Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Reliability based multiobjective optimization for design of structures subject to random vibrations

Giuseppe Carlo MARANO
(*Department of Environmental Engineering and Sustainable Development, Technical University of Bari, viale del Turismo, 10-74100, Taranto, Italy*)
E-mail: gmarano@poliba.it

Received June 7, 2007; revision accepted Sept. 11, 2007; published online Dec. 14, 2007

Abstract: Based on a multiobjective approach whose objective function (OF) vector collects stochastic reliability performance and structural cost indices, a structural optimization criterion for mechanical systems subject to random vibrations is presented for supporting engineer's design. This criterion differs from the most commonly used conventional optimum design criterion for random vibrating structure, which is based on minimizing displacement or acceleration variance of main structure responses, without considering explicitly required performances against failure. The proposed criterion can properly take into account the design-reliability required performances, and it becomes a more efficient support for structural engineering decision making. The multiobjective optimum (MOO) design of a tuned mass damper (TMD) has been developed in a typical seismic design problem, to control structural vibration induced on a multi-storey building structure excited by nonstationary base acceleration random process. A numerical example for a three-storey building is developed and a sensitivity analysis is carried out. The results are shown in a useful manner for TMD design decision support.

Key words: Structural optimization, Multiobjective optimization (MOO), Random vibration, Tuned mass damper (TMD)
doi:10.1631/jzus.A072128 **Document code:** A **CLC number:** O32

INTRODUCTION

In the field of structural engineering, decisions regarding design making consist in applying the solution which best satisfies the required performance given the resources available. The typical approach used by many engineers is based on indirect or intuitive methods which depend on past experiences, subconscious motives, incomplete logical schemes, random selections and sometimes even intuitive simplified mechanical schemes. Such methods, maybe extremely pragmatic and applicable, generally do not offer high optimal performance solutions, which means that they only satisfy the given design requirement without effectively minimizing the required resources. An alternative approach in structural design is the optimal structural design (OSD), which consists in applying only a logical process mathematically expressed in support of decision making. The standard single objective optimization (SOO) consists in minimizing or maximising one objective function (OF) capable of describing system performances. In addition, it may be necessary to satisfy given constraints. The OF is defined by construction and/or failure costs, total weight or one structural performance index. This alternative approach can provide at least one single optimal solution. The multiobjective optimization (MOO) approach is based on an OF vector whose dimension is greater than 1 and elements include different structural costs and performance indices. Unlike the SOO, the MOO produces a set of possible solutions and the designer must select only one possible solution that better agrees with the designer's own decisions.

Regarding structural problems where dynamic loads are intrinsically random, both OF and constraints may be expressed by probabilistic entities, such as covariances, spectral moments, probability of failures and similar entities (Nigam, 1972). In this field, there is a wide class of structural engineering problems involving structural systems thought and designed to sustain dynamic actions, which can be suitably modelled as random events rather than deterministic ones, such as earthquakes, winds pressure, sea waves and rotating machinery induced vibrations. Structural responses to these actions are random processes, and thus the random vibration theory is the most reliable way to assess structural response in a probabilistic manner.

Random dynamic analysis seems to be the most useful method to obtain suitable information concerning structure response and reliability. In the field of structural engineering, probabilistic methodologies have gained increasing importance and are now frequently used to assess structural safety problems. Probabilistic approaches can take into consideration structural parameters or loads and the effects of uncertainty on structural response in all cases where mechanical and excitation parameters are intrinsically random quantities. Even if this approach may considerably increase the difficulties in analysis, it is the only approach that can offer some essential design information that is not usually directly available by more conventional and less complicated deterministic approaches. Confident with these reasons and given the 60 years experiences in the field of structural dynamics, the deterministic approach (in which forces and structural responses are assumed as quantities exactly known) has been replaced with the stochastic approach, which allows a more representative and detailed structural response and safety evaluation.

In the meantime, optimization methods have gained increasing importance in the field of structural design, typically based on the implicit assumption that all involved variables are deterministic. This "conventional" approach could fail when the real uncertain nature of some structural parameters is properly considered, reducing the optimal performance or at least making unfeasible the expected optimal goal. In the last decades, different approaches have been proposed essentially by using probabilistic methodologies due to computational and conceptual difficulties involved in treating uncertainty properly in structural optimization. In standard reliability based design optimization (RBDO) (Rackwitz *et al*., 1995; Kuschel and Rackwitz, 2000), the OF is minimized under probabilistic constraints instead of conventional deterministic ones. The proposed approaches for RDBO are essentially referred to time-invariant cases (Pedersen and Thoft-Christensen, 1995; Polak *et al.*, 1997; Gasser and Schueller, 1997). Only a few contributions deal with time-variant aspects (Rosenblueth and Mendoza, 1971; Kuschel and Rackwitz, 2000), in which reliability is determined by the out-crossing approach and by the context of well-known FORM or SORM.

A simplified approach in structural optimization dynamic problems consists in assuming that loads are the only uncertain sources of uncertainty, when they have a clear undeterministic nature as in the case of earthquakes or wind actions. In the case of earthquakes or wind actions, these loads are suitably modelled by stochastic processes and the standard random vibrations theory can be adopted if all the other involved quantities are assumed as deterministic. Structural response characterization is so completely described by stochastic processes with deterministic parameters. With reference to seismic engineering and seismic protection devices, many optimization applications have been developed in the last twenty years (Wirsching and Campbell, 1974; Constantinou and Tadjbakhsh, 1983a; 1983b; 1985; Park *et al*., 2004; Marano *et al*., 2006; 2007a). A complete stochastically defined optimum design method is also proposed by Marano *et al*. (2007b), in which a reliability based optimum criterion was developed adopting a covariance approach. Both OF and constraints are defined in a stochastic way, where the latter imposes a limit to the failure probability associated to the first threshold crossing of structural displacement over a given value. A reliability based methodology for the robust optimal design of uncertain linear structural systems subjected to stochastic dynamic loads was also presented by Papadimitriou *et al*.(1997) and Papadimitriou and Ntotsios (2005). System safety referred to structural displacements was used as structural performance index, under stationary white noise input conditions. The methods proposed by the authors also deal with robust solution evaluating both mean and covariance of OF, by using an MOO robust design.

Moreover, all proposed methods for seismic devices optimization are based on the minimization of a single OF that quantifies the protected systems response reduction in respect of the unprotected configuration. Moreover, the OFs are expressed in terms of covariances, and the main limitation is the lack of information about final structural performance, which is unknown when expressed in terms of reliability. For instance, in the case of vibration protection devices, the ratio between protected and unprotected structural displacement (or inertial acceleration) covariance is commonly used as OF. It is not at all possible to evaluate if a given required performance, commonly expressed as a limitation on maximum main system displacement or similar response measures, even if it is possible to immediately indicate the advantages in adopting a specific seismic protection device, is really achieved by using the protection strategy adopted.

For this specific reason, the present work is focused on the structural optimum design criteria that directly involves in a performance based design (PBD) in the random vibrating structural problem. Without loss in generality, the optimum design of a vibrations control device is analysed as a case study regarding structures subject to seismic actions. Moreover it must also be taken into account that several OFs are involved in design decisions different from conventional optimization (single OF). These functions are often in conflict with each other and for them it is not possible to define a universally approved criterion for "optimum" design as occurs in SOO. For this reason, Pareto dominance and Pareto optimality constitute very important notions in MOO problems, not only being able to furnish a single defined optimal solution (as in SOO), but giving a set of possible optimal solutions satisfying, at the same time, with different performances, all designers' objectives.

In this work, an MOO procedure is adopted for the optimum design of seismic devices for linear structures subject to random seismic loads. This procedure adopts a 2D OF vector that is defined by using both standard deterministic costs and structural survival probability indices. An example is developed with the first OF element assumed as a deterministic device cost and the second one being the system failure probability. The failure is defined as the first crossing out of an admissible domain of one structural response during all seismic actions. Without loss in generality, the failure is the allowable top floor displacement, but other structural responses could be easily used.

The reliability evaluation is developed by using the state space covariance analysis, and the Poisson hypothesis is adopted to evaluate the mean threshold crossing rate for the safe domain. The device analysed for seismic protection is the standard tuned mass damper (TMD). A single TMD located at the top of a multi-degree of freedom (DOF) linear system, in which a multi-storey building is analysed. In detail, the base acceleration representing seismic actions are modelled by a nonstationary filtered white noise that can yield a quite realistic seismic loads model. In the optimization problem, the design vector is the collection of TMD mechanical parameters including frequency, mass ratio and damping ratio. As stated before, the main innovation of the proposed approach consists in adopting the performance based seismic design (PBSD) in an MOO problem for the optimum design of a TMD in accordance with modern seismic technical codes. In addition, unlike the SOO approach, the proposed approach can give predesign information, extremely useful in initial designer decisions and as the level of failure probability reduction by using a specific seismic control strategy.

Using the MOO proposed in this work, the designer can control performances and costs in different Pareto front locations, and define solution types to be adopted according to sensibilities and decisions. With more details and with reference to a TMD device, a piece of suitable information for designer is the minimum mass ratio (that is defined as the ratio between TMD and main structural masses) necessary to increase reliability under a given level structure. This is a fundamental element for deciding whether this mass ratio can be practically adopted or not. As an application of the proposed strategy, a multi-DOF system, representing a multi-storey plane frame in a simplified way, is protected by a TMD against earthquake loads. The TMD optimal solution has been obtained for different levels of admissible top floor maximum lateral displacement.

STOCHASTIC ANALYSIS OF MULTI-DOF LINEAR SYSTEM SUBJECT TO RANDOM LOADS

For a generic linear n DOF system excited by a forcing vector $\boldsymbol{f}(t)$ whose related stochastic process is the Gaussian with null mean value stochastic vector $\boldsymbol{F}(t)$, the well known differential matrix motion equation is

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t)+\boldsymbol{C}\dot{\boldsymbol{y}}(t)+\boldsymbol{K}\boldsymbol{y}(t)=\boldsymbol{f}(t), \quad (1)$$

where ***M***, ***C*** and ***K*** are the deterministic mass, damping and stiffness matrices, respectively; $\ddot{\boldsymbol{y}}(t)$, $\dot{\boldsymbol{y}}(t)$, $\boldsymbol{y}(t)$ are the acceleration, velocity and displacement vectors, respectively, and the related stochastic processes are $\ddot{\boldsymbol{Y}}(t)$, $\dot{\boldsymbol{Y}}(t)$ and $\boldsymbol{Y}(t)$, respectively.

The motion Eq.(1) can be written as a first order differential matrix equation by introducing the space state vector $\boldsymbol{z}(t)=(\boldsymbol{y}(t)\ \ \dot{\boldsymbol{y}}(t))^{\mathrm{T}}$, and in the hypothesis of zero mean Gaussian input (as commonly assumed for earthquakes), the stochastic response is completely described by state space covariance matrix knowledge $\boldsymbol{R}(t)$. It can be evaluated by means of the Lyapunov Covariance Matrix Equation (Soong and Grigoriu, 1992) shown as follows:

$$\dot{\boldsymbol{R}}(t)=\boldsymbol{A}\boldsymbol{R}(t)+\boldsymbol{R}(t)\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}(t), \quad (2)$$

where the matrix $\boldsymbol{B}(t)=\langle\widehat{\boldsymbol{f}}\boldsymbol{z}^{\mathrm{T}}\rangle+\langle\boldsymbol{z}\widehat{\boldsymbol{f}}^{\mathrm{T}}\rangle$, and the system matrix and input vector are

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C}\end{bmatrix},\quad \widehat{\boldsymbol{f}}(t)=\begin{Bmatrix}\boldsymbol{0}\\ \boldsymbol{M}^{-1}\boldsymbol{f}(t)\end{Bmatrix}. \quad (3)$$

Furthermore, in order to find the best design under a given stochastic load process, the n_b elements design vector (DV) ***b*** will be introduced to determine shape, sections or other mechanical structural features and, hence, determine the actual value of the stiffness, mass and damping matrices.

With reference to a defined PBD index in a stochastic way, the mechanical safety or reliability $r(T)$ at time T is a natural solution. It is defined as the failure survival probability, in which the failure is a partial or total damage in the interval [0, T]. With reference to a variety of interpretations (generally not only of a mechanical nature) of this condition, it is obvious that the definition of failure plays a central role in the reliability evaluation. Usually, the collapse can be associated to the threshold crossing probability and is more precisely determined by the first time crossing of a structural response parameter $s(t)$ through a given threshold value β.

OPTIMIZATION CRITERIA

In all engineering fields, designers attempt to find solutions that conjugate performance and satisfaction of several requirements. Designers can obtain the optimum within the imposed conditions using standard optimization techniques. In the field of structural engineering, structures designed in this way are safer, more reliable and less expensive than traditionally designed structures. Generally speaking, the structural optimization problem could be formulated as the selection of a set of design variables (that are the design parameters that characterize the structural configuration), collected in the above so-called DV ***b***, over a possible admissible domain Ω_b. With reference to SOO problem, the optimal DV can minimize a given OF and satisfy the assigned constraint conditions. Deterministic-based optimization is aimed to minimize structural weight or volume, subject to given deterministic constraints generally referred to general stresses and/or displacements. An additional probabilistic constraint is considered in the case of reliability-based design, related to structural performance. Reliability theory is afterwards introduced into structural engineering and structural optimization in order to consider all existing sources of uncertainty in a more rational way. These sources can influence structural response as well as the circumstance that the loadings applied to a structure are not quantities exactly known. Therefore, the reliability is recognized as a performance constraint in structural engineering, and an optimum design should generally balance both cost and performance which concerns structural reliability. Probabilistic constraints in SOO usually define the feasible region of the design space by restricting the probability that a deterministic constraint is violated within the allowable probability of violation.

Moreover, not a single "efficiency" index (as in SOO) in many real engineering problems are involved, but several ones that could be related to structural cost or weight, structural performances and other similar criteria. Each of these indices is typically in conflicting with the others, and it is not possible to define a universally approved criterion of "optimum" as in SOO, where optimization is achieved by assuming that one "efficiency" index must be minimised and the other ones must be considered as problem constraints. Moreover, the choice of the index to be minimised and the indices to be transformed into constraints is questionable. The abovementioned question depends strongly on designer's opinion and experience. On the contrary, the MOO gives the opportunity to the designer to evaluate a set of possible solutions that satisfy more than one index, but with different performances. The definitions of these solutions are usually known as the Pareto dominance and Pareto optimality criterion, and constitute a fundamental point in the MOOPs. Regarding the Pareto optimality definition, it is assumed that a design vector $\boldsymbol{b}^*$ is Pareto optimal if no feasible vector $\boldsymbol{b}$ exists, which would decrease some criteria without causing a simultaneous increase in at least one other criterion. Unfortunately, this concept almost always fails to give a single solution, but rather a set of solutions called the Pareto optimal set. The vector $\boldsymbol{b}^*$ corresponding to the solutions included in the Pareto optimal set is called non-dominated. Essentially, defining the generic "efficiency" index as $OF_i(\bar{\boldsymbol{b}})$, a typical minimization-based MOOP is assumed as

$$\min_{\bar{\boldsymbol{b}}\in\Omega_{\bar{b}}}\{OF_1(\bar{\boldsymbol{b}}),OF_2(\bar{\boldsymbol{b}}),\ldots,OF_M(\bar{\boldsymbol{b}})\}. \tag{4}$$

Given two candidate solutions $\{\bar{\boldsymbol{b}}_j,\bar{\boldsymbol{b}}_k\}\in\Omega_{\bar{b}}$, if

$$\forall i\in\{1,\ldots,M\},\quad OF_i(\bar{\boldsymbol{b}}_j)\le OF_i(\bar{\boldsymbol{b}}_k)\wedge \exists i\in\{1,\ldots,M\}: OF_i(\bar{\boldsymbol{b}}_j)< OF_i(\bar{\boldsymbol{b}}_k), \tag{5}$$

and defined the two objective vectors

$$\boldsymbol{v}(\bar{\boldsymbol{b}}_j)=\{OF_1(\bar{\boldsymbol{b}}_j),\ldots,OF_M(\bar{\boldsymbol{b}}_j)\}, \tag{6}$$

$$\boldsymbol{v}(\bar{\boldsymbol{b}}_k)=\{OF_1(\bar{\boldsymbol{b}}_k),\ldots,OF_M(\bar{\boldsymbol{b}}_k)\}. \tag{7}$$

Vector $\boldsymbol{v}(\bar{\boldsymbol{b}}_j)$ is said to dominate vector $\boldsymbol{v}(\bar{\boldsymbol{b}}_k)$ (denoted by $\boldsymbol{v}(\bar{\boldsymbol{b}}_j)\prec\boldsymbol{v}(\bar{\boldsymbol{b}}_k)$). Moreover, if no feasible solution $\boldsymbol{v}(\bar{\boldsymbol{b}}_k)$ exists that dominates solution $\boldsymbol{v}(\bar{\boldsymbol{b}}_j)$, $\boldsymbol{v}(\bar{\boldsymbol{b}}_j)$ is classified as a non-dominated or Pareto optimal solution. In other terms, the candidate solution $\bar{\boldsymbol{b}}_j\in\Omega_{\bar{b}}$ is a Pareto optimal solution if and only if

$$\nexists\bar{\boldsymbol{b}}_k\in\Omega_{\bar{b}}:\boldsymbol{v}(\bar{\boldsymbol{b}}_k)\prec\boldsymbol{v}(\bar{\boldsymbol{b}}_j). \tag{8}$$

More simply, $\bar{\boldsymbol{b}}_j\in\Omega_{\bar{b}}$ is a Pareto optimal solution if a feasible vector $\bar{\boldsymbol{b}}_k\in\Omega_{\bar{b}}$, which would decrease some criteria without causing a simultaneous increase in at least one other criterion (Coello, 1999), does not exist. Unfortunately, the Pareto optimum almost always does not give a single solution but rather a set of solutions and it cannot proceed in an analytical way. The collection of all Pareto optimal solutions is known as the Pareto optimal set or Pareto efficient set. The corresponding objective vectors are instead described as the Pareto front or trade-off surface. Normally, the decision on the "best solution" to be adopted is formulated by the so-called (human) decision maker (DM). The case in which the DM does not have any role and a generic Pareto optimal solution is considered acceptable (no-preference based methods) is extremely rare. On the other hand, several preference-based methods exist in literature, although this particular aspect of research tends to have been somewhat overlooked.

In this case, the MOOP is defined by

$$\text{find } \boldsymbol{b}\in\Omega_{b}, \tag{9}$$

$$\text{which minimizes } \overline{OF}(\boldsymbol{b},t). \tag{10}$$

The OF vector is defined as

$$\overline{OF}(\boldsymbol{b},t)=\{OF_1(\boldsymbol{b},t),OF_2(\boldsymbol{b},t)\}, \tag{11}$$

where

$$OF_2(\boldsymbol{b},t)=Pr\left(G(\boldsymbol{S}(\boldsymbol{b},\tau))\ge 0\mid \tau<T\right), \tag{12}$$

and $G(\boldsymbol{S}(\boldsymbol{b},\tau))$ is the required reliability limit state function for the structural system.

MULTIOBJECTIVE PERFORMANCE RELIABILITY OPTIMIZATION OF TMD IN SEISMIC PROTECTION

Traditional optimum design of TMD is based on protected system mean-square response. In this study, a performance reliability optimum design is developed for a TMD positioned on a simple 1 DOF linear structural system. The innovation of this approach is that the optimization has to be performed by satisfying a design performance expressed in a full stochastic way by a limitation on failure probability. It is well-known that a TMD can be designed to control a single structural model only. Given the properties of the mode that needs to be controlled, the problem is essentially the same as designing a TMD for a SDOF structure. Therefore, structure is described by means of a single DOF system and is equipped with a linear single TMD with the aim of reducing undesirable vibrations caused by dynamic loads acting at its foundation, which are here modelled by means of a general filtered white noise stationary stochastic process.

In order to improve TMD efficiency, it is imperative to define the optimum mechanical parameters (i.e., the optimum tuning frequency, damping and mass ratio) of TMD. Although the basic design concept of TMD is quite simple, the parameters of TMD system must be obtained through an optimal design procedure in order to satisfy performance requirements. For these reasons, the determination of optimal design parameters of TMD to enhance the control effectiveness has become very crucial.

A performance reliability based optimization is adopted to carry out an optimum TMD design. More precisely, a minimum of the mass ratio, that is, the ratio of the added mass on the mass of the structure, is investigated, together with the minimization of a performance index on structural reliability. The choice of the mass ratio as function to be minimized depends on the fact that this quantity can be strictly related to the total cost of the vibration control device. In general, it is evident that limiting the TMD mass is a primary necessity of the designer, both in mechanical and in civil engineering. Increasing TMD mass will certainly increase location volume and total structure vertical dead load, and will also increase stiffness and damping connections and similar connections. So, a primary strategy in TMD design is to evaluate the minimum mass that this device needs to satisfy the given required performances. Moreover, this aspect is directly related to the circumstance that in the usual range of mass ratio between TMD and primary structure, by increasing this parameter, vibrations control efficiency will increase. This tendency is not strictly monotonic, because a minimum exists and corresponds to the optimal mass ratio, but usually the value is too great to be realistically and economically applicable in applied engineering (Marano *et al*., 2007b).

The second OF vector element that is expressed as a structure reliability performance index is related to the failure probability associated to the crossing of the protected system displacement over an allowable limit, and is a function of designer's decision. The minimum reliability level utilized to define the constraint is also assigned according to designer's decisions that depend on the risk level assumed as being acceptable for each given condition.

STATE SPACE MODEL OF THE SYSTEM

A standard way in modelling TMD is carried out by a mass-dashpot-spring system (the secondary system) attached to the top of a linear multi-DOF system (Fig.1). The main scope is to reduce unacceptable vibrations in the main structural system, i.e., the damage level and failure probability. In this specific case, the base excitation acting on the building is treated as a nonstationary filtered stochastic process. It is quite important to represent the evolutive nature of response processes, given the effect that this characteristic has on structural reliability. Simplicity and less computing cost could be obtained by treating the process as a stationary one, but it could overestimate the real final reliability, so that the engineering decision based on the optimization criteria could be strongly different from the real physical phenomenon. Therefore, a time-modulated input process is here adopted for base acceleration description, and the system motion equations of the system in Fig.1 are

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t)+\boldsymbol{C}\dot{\boldsymbol{X}}(t)+\boldsymbol{K}\boldsymbol{X}(t)=-\boldsymbol{M}\boldsymbol{r}\ddot{\boldsymbol{X}}_b, \tag{13}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are, respectively, the deterministic mass, damping and stiffness $(n+1)\times(n+1)$ matrices. The $(n+1)$ vectors $\boldsymbol{X}=(x_1,x_2,\ldots,x_n,x_\mathrm{T})^\mathrm{T}$, $\dot{\boldsymbol{X}}=(\dot{x}_1,\dot{x}_2,\ldots,\dot{x}_n,\dot{x}_\mathrm{T})^\mathrm{T}$ and $\ddot{\boldsymbol{X}}=(\ddot{x}_1,\ddot{x}_2,\ldots,\ddot{x}_n,\ddot{x}_\mathrm{T})^\mathrm{T}$ collect the displacements, velocities and accelerations of n floors and of the TMD relative to the ground, and finally $\boldsymbol{r}=(1,\ldots,1)^\mathrm{T}$.

The TMD mechanical characteristics are described by parameters m_T, k_T and c_T, respectively, the mass, the stiffness and the damping of the TMD.

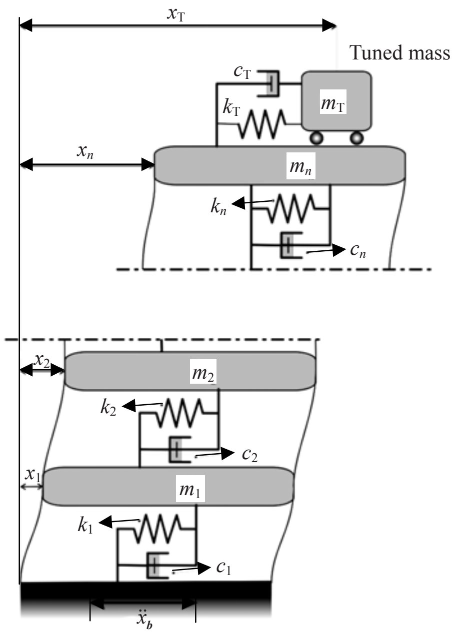

Fig.1 Schematic model of a multi-DOF structural system equipped with a TMD

Introducing the space state vector $\boldsymbol{Z}=(\boldsymbol{X}\ \ \boldsymbol{X}_{\mathrm{f}}\ \ \dot{\boldsymbol{X}}\ \ \dot{\boldsymbol{X}}_{\mathrm{f}})^{\mathrm{T}}$, the system matrix $\boldsymbol{A}$ is

$$\boldsymbol{A}=\begin{bmatrix} \boldsymbol{0}^{(n+2)(n+2)} & \boldsymbol{I}^{(n+2)(n+2)} \\ -\boldsymbol{H}_{\boldsymbol{K}} & -\boldsymbol{H}_{\boldsymbol{C}} \end{bmatrix}, \quad (14)$$

where the two sub-matrices $(n+2)(n+2)$ $\boldsymbol{H}_{\boldsymbol{K}}$ and $\boldsymbol{H}_{\boldsymbol{C}}$ are respectively

$$\boldsymbol{H}_{\boldsymbol{K}}=\begin{bmatrix} & \omega_{\mathrm{f}}^2 \\ (\boldsymbol{M}^{-1}\boldsymbol{K})^{(n+1)(n+1)} & \omega_{\mathrm{f}}^2 \\ & \cdots \\ & \omega_{\mathrm{f}}^2 \\ 0\ \ ..\ \ ..\ \ 0 & -\omega_{\mathrm{f}}^2 \end{bmatrix}, \quad (15a)$$

$$\boldsymbol{H}_{\boldsymbol{C}}=\begin{bmatrix} & 2\xi_{\mathrm{f}}\omega_{\mathrm{f}} \\ (\boldsymbol{M}^{-1}\boldsymbol{C})^{(n+1)(n+1)} & 2\xi_{\mathrm{f}}\omega_{\mathrm{f}} \\ & \cdots \\ & 2\xi_{\mathrm{f}}\omega_{\mathrm{f}} \\ 0\ \ ..\ \ ..\ \ 0 & -2\xi_{\mathrm{f}}\omega_{\mathrm{f}} \end{bmatrix}. \quad (15b)$$

To describe the earthquake ground acceleration, the nonstationary Kanai-Tajimi (K-T) stochastic seismic model (Tajimi, 1960) is used in this work. This model has found wide application in random vibration analysis of structures because it provides a simple way to describe ground motions characterized by a single dominant frequency. The model is obtained by the use of a simple filtered white noise linear oscillator, which in its original formulation treats earthquakes as stationary random processes. However, accelerograms clearly show their strongly nonstationary nature both in amplitude and frequency contents, so that a generalized nonstationary K-T model is given by enveloping the stationary input stochastic process (in this case a stationary Gaussian white noise process $w(t)$, which is supposed to be generated at the bed rock) through a deterministic temporal modulation function $\varphi(t)$, which controls the time amplitude variation without affecting the earthquake frequency content.

Following the above considerations, the total acceleration $\overline{\ddot{\boldsymbol{X}}}_b(t)$ acting at the base of the structure is given by summing the contribution of inertial force $\ddot{\boldsymbol{X}}_{\mathrm{f}}(t)$ of the K-T filter and the time-modulated white noise excitation $\varphi(t)w(t)$ as follows,

$$\begin{cases} \overline{\ddot{\boldsymbol{X}}}_b(t)=\ddot{\boldsymbol{X}}_{\mathrm{f}}(t)+\varphi(t)w(t), \\ \ddot{\boldsymbol{X}}_{\mathrm{f}}(t)+2\xi_{\mathrm{f}}\omega_{\mathrm{f}}\dot{\boldsymbol{X}}_{\mathrm{f}}(t)+\omega_{\mathrm{f}}^2\boldsymbol{X}_{\mathrm{f}}(t)=-\varphi(t)w(t), \end{cases} \quad (16)$$

where $\boldsymbol{X}_{\mathrm{f}}(t)$ is the displacement response of the K-T filter, ω_{f} is the K-T filter natural frequency and ξ_{f} is the K-T filter damping coefficient.

Regarding the modulation function $\varphi(t)$, different formulations have been proposed in literature. In this paper, the one proposed by Jennings (1964) is used and has the following form:

$$\varphi(t)=\begin{cases} (t/t_1)^2, & t<t_1; \\ 1, & t_1\le t\le t_2; \\ \mathrm{e}^{-\beta(t-t_2)}, & t>t_2, \end{cases} \quad (17)$$

where $t_d=t_2-t_1$ is the time interval, and the peak excitation is constant. Parameters are assumed as t_1=3 s, t_2=15 s and β=0.4 s^{-1}.

The Power Spectral Density (PSD) intensity constant S_0 can be related to the standard deviation $\sigma_{\ddot{x}_b}$ of ground acceleration (Crandall and Mark, 1963) by means of the following relation: $S_0=2\xi_{\mathrm{f}}\sigma_{\ddot{x}_b}^2\,/$

$[\pi(1+4\xi_f^2)\omega_f]$, and assuming PGA = $3\sigma_{\ddot{x}_b}$. The relation between PGA and spectral density is

$$S_0=\frac{2\xi_f(PGA)^2}{3^2\pi(1+4\xi_f^2)\omega_f}=0.0707\frac{\xi_f(PGA)^2}{(1+4\xi_f^2)\omega_f},\quad (18)$$

where ξ_f and ω_f are the damping ratio and pulsation frequency of the filter.

In the present study, in order to perform the sensitivity analysis, a stochastic model is considered for a typical earthquake expected on the ground, having moderate-high flexibility. The given earthquake is characterized by an energy content concentrated in the range of 1~4 Hz with PGA equal to 0.35g (the value that generally represents a ground motion of high intensity). The K-T model has values of ω_f=3π rad/s, ξ_f=0.45 and S_0=175.5 m^2/s^3.

The Liapunov Eq.(2) can be solved by numerical way obtaining the evolutionary response covariance matrix. As stated above, a performance reliability criterion is here adopted to perform the optimum design of a TMD device in the protection of a general multi-storey building subject to a filtered nonstationary base acceleration input process. The structural performance required concerns structure reliability associated to the maximum lateral building displacement. The possibility to satisfy a required limitation of maximum lateral displacement has been investigated with a TMD device placed at the top storey of the building whose cost has to be limited by minimising its mass.

The optimum design is aimed to define TMD mechanical characteristics such as the frequency ω_T and the damping ratio ξ_T, which are collected in the design vector $\boldsymbol{b}=[\omega_T, \xi_T]^T$.

Indicating with γ_m the mass ratio is defined as the TMD mass with respect to the total building one

$$\gamma_m=m_{TMD}/\sum_{i=1}^{n_f}m_i,\quad (19)$$

where n_f is the total floors number and m_i is the storey of each mass.

Possible strategy that could be adopted for the structural optimization of TMD mechanical parameters is the minimization of γ_m and of the system failure probability, here related to the crossing of the top storey lateral displacement over a fixed allowable value. In this case, indicating with $P_f(\boldsymbol{b}, x_{adm}, T)$ the structure failure probability at time T (the end of structural vibrations), it is assumed that the conventional structural failure takes place when the building top storey lateral displacement x_n crosses a fixed threshold value x_{adm}. This performance index (or its complementary reliability $r(\boldsymbol{b}, x_{adm}, T)=1-P_f$), with respect to the first exceeding of a threshold value x_{adm} must be evaluated. Giving the assumption that $r(\boldsymbol{b}, x_{adm}, 0)=1$ at the beginning of the seismic action, the approximate Poisson formulation of the exact Rice (1944; 1945)'s formula for a symmetric barrier gives

$$P_f(\boldsymbol{b},x_{adm},T)=1-e^{-2\int_0^T\nu^+(\boldsymbol{b},x_{adm},\tau)d\tau},\quad (20)$$

where the threshold crossing rate $\nu^+(\boldsymbol{b}, x_{adm}, T)$ is, assuming that the above stochastic processes are Gaussian with null mean values,

$$\nu_S^+(\boldsymbol{b},x_{adm},t)=\frac{1}{2\pi}a^{(1)}(\boldsymbol{b},t)a^{(2)}(\boldsymbol{b},t)a^{(3)}(\boldsymbol{b},x_{adm},t)\cdot\chi\left[d_S(\boldsymbol{b},x_{adm},t)\right],\quad (21)$$

where

$$a^{(1)}(\boldsymbol{b},t)=\sigma_{\dot{X}_n}(\boldsymbol{b},t)/\sigma_{X_n}(\boldsymbol{b},t),\quad (22)$$

$$a^{(2)}(\boldsymbol{b},t)=\sqrt{1-\rho_{X_n\dot{X}_n}^2(\boldsymbol{b},t)},\quad (23)$$

$$a^{(3)}(\boldsymbol{b},x_{adm},t)=\exp\left\{-\frac{1}{2}\left(\frac{x_{adm}}{\sigma_{X_n}(\boldsymbol{b},t)}\right)^2\right\},\quad (24)$$

$$\chi[d_S(\boldsymbol{b},x_{adm},t)]=\exp(-d_X(\boldsymbol{b},x_{adm},t)^2/2)+d_X(\boldsymbol{b},x_{adm},t)\sqrt{2\pi}[1-\Phi\{d_X(\boldsymbol{b},x_{adm},t)\}],\quad (25)$$

$$d_X(\boldsymbol{b},x_{adm},t)=\frac{x_{adm}}{\sigma_{X_n}(\boldsymbol{b},t)}\frac{\rho_{X_n\dot{X}_n}(\boldsymbol{b},t)}{\sqrt{1-\rho_{X_n\dot{X}_n}^2(\boldsymbol{b},t)}}.\quad (26)$$

$\sigma_{X_n}(\boldsymbol{b},t)$ and $\sigma_{\dot{X}_n}(\boldsymbol{b},t)$ are the standard deviations of $\boldsymbol{X}_n(\boldsymbol{b},t)$ and $\dot{\boldsymbol{X}}_n(\boldsymbol{b},t)$ and $\rho_{X_n\dot{X}_n}(\boldsymbol{b},t)$ is their correlation factor.

The MOO problem is hence defined by collecting in an OF vector both the deterministic cost index and the reliability measure, so that the multi-objective optimal criteria could be stated as

$$\text{find } \boldsymbol{b} \in \Omega_b, \tag{27}$$

$$\text{which minimizes } \overline{OF}(\boldsymbol{b},T) = \{\gamma_m, P_f(\boldsymbol{b},T)\}, \tag{28}$$

yielding a numerical problem that can be solved with the abovementioned methods. Due to the relatively regular solution of the problem, the standard weight method has been here adopted in the Pareto optimal set definition.

NUMERICAL RESULTS

Numerical results of TMD optimum design are shown in this section. Mechanical characteristics regarding the storey masses and lateral stiffness of the main structure with three DOF are reported in Table 1. The damping matrix is assumed as a proportional one. System natural periods are respectively 0.47, 0.18 and 0.14 s.

Table 1 Mechanical characteristics regarding the storey masses and lateral stiffness of the main structure with three DOF

	Stiffness ($\times 10^6$ N/m)	Mass ($\times 10^3$ kg)
First floor	5	6
Second floor	4	6
Third floor	3	4.2

The optimum design of TMD is aimed to evaluate the design vector $\boldsymbol{b}$, $\boldsymbol{b}=[\omega_T, \xi_T]^T$, that simultaneously minimizes the performances expressed by Eq.(28). A Pareto optimum front is obtained by solving the original problem.

The plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the Pareto front. Fig.2 shows the Pareto fronts. The minimized mass ratio γ_m is plotted on the x-axis and corresponds by Pareto standard to the minimum of failure probability P_f plotted on the y-axis. The other optimum TMD parameters, ω_T and ξ_T, are respectively plotted in Figs.3a and 3b. Different lines in Figs.2~3 correspond to various admissible structural roof displacements x_{adm}, which correspond to 8 and 9 cm. Results obtained can be therefore adopted to develop a performance-reliability based optimum design of TMD. More precisely, for a given limit value of $\tilde{P}_f$ the optimum solution $\boldsymbol{b}^{opt}$ is obtained, which minimizes γ_m and satisfies the required performance (for example the horizontal line for $\tilde{P}_f$ is 10^{-2} which corresponds to r_{min}=0.99). It can be observed that a higher performance level in terms of reliability requires a higher mass ratio. Moreover, it is possible to deduce that for some values of maximum roof displacement, the optimality cannot be reached by using a TMD. This means that a solution does not always exist. On the other hand, in some cases, the required performance is attained without using the TMD vibrations control strategy. It is obvious that different results depend on the particular values of $\tilde{P}_f$ and x_{adm}. Therefore, the proposed method represents a useful support for designer decisions, offering a complete and clear scenario of all possible solutions regarding both limit displacements and required reliability.

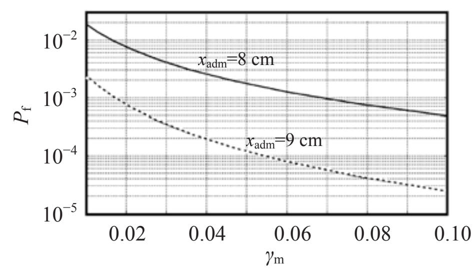

Fig.2 Pareto fronts for different admissible displacements

The optimum design vector elements values are reported in Figs.3a and 3b. In Fig.3a the optimum TMD frequency ratio is reported on the y-axis, i.e., the ratio of the optimum TMD frequency ω_{TMD}^{opt} with respect to the structural frequency ω_s. The x-axis gives the optimum mass ratio γ_m. In Fig.3b, the optimum TMD damping ratio ξ_{TMD}^{opt} is plotted on the y-axis, whereas the x-axis furnishes the optimum mass ratio γ_m.

In Figs.3a and 3b, it can be deduced that all optimum solutions depend only on the mass ratio but not on the admissible displacement x_{adm}. This result is quite reasonable given that the optimum solution is essentially related to the mass ratio, and in any case tends to find the couple of optimal TMD mechanical parameters capable of maximizing the vibrations reduction. On the contrary, the failure level for a given mass ratio depends passively only on the admissible displacement, so that the optimal solution is not directly related to failure probability.

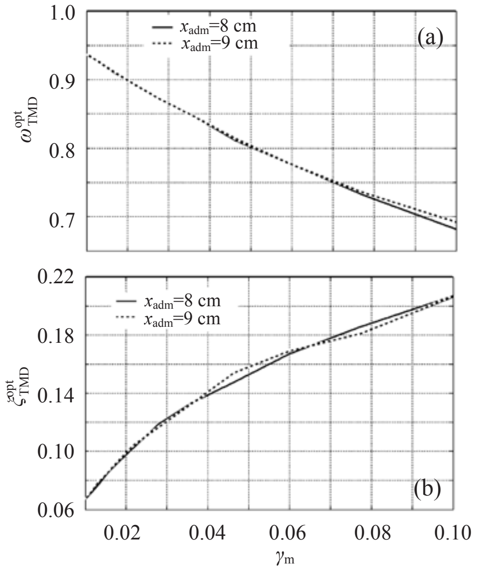

Fig.3 Optimum TMD frequency ratio (a) and optimum TMD damping ratio (b) for different admissible displacements

Moreover, by observing these figures, it is possible to notice that when γ_m grows, two different trends can be noticed for the DV elements. Firstly, the optimum TMD frequency ratio (i.e., the ratio of the optimum TMD frequency ω_{TMD}^{opt} with respect to the structural frequency ω_s) decreases. It starts from a value quite close to 0.95 (TMD is nearly in resonance with the main structure) and reaches a value of about 0.70. On the contrary, the optimum TMD damping ratio ξ_{TMD}^{opt} grows as mass ratio increases, from 0.07 up to about 0.21 (for γ_m=0.10), following an approximately parabolic law.

CONCLUSION

This study has focused on a performance reliability-based optimum design of linear elastic multi-DOF structures subject to random loads. Unlike traditional design methods based on the minimization of system mean-square response, a reliability-based performance index is considered in the proposed design to be a more useful and efficient support in making engineering decision. This approach has been adopted for defining an MOO criterion based also on system performance reliability.

As a case study, the optimum design of mechanical characteristics of TMD has been analysed. The criterion selected for the optimum design is based on the minimization of the mass of the vibrations control device and on a performance reliability associated to the system displacement crossing beyond a given allowable displacement.

The original 1D optimum design has been transformed into a multi-dimensional criterion. The Pareto fronts have then been obtained. The sensitivity analyses carried out by varying the admissible displacement have shown that the optimum solution does not always exist, and that in some cases the required performance is obtained without using the TMD strategy. The results obtained can be used as a suitable decision making support for designers in evaluating the efficiency of TMD systems to obtain assigned required performances in vibrations control.

ACKNOWLEDGEMENT

The author would like to thank the Provincia di Taranto, Engineering Faculty of Taranto, Technical University of Bari for the financial supporting of this research.

References

Coello, C.A., 1999. A comprehensive survey of evolutionary-based multiobjective optimization techniques. *Knowledge and Information Systems*, **1**:269-308.

Constantinou, M.C., Tadjbakhsh, I.G., 1983a. Optimum design of a first story damping system. *Computer and Structures*, **17**(2):305-310. [doi:10.1016/0045-7949(83)90019-6]

Constantinou, M.C., Tadjbakhsh, I.G., 1983b. Probabilistic optimum base isolation of structures. *J. Struct. Engrg. Div., ACSE*, **109**:676-689.

Constantinou, M.C., Tadjbakhsh, I.G., 1985. Optimum characteristics of isolated structures. *ASCE J. Struc. Engrg.*, **111**(12):2733-2750.

Crandall, S.H., Mark, W.D., 1963. Random Vibration in Mechanical Systems. Academic Press.

Gasser, M., Schueller, G.I., 1997. Reliability-based optimization of structural systems. *Mathematical Methods of Operations Research*, **46**(3):287-308. [doi:10.1007/BF01194858]

Jennings, P.C., 1964. Periodic response of a general yielding structure. *J. Engrg. Mech. Div., ASCE*, **90**(EM2):131-166.

Kuschel, N., Rackwitz, R., 2000. Optimal design under time-variant reliability constraints. *Struct. Safety*, **22**(2): 113-127. [doi:10.1016/S0167-4730(99)00043-0]

Marano, G.C., Trentadue, F., Greco, R., 2006. Optimum design criteria for elastic structures subject to random dynamic loads. *Engineering Optimization*, **38**(7):853-871.

[doi:10.1080/03052150600913028]

Marano, G.C., Trentadue, F., Greco, R., 2007a. Stochastic optimum design criterion for linear damper devices for building seismic protection. *Structural and Multidisciplinary Optimization*, **33**(6):441-455. [doi:10.1007/s00158-006-0023-0]

Marano, G.C., Greco, R., Trentadue, F., Chiaia, B., 2007b. Constrained reliability-based optimization of linear tuned mass dampers for seismic control. *International Journal of Solids and Structures*, **44**(22-23):7370-7388. [doi:10.1016/j.ijsolstr.2007.04.012]

Nigam, N.C., 1972. Structural optimization in random vibration environment. *AIAA J.*, **10**(4):551-553.

Papadimitriou, C., Ntotsios, E., 2005. Robust Reliability-based Optimization in Structural Dynamics Using Evolutionary Algorithms. Proc. 6th International Conference on Structural Dynamics, Paris, p.735-739.

Papadimitriou, C., Katafygiotis, L.S., Siu, K.A., 1997. Effects of structural uncertainties on TMD design: A reliability-based approach. *Journal of Structural Control*, **4**(1):65-88. [doi:10.1002/stc.4300040108]

Park, K.S., Koh, H.M., Hahm, D., 2004. Integrated optimum design of viscoelastically damped structural systems. *Engineering Structures*, **26**(5):581-591. [doi:10.1016/j.engstruct.2003.12.004]

Pedersen, C., Thoft-Christensen, P., 1995. Interactive Structural Optimization with Quasi-Newton-algorithms. Proceedings of the 6th IFIP WG 7.5 Conference, Assisi, Italy. Chapman & Hall, London, p.225-232.

Polak, E., Kirjner-Neto, C., der Kiureghian, A., 1997. Structural Optimization with Reliability Constraints. Proceedings of the 7th IFIP WG 7.5 Conference Boulder '96. Pergamon, New York, p.289-296.

Rackwitz, R., Augusti, G., Borri, A. (Eds.), 1995. Reliability and Optimization of Structural Systems, Proc. IFIP WG 7.5 Working Conference, Assisi, Italy. Chapman & Hall, London.

Rice, S.O., 1944. Mathematical analysis of random noise. *Bell System Technical Journal*, **23**:282-332.

Rice, S.O., 1945. Mathematical analysis of random noise—Conclusion. *Bell System Technical Journal*, **24**:46-156.

Rosenblueth, E., Mendoza, E., 1971. Reliability optimization in isostatic structures. *J. Engrg. Mech. Div., ASCE*, **97**(EM6):1625-1642.

Soong, T.T., Grigoriu, M., 1992. Random Vibration of Mechanical and Structural Systems. Prentice-Hall, Upper Saddle River, NJ.

Tajimi, H., 1960. A Statistical Method of Determining the Maximum Response of a Building During Earthquake. Proc. of 2nd World Conf. on Earthquake Engineering, Tokyo, Japan.

Wirsching, P.H., Campbell, G.W., 1974. Minimal structure response under random excitation using vibration absorber. *Earthquake Engineering & Structural Dynamics*, **2**(4):303-312. [doi:10.1002/eqe.4290020402]