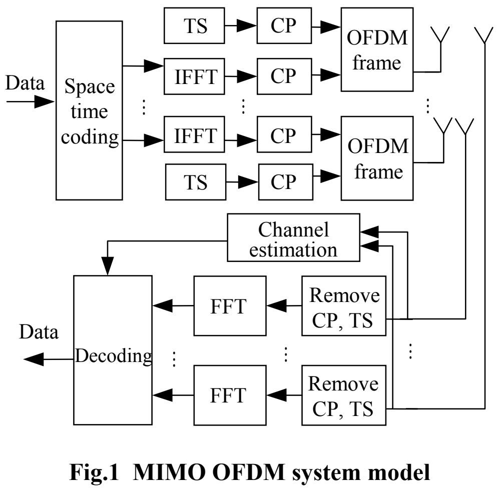

Fig.1 MIMO OFDM system model

The channel impulse response (CIR) for each transmitter-receiver antenna pair is assumed to be quasi-static over one OFDM frame and the maximum channel delay is L samples. Then the CIR from the pth transmitter antenna to the qth receiver antenna can be expressed as

$$h_{(p,q)}(n)=\sum_{l=0}^{L-1}h_{(p,q)}(l)\delta(n-l),$$

where $h_{(p,q)}(l)$ is the lth path gain from the pth transmitter antenna to the qth receiver antenna. For simplicity, the fades between different transmitter and receiver antennas are assumed to be independent and identically distributed (i.i.d). Define

$$\boldsymbol{H}_{(p,q)}=[h_{(p,q)}(0),h_{(p,q)}(1),\dots,h_{(p,q)}(L-1)]^{\mathrm{T}}.$$

Then, the signal at the qth receiver antenna after the CP removal can be given by

$$\boldsymbol{R}_q=\sum_{p=1}^{P}\boldsymbol{S}_p\boldsymbol{H}_{(p,q)}+\boldsymbol{W}_q, \qquad (1)$$

where $\boldsymbol{W}_q$ is a K-vector complex Gaussian noise with zero mean and variance σ_w^2.

TIME-DOMAIN TRAINING SEQUENCE DESIGN FOR CHANNEL ESTIMATION

The challenge associated with channel estimation in MIMO OFDM systems is that the received signal is the superposition of training sequences from all transmitter antennas through independent channels corrupted by noise, which can be seen from Eq.(1). Define

$$\boldsymbol{S}=[\boldsymbol{S}_1,\boldsymbol{S}_2,\dots,\boldsymbol{S}_P],\ \ \boldsymbol{H}_q=[\boldsymbol{H}_{(1,q)},\boldsymbol{H}_{(2,q)},\dots,\boldsymbol{H}_{(P,q)}]^{\mathrm{T}}.$$

Then Eq.(1) can be expressed in matrix form as

$$\boldsymbol{R}_q=\boldsymbol{S}\cdot\boldsymbol{H}_q+\boldsymbol{W}_q. \qquad (2)$$

Optimal design criteria

Based on Eq.(2), the least square channel estimation (also maximum likelihood), assuming $\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S}$ has full rank, is given by

$$\hat{\boldsymbol{H}}_q = (\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\boldsymbol{S}^{\mathrm{H}}\boldsymbol{R}_q. \quad (3)$$

From Eq.(3), the MSE of the LS channel estimation is given by

$$\begin{aligned} MSE &= \frac{1}{LP}E\left\{|\hat{\boldsymbol{H}}_q - \boldsymbol{H}_q|^2\right\} \\ &= \frac{1}{LP}E\left\{\left|(\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\boldsymbol{S}^{\mathrm{H}}\boldsymbol{W}_q\right|^2\right\} \\ &= \frac{\sigma_w^2}{LP}\mathrm{tr}\left((\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\right). \end{aligned} \quad (4)$$

Using a similar argument as (Tung *et al*., 2001), we can show that in order to obtain the minimum MSE of the LS channel estimation subject to a fixed energy E_{n} dedicated for training, we require

$$\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S} = E_{\mathrm{n}}\boldsymbol{I}_{LP}. \quad (5)$$

The minimum MSE (MMSE) is given by

$$MMSE = \delta_w^2 / E_{\mathrm{n}}. \quad (6)$$

Under the same total training signal energy constraint, the method can acquire the same performance as that using optimal frequency-domain training sequence (Barhumi *et al*., 2003). The condition of Eq.(5) can be equivalently stated as

Condition I: For $m,l = 0,1,...,L-1$

$$\sum\nolimits_{n=0}^{K-1} s_i(n-m)s_i(n-l) = \begin{cases} E_{\mathrm{n}}, & m = l, \\ 0, & m \neq l, \end{cases} \quad (7)$$

Condition II: For $m,l = 0,1,...,L-1$

$$\sum\nolimits_{n=0}^{K-1} s_i(n-m)s_j(n-l) = 0, \quad i \neq j. \quad (8)$$

Condition I means that the autocorrelation of the time-domain training sequence is an impulse function. Condition II means that the correlation between training sequences transmitted from different antennas is zero with its different cyclic shifts. It is impossible to find a group of sequences that also satisfy ideal autocorrelation and cross-correlation. But it can be seen from Eqs.(7) and (8) that orthogonality should be satisfied only when m, $l\in\{0, 1, \ldots, L-1\}$. For frequency-domain training sequence design, it means constant-modulus modulation; for time-domain training sequence design, it means different cyclic shifts of an initial training sequence where autocorrelation is an impulse function.

Optimal time-domain training sequence

Many sequences, e.g., PN (pseudo-noise) sequence, etc., have good autocorrelation and can be used as the initial sequence. In this paper, modulatable orthogonal sequence proposed by Suehiro and Hatori (1988) is used as the initial training sequence due to its good autocorrelation and cross-correlation properties. For a prime number $\sqrt{K}$, there are $\sqrt{K}-1$ orthogonal sequences of period K. The sequence can be represented by

$$s(n) = \sqrt{E_{\mathrm{n}}/K}\cdot b_{i_1}\cdot\exp\left(\mathrm{j}2\pi i_0 i_1/\sqrt{K}\right),$$

where $0 \le i_0,\ i_1 \le \sqrt{K}-1$, $n = i_0\sqrt{K}+i_1$. The elements b_{i_1} are complex coefficients of magnitude 1.0. The autocorrelation function can be expressed as

$$\phi(k) = \sum_{n=0}^{K-1} s^{\mathrm{H}}(n)s\left((n+k)_K\right) = \begin{cases} E_{\mathrm{n}}, & k = 0, \\ 0, & k \neq 0, \end{cases}$$

where $(\cdot)_K$ donotes cyclic shift. Training sequences from different transmitter antennas are formed by different cyclic shifts of the initial training sequence. For example, for the training sequence transmitted from the pth antenna, the cyclic shift is $(p-1)\times L$. Since we need to ensure that the L multipath spread signals from one transmitter antenna will not overlap with the multipath spread signals from other antennas, the length of the training sequence must satisfy $K\ge LP$. Then all of the training sequences can be expressed as

Antenna 1: $\{s(0),\ldots,s(L-1),s(L),\ldots,s(K-1)\}$;
Antenna 2: $\{s(L),\ldots,s(K-1),s(0),\ldots,s(L-1)\}$;
…
Antenna P: $\{s[(P-1)L],\ldots,s(K-1),s(0),\ldots,s[(P-1)L-1]\}$.

Because the autocorrelation of the sequence is an impulse function, Conditions I and II can be satisfied. The channel estimation of Eq.(3) can be simplified by

$$\hat{\boldsymbol{H}}_q = \boldsymbol{S}^{\mathrm{H}}\boldsymbol{R}_q / E_{\mathrm{n}}. \quad (9)$$

From Eq.(9) it can be seen that the proposed method only needs a matrix multiplication and the length of training sequence is $P\times L$. For traditional frequency-domain training sequences, P OFDM training symbols that consist completely of pilot symbols are needed. In these systems the length of training sequence is $P\times N$. For OFDM systems, the number of subcarriers N is much larger than the channel maximum delay L. In order to get channel state information, significant-tap-catching (STC) estimator (Li *et al.*, 1999) needs a matrix inversion. So the channel estimator using optimal time-domain training sequence has low computation complexity and high transmission efficiency.

Suboptimal time-domain training sequence

The method using optimal time-domain training sequence has higher transmission efficiency than that using frequency-domain training sequence. However, when a large number of transmitter antennas are employed, the transmission efficiency decreases seriously. In order to improve the transmission efficiency, suboptimal time-domain training sequence is proposed in this subsection. The length of optimal time-domain training sequence should be longer than $P\times L$ because the cross-correlation between sequences from different antennas should be zero. If we tolerate the interference and use sequences whose cross-correlation is not zero but satisfies the mathematical lower bound to estimate MIMO channels, the length of training sequences should only be longer than L no matter how many antennas are used. Though interference will be induced by non-ideal cross-correlation, this interference is very small and most can be removed. Abovementioned modulatable orthogonal sequences have good autocorrelation and cross-correlation functions. Then we still use these sequences to form suboptimal time-domain training sequence structure. The sequence transmitted from antenna p is represented by $s_p(n)=\sqrt{E_\mathrm{n}/K}\cdot b_{i_1}\cdot\exp\left(\mathrm{j}2\pi p i_0 i_1/\sqrt{K}\right)$.

The cross-correlation function can be expressed as

$$\psi(k)=\left|\sum_{n=0}^{K-1}s_p^\mathrm{H}(n)\cdot s_{p'}\left((n+k)_K\right)\right|\le\frac{E_\mathrm{n}}{K},\ \forall p\ne p'. \quad (10)$$

Based on Eq.(9), we first define an $LP\times 1$ vector

$$\begin{aligned}\boldsymbol{M}_q&=\boldsymbol{S}^\mathrm{H}\boldsymbol{R}_q/E_\mathrm{n}=(\boldsymbol{S}^\mathrm{H}\boldsymbol{S}\boldsymbol{H}_q+\boldsymbol{S}^\mathrm{H}\boldsymbol{W}_q)/E_\mathrm{n}\\&=(\boldsymbol{I}_{LP}+\boldsymbol{E}/E_\mathrm{n})\boldsymbol{H}_q+\boldsymbol{S}^\mathrm{H}\boldsymbol{W}_q/E_\mathrm{n},\end{aligned} \quad (11)$$

where $\boldsymbol{S}^\mathrm{H}\boldsymbol{S}=E_\mathrm{n}\boldsymbol{I}_{LP}+\boldsymbol{E}$. $\boldsymbol{E}$ is the interference induced by non-ideal cross-correlation between sequences from different antennas. If we can remove the interference, the channel can be estimated. The interference can be expressed as

$$\boldsymbol{E}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{S}_1^\mathrm{H}\boldsymbol{S}_2 & \cdots & \boldsymbol{S}_1^\mathrm{H}\boldsymbol{S}_{P-1} & \boldsymbol{S}_1^\mathrm{H}\boldsymbol{S}_P\\ \boldsymbol{S}_2^\mathrm{H}\boldsymbol{S}_1 & \boldsymbol{0} & \cdots & \boldsymbol{S}_2^\mathrm{H}\boldsymbol{S}_{P-1} & \boldsymbol{S}_2^\mathrm{H}\boldsymbol{S}_P\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \boldsymbol{S}_{P-1}^\mathrm{H}\boldsymbol{S}_1 & \boldsymbol{S}_{P-1}^\mathrm{H}\boldsymbol{S}_2 & \cdots & \boldsymbol{0} & \boldsymbol{S}_{P-1}^\mathrm{H}\boldsymbol{S}_P\\ \boldsymbol{S}_P^\mathrm{H}\boldsymbol{S}_1 & \boldsymbol{S}_P^\mathrm{H}\boldsymbol{S}_2 & \cdots & \boldsymbol{S}_P^\mathrm{H}\boldsymbol{S}_{P-1} & \boldsymbol{0}\end{bmatrix}.$$

According to Eq.(10), the value of $|\boldsymbol{E}|$ is very small. Then $\boldsymbol{M}_q$ can be used to identify significant taps in the channel. We define an $LP\times 1$ vector $\boldsymbol{\Gamma}_q$ that denotes the estimated significant taps. The ith element is expressed as

$$\boldsymbol{\Gamma}_q(i)=\begin{cases}\boldsymbol{M}_q(i), & |\boldsymbol{M}_q(i)|\ge\lambda,\\ \boldsymbol{0}, & |\boldsymbol{M}_q(i)|<\lambda,\end{cases} \quad (12)$$

where $\boldsymbol{M}_q(i)$ denotes the ith element of $\boldsymbol{M}_q$, λ denotes the amplitude threshold of selected taps. The threshold is affected by the channel delay profile. For simplicity, we usually assume $\lambda=0.1$. If we choose an appropriate threshold, all of the significant paths can be identified. The amplitude of nonsignificant taps is very small, and they can be ignored. Then the interference can be removed and the channel can be estimated by

$$\hat{\boldsymbol{H}}_q=(\boldsymbol{S}^\mathrm{H}\boldsymbol{R}_q-\boldsymbol{E}\boldsymbol{\Gamma}_q)/E_\mathrm{n}\approx\boldsymbol{H}_q+\boldsymbol{S}^\mathrm{H}\boldsymbol{W}_q/E_\mathrm{n}. \quad (13)$$

It can be seen that the length of the proposed suboptimal time-domain training sequence only needs to be longer than L to estimate the MIMO channels. However, for optimal frequency-domain training sequence, Barhumi *et al.*(2003) and Minn and Al-Dhahir (2006) showed that the length needs to be more than $L\times P$. So the system using suboptimal time-domain training sequence has the highest transmission efficiency.

SIMULATION RESULTS

In our simulation, two transmitter antennas and two receiver antennas are used for diversity. We assume channels with 8 paths and channel maximum delay L=16, Doppler frequency of 5 Hz and 100 Hz are used to represent different mobile environments. These paths are simulated as i.i.d and correlated in time with a correlation function according to Jakes' model $r_{hh}(\tau)=\sigma_h^2 J_0(2f_d\tau)$, where f_d denotes Doppler frequency. The links between different transmitter and receiver antennas are independent. Parameters of the simulated OFDM system are set as follows. The total channel bandwidth of 800 kHz is divided into 128 subchannels and the length of CP is G=32. A 16-state space-time code with QPSK modulation is applied. To make the tones orthogonal to each other, the symbol duration is 160 μs. This results in a total OFDM symbol length 200 μs and a subchannel symbol rate 5 kBd. The lengths of optimal and suboptimal time-domain training sequence (TDTS) are 49 and 25 samples, respectively. The length of optimal frequency-domain training sequence (FDTS) is 128. The FDTS method uses one OFDM symbol and is designed by frequency-division multiplexing (FDM) method (Minn and Al-Dhahir, 2006).

The MSE as a function of signal-to-noise ratio (SNR), which is defined as $SNR=10\cdot\lg(\sigma_w^2/E_s)$, where $E_s = E\left\{\sum_{p=1}^{P}\left|\boldsymbol{S}_p\boldsymbol{H}_{(p,q)}\right|^2\right\}$, is shown in Fig.2. We compare the MSE performance of channel estimators that adopted optimal FDTS, optimal and suboptimal TDTS. From the figure, one can see that at low *SNR* (*SNR*≤10 dB) the method using optimal TDTS has better performance than that using optimal FDTS. At high *SNR*, they can acquire the same performance. For the estimator using suboptimal TDTS, there is an around 2-dB degradation in the required *SNR*.

Another performance criterion is bit error ratio (BER), as shown in Fig.3. One-tap equalization in the frequency domain is used in the simulation. When the Doppler frequency is 5 Hz, to achieve a $BER=1\times10^{-3}$, the required *SNR* are about 9 dB and 10 dB for optimal and suboptimal training sequences, respectively. When Doppler frequency is 100 Hz, the required *SNR* are about 14 dB and 15 dB for optimal and suboptimal training sequences, respectively. It is approximately 1 dB away from the optimal performance curves. It also can be seen that the performance degraded seriously with the increase of Doppler frequency.

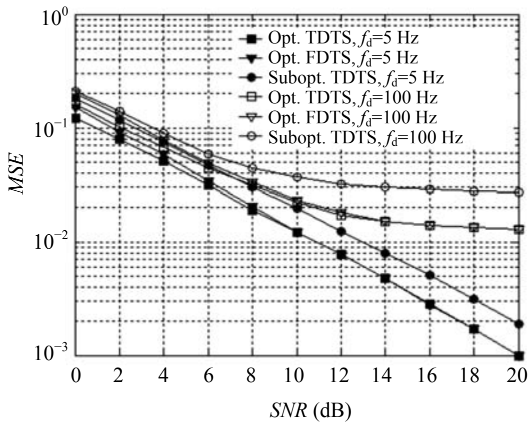

Fig.2 *MSE* performance of a MIMO OFDM system

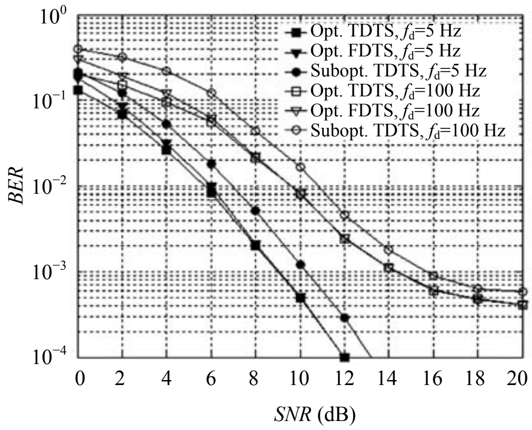

Fig.3 *BER* performance of a MIMO OFDM system

CONCLUSION

In this paper, we describe a Least Squares (LS) channel estimation scheme for MIMO OFDM systems. Based on minimum mean square error, the optimal and suboptimal time-domain training sequences are proposed. Compared to estimator using optimal frequency-domain training sequence, the proposed methods have low computational complexity and high transmission efficiency. At low *SNR*, the performance of estimator using optimal time-domain training sequence is better than that of estimator using optimal frequency-domain training sequence.

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