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Study on determination of the stable slope configuration for deep open pit mine

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Abstract: The space effects of deep pit slope are analyzed by an elastic mechanics principle. The interaction among the critical slide angle, the friction coefficient, the cohesion, and the horizontal radius of the deep pits is derived in this paper. It indicates that the deeper the pit is excavated, the greater the critical slide angle is. Both the theory for reducing stripping waste rock in deep pit and the approach to determining the configuration of the stable slope are developed from the interaction. The theory in this paper comprises the preceding principles of stability analysis of slopes and is suitable for analyzing that of deep pit.

Key words: Elastic mechanics, Deep pit, Slope stability, Critical slide angle, Ratio of stripping-to-ore
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INTRODUCTION

Excavation of open pits is the greatest man-made project in the world (Stewart and Seegmiller, 1971), and gets deeper and deeper (Moffitt *et al.*, 1971), i.e., turns an open pit into a deep pit (Rassam and Williams, 1999). Because of deep excavation, the great ratio of stripping-to-ore has made a lot of mines difficult (Biswas and Peng, 1999). Hence searching for the methods to reduce stripping waste rocks in deep pits is more and more urgent (Zhang, 1998; Bye and Bell, 2001).

Because the horizontal curvature radius of deep pit turns smaller with the deepening of excavation, the rock arch with a smaller curvature radius has better stability (Zhu and Zhang, 2002; Zhang *et al.*, 2002; Cai *et al.*, 2007), and the great deep pit's bottom can transfer and bear the virgin horizontal stresses of the lower slope, which is favorable for the stability of the latter (Zhu, 2003; Zhu and Zhang, 2003a). However, the preceding analysis used for stability of deep pit's slope is based on the principle of plain strain and regards the slope of deep pit as a straight line. So the convex slope of deep pit not only can reduce waste excavations gradually, but also agrees with nature (Zhu and Zhang, 2003b; 2004). This paper focuses on the law of changing of the stability for the deep pit's slope as the development of excavation by the elastic mechanics and the theory in determining the configuration of stable slope for deep pits.

BASIC SUPPOSITION

The configuration of deep pit's slope is complicate, which is composed of serial steps in general. For convenience of mechanical analysis, the configuration of deep pit's slope is looked as a convex slope, so that it is shown as a circle or a oval-shaped circle on flat surface and the rock mass is homogeneous, continuous and elastic that is only loaded by its dead weight.

MECHANICAL ANALYSIS

Take the circle rock on the slope of a deep pit as an example, as shown in Fig.1. It is supposed that average radius of the circle rock approximates to the horizontal radius of the deep pit and the slope angle of positioned circle rock is α. While the circle rock moves down to an increment ΔZ, the changing quantity of the average radius r is

$$\Delta r=\Delta Z\cot\alpha. \quad (1)$$

The strain of the rock circle in circle direction is

$$\varepsilon_\theta=\cot\alpha\Delta Z/r, \quad (2)$$

then the elastic stress can be expressed as

$$\sigma_\theta=E_\theta\cot\alpha\Delta Z/r, \quad (3)$$

where E_θ is the elastic model of the rock circle in the circle direction.

When the segment ΔV of the rock circle is in critical slide state, the forces acting on it are shown in Fig.2. ΔS is the area of the rock circle's cross-section, thus ΔV is

$$\Delta V=\Delta Sr\mathrm{d}\theta. \quad (4)$$

If ΔV is in balance state in slide direction, the following equation can be established

$$\begin{aligned}&\Delta V\gamma\sin\alpha-\Delta S\sigma_\theta\mathrm{d}\theta\cos\alpha\\&\quad=(\Delta V\gamma\cos\alpha+\Delta S\sigma_\theta\mathrm{d}\theta\sin\alpha)f,\end{aligned} \quad (5)$$

where γ is the rock unit weight and f is the friction coefficient. Substituting Eqs.(3) and (4) into Eq.(5), yields

$$\begin{aligned}&\gamma\sin\alpha-E_\theta\cot\alpha\cos\alpha\Delta Z/r^2\\&\quad=(\gamma\cos\alpha+E_\theta\cot\alpha\sin\alpha\Delta Z/r^2)f.\end{aligned} \quad (6)$$

When the rock circle gets critical yielding, i.e., $\sigma_\theta=\sigma_{\theta\max}$, take the principal stresses as

$$\begin{cases}\sigma_1=\sigma_{\theta\max},\\ \sigma_3=0.\end{cases} \quad (7)$$

Then, according to Coulomb's shear strength criterion, the following equation can be derived as

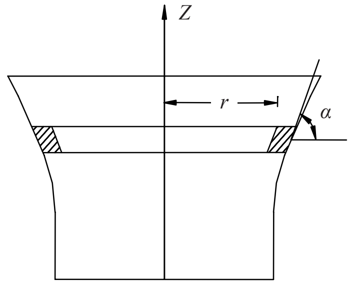

Fig.1 Diagram of slope deep pit

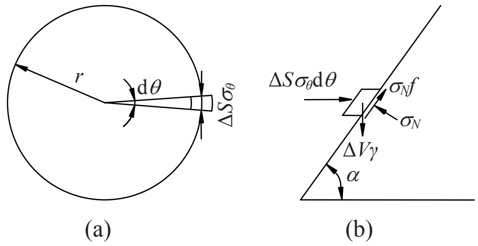

Fig.2 Forces on differential element ΔV of a circle. (a) Forces in horizontal plane; (b) Forces in vertical plane

$$\sigma_{\theta\max}=\frac{2c\cos\varphi}{1-\sin\varphi}, \quad (8)$$

where c and φ are cohesion and angle of internal friction, respectively.

Substituting Eq.(3) into Eq.(8) and rearranging Eq.(8) gives $\Delta Z_{\max}$ as

$$\Delta Z_{\max}=\frac{2rc\cos\varphi}{E_\theta\cot\alpha(1-\sin\varphi)}. \quad (9)$$

Substituting Eq.(9) into Eq.(6), $\tan\alpha$ is developed as

$$\tan\alpha=f+\frac{2c(1+f^2)\cos\varphi}{\gamma r(1-\sin\varphi)-2fc\cos\varphi}. \quad (10)$$

When $f=\tan\varphi$, Eq.(10) is changed to

$$\tan\alpha=\tan\varphi+\frac{2c}{\gamma r(1-\sin\varphi)\cos\varphi-2c\sin\varphi\cos\varphi}, \quad (11)$$

which is the basic equation derived in this paper.

From Eq.(11), we can conclude as follows:

(1) When the physical parameters of the rock mass are constant, the shorter the horizontal radius of the deep pit's slope is, or the deeper the deep pit is excavated, the greater the stable slope angle α is. If the horizontal radius approaches infinite, the stable slope angle approaches the limit φ. This condition is the same as the linear slope.

(2) When the horizontal radius is

$$r=\frac{2c\sin\varphi}{\gamma(1-\sin\varphi)}, \tag{12}$$

the stable slope angle may reach π/2. This condition means that a well with a small diameter under good geologic condition is still stable without any prop upon the wall.

(3) The greater the cohesion and the angle of internal friction are, the greater the stable slope's angle is. But the greater the rock unit weight is, the less the stable slope's angle is.

If the coordinate system of the slope of deep pit is taken as that in Fig.3, tanα can be expressed as the differential coefficient of the height of the slope with respect to the horizontal radius, i.e.,

$$\mathrm{d}H/\mathrm{d}r=\tan\alpha. \tag{13}$$

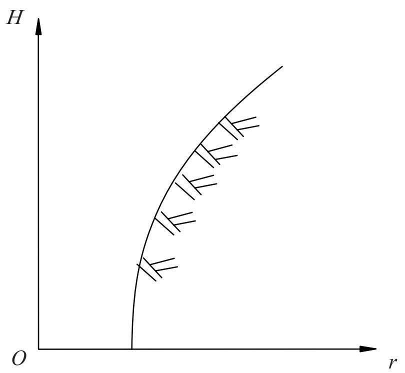

Fig.3 Coordinate system of slope

Then, Eq.(11) can be expressed as

$$\frac{\mathrm{d}H}{\mathrm{d}r}=\tan\varphi+\frac{2c}{\gamma r(1-\sin\varphi)\cos\varphi-c\sin2\varphi}. \tag{14}$$

By integrating Eq.(14), the relationship between the stable slope's height and the horizontal radius is given by

$$H=(r-r_0)\tan\varphi+\frac{2c}{\gamma(1-\sin\varphi)\cos\varphi}\times\ln\frac{\gamma r(1-\sin\varphi)\cos\varphi-2c\sin\varphi\cos\varphi}{\gamma r_0(1-\sin\varphi)\cos\varphi-2c\sin\varphi\cos\varphi}, \tag{15}$$

where c, φ and γ are assumed as constants. Taking γ=27 kN/m^3, f=1.5, c=0.1 MPa, the horizontal radius r ranging from 50 to 800 m, the height of the deep pit's slope can be calculated as shown in Fig.4.

From Fig.4 it can be seen that the ratio of the slope is larger as the horizontal radius r is lesser and when the r approaches infinite, the ratio of the slope approaches the friction coefficient f. With the increase in excavation depth of a deep pit, the horizontal radius of the deep pit's slope decreases gradually, while the stable slope's angle increases. It is found that the slope curve is gently upward and steeply downward, which fits with the real conditions of the rock mass. For the damage extent of the lower excavation rock mass by blasting and weathering it is less with excavation depth increasing (Pariseau *et al.*, 2008), i.e., the cohesion c and the angle of internal friction φ are still higher, which can cause increase in stable slope's angle, too.

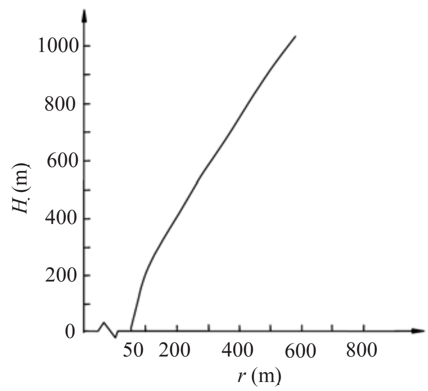

Fig.4 Curve of *H*-*r* for some rock slope

This is the space mechanics principle about increasing stable slope's angle gradually while excavation depth increases. It may establish the theory foundation for selecting the convex slope to reduce stripping waste rock in the open pit mining, especially in the deep pit.

If the deep open pit is an oval-shaped circle on flat surface, when the circle rock which is on the slope of the deep pit slides down to an increment ΔZ, the elastic stress in circle direction can be expressed as

$$\sigma_\theta = E_\theta \cot\alpha \Delta Z / r. \tag{16}$$

It can be seen that if all the slope angles are the same on the same horizontal plane, σ_θ caused by ΔZ is different at different points with different horizontal radii of curvature; the elastic stresses in circle direction at the end of the major and the minor semi-axes are expressed as

$$\sigma_{\theta\mathrm{l}} = E_\theta \cot\alpha \frac{\Delta Z}{b^2} a, \tag{17}$$

$$\sigma_{\theta\mathrm{s}} = E_\theta \cot\alpha \frac{\Delta Z}{a^2} b, \tag{18}$$

where α is the major semi-axis of the ellipse and b is the mirror one as shown in Fig.5. Because $a>b$, it can be found that

$$\sigma_{\theta\mathrm{l}} > \sigma_{\theta\mathrm{s}}. \tag{19}$$

Because the radius of curvature at the end of the major semi-axis is the least and that at the end of the minor semi-axis is the largest, the elastic stress σ_θ satisfies

$$\sigma_{\theta\mathrm{s}} \le \sigma_\theta \le \sigma_{\theta\mathrm{l}}. \tag{20}$$

It is known that the larger the stress σ_θ is, the more stable the slope is. Thus, it can be inferred that the slope at the end of major semi-axis of the ellipse is the most stable, while that at the end of minor semi-axis is the most unstable. The position of the slope's sliding region of Jinchuai open pit in China is a practical example to prove the inference (Xu *et al.*, 1985).

When the segment ΔV of the rock circle is in critical slide state, the forces acting on it are shown in Fig.5.

If the slope angle of an oval-shaped deep pit is changed to satisfy the condition that stress σ_θ at every point on the same horizontal plane is equivalent, i.e.,

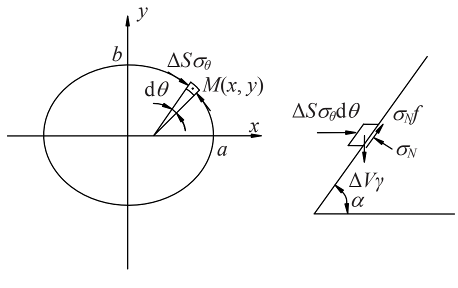

Fig.5 Forces on differential element ΔV of oval-shaped circle. (a) Forces in horizontal plane; (b) Forces in vertical plane

the slope on the same plane is in the same stable state, the slope angle at point $M(x, y)$ can be derived as

$$\tan\alpha = \tan\varphi + 2ca^4 b / \{\gamma[a^4 + x^2(b^2 - a^2)]^{3/2} \cdot (1-\sin\varphi)\cos\varphi - 2ca^4 b\sin\varphi\cos\varphi\}. \tag{21}$$

Then, by integrating $\mathrm{d}H/\mathrm{d}r = \tan\alpha$, the stable slope's height with the horizontal radius is given as

$$H = (r - r_0)\tan\varphi + \frac{2c}{\gamma(1-\sin\varphi)\cos\varphi} \times \ln\frac{\gamma r(1-\sin\varphi)\cos\varphi - c\sin 2\varphi}{\gamma r_0(1-\sin\varphi)\cos\varphi - c\sin 2\varphi}, \tag{22}$$

where c, φ and γ are supposed to be constants, r_0 is the horizontal radius of curvature of the point at the slope's bottom corresponding to the point $M(x, y)$. Having known the slope's height H, taking r_l as the horizontal radius of curvature at the end of the major semi-axis, and r_s as the one at the end of minor semi-axes, from Eq.(22), the horizontal radii of curvature at the end of the major and minor semi-axis of the slope bottom ellipse, i.e., $r_{0\mathrm{l}}$ and $r_{0\mathrm{s}}$, can be obtained. However, according to the traditional theory of slope stability, the relation between the slope's height and the horizontal radius of curvature is

$$H = (r - r_0)\tan\varphi, \tag{23}$$

Taking γ=27 kN/m^3, c=0.2 MPa and φ=45°, the major and minor semi-axes of slope's bottom ellipse as a_0=150 m and b_0=100 m, the slope's height H=500 m. The area of the ellipse at height H is

$$S=ab\pi=r_1r_2\pi. \quad (24)$$

However, the area S^* determined by the traditional theory is

$$S^*=(H+66.6)(H+255)\pi. \quad (25)$$

Thus, if taking $\Delta S=S^*-S$ as the integrand and the slope's height as the variable of integration, the quantity of stripping waste rock reduced by comparing the configuration of limit equilibrium slope determined by the traditional theory of slope stability with that determined in this paper, V can be described as

$$V=\int_0^H \Delta S\mathrm{d}h. \quad (26)$$

According to this condition, the result of calculation is $V=1.11\times10^8$ m^3.

It can be seen that a great quantity of stripped waste rock can be reduced by the slope configuration determined by this theory.

DISCUSSION AND CONCLUSION

To apply the theory of elasticity, the homogeneous, isotropic and elastic behaviors of material are assumed. In reality, the rock mass properties are very complicated. It is necessary to resort to the numerical method for the further study on this question.

However, the work in this paper has proved the feasibility of the convex-form slope theoretically, and the following conclusions can be drawn:

(1) Traditional analysis about stability of slope is based on the principle of plain strain. According to the principle, slope configuration is determined as a straight line in vertical plane, which only adapts to linear slopes or shallow open pit mining.

(2) Suppose the rock mass is homogeneous and elastic that is only loaded by its dead weight and is excavated as a trumpet or an oval-shaped trumpet, the space effect on deep pit's slope is analyzed. This paper derives the interaction of stable slope angle, the rock's physical parameters and the horizontal radius of the deep pit. The interaction shows that the stable slope angle of the rock mass excavated increases gradually with the increase in excavation depth, thus the stability of the lower slope's rock mass increases simultaneously, so that the theory base of convex slope selection is set up to reduce stripping waste rock in deep pit and the preliminary way is given to determine reasonable configuration for other similar slopes (Yue *et al.*, 2004).

(3) This theory containing traditional analysis about stability of linear slope is also good at analyzing other rock slope's stability. A large amount of stripped waste rock can be reduced by the proposed model, as compared with that determined by the traditional theory of slope stability.

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