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Theoretical and experimental studies of heat transfer with moving phase-change interface in freezing and thawing of porous potting soil*

Michael K. H. LEUNG†, Kit-ying CHAN
(*Department of Mechanical Engineering, the University of Hong Kong, Hong Kong, China*)
†E-mail: mkhleung@hku.hk

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Abstract: Soil in a cold region is subject to frequent freezing and thawing cycles. Soil frozen for a prolonged period may cause adverse freeze damage to the plants due to cell dehydration or root cell rupture. It is important to understand the detailed heat transfer behaviors of the freezing and thawing processes to prevent freeze damage, and to devise proper mitigation measures for effective pot planting in cold regions. A theoretical model was developed to analyze the transient moving phase-change interface heat transfer in the freezing and thawing of porous potting soil. The theoretical derivation is based on the assumption that the soil freezes completely at a single temperature. Microscopic poromechanic effects on heat transfer behavior were ignored. The spatial domain of the problem was simplified to a 1D spherical coordinate system with variation in the radial direction. Green's function was applied to solve for the time-dependent body temperature. Experiments were conducted for validation of the theoretical model. Reasonable agreement between the theoretical predictions and experimental measurements was obtained. The theoretical model developed can be easily used to determine the sensitivity of various parameters in the freezing/thawing processes, e.g., thermal properties of soil, ambient temperature, and planting pot size.

Key words: Porous soil, Freezing, Thawing, Moving phase-change interface, Theoretical model
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INTRODUCTION

Soil has a porous matrix and the voids are filled with air and water providing a supply of oxygen and water for plant root growth. When the ambient temperature falls below the freezing point, the soil will be frozen as the liquid water crystallizes to solid phase (Hohmann, 1997; Coussy, 2005). Conversely, frozen soil exposed to temperatures above the melting point will be thawed. During the freezing or thawing process, a water-ice phase-change interface is formed as latent heat is involved. The interface travels until complete phase change or steady state is reached (Kozlowski, 2004).

It is important to understand such transient heat transfer phenomena in the freezing and thawing of soil, especially in cold regions where soil is subject to frequently freezing and thawing cycles. Freeze damage occurs when the frozen soil causes cell dehydration of plant roots, or when frozen plant roots suffer from root cell rupture. The degree of freeze damage is proportional to the duration of the frozen condition. Knowledge of the details of the heat transfer behaviors may enable appropriate measures to be devised for growing plants in cold regions effectively.

Previous studies were conducted to analyze freezing and thawing of soil on the land surface (Hohmann, 1997; Sjursen *et al.*, 2005; Hansson and Lundin, 2006). These studies are highly relevant to agriculture and forestry in cold regions. In the present

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study, the primary concerns were freezing and thawing of potting soil. A theoretical model was developed to predict the heat transfer in the freezing and thawing of soil. It is also applicable to other processes of a similar nature involving a moving phase-change interface in porous media, including frozen food processing (Huan *et al*., 2003; Leung *et al*., 2007), cryopreservation (Wolfe and Bryant, 2001) and phase-change-material energy storage (Mesalhy *et al*., 2005). The details of the theoretical derivation and experiments conducted for model validation are presented in this paper.

THEORETICAL MODEL

In this theoretical analysis, potting soil was considered to be a porous spherical body with water-containing pores and air voids. When the body is exposed to an environment below its freezing point, the water content near the body surface will freeze first and a water-to-ice phase-change interface will be formed. As the body is cooled continuously, the interface will move inwards towards the center until the body is completely frozen. Conversely, when a frozen porous body is exposed to an environment above its melting point, the body will be thawed in a similar manner. The processes are illustrated in Fig.1.

Generally, freezing or thawing of a porous material involves complex microscopic poromechanics, including pore pressurization due to water crystallization and thermomechanical effects between the solid matrix and water (Coussy, 2005; Coussy and Fen-Chong, 2005). In a cooling process, a porous material should freeze progressively over a range of temperature due to capillary effects and thus the latent heat effect is distributed over a partially frozen region (Boukpeti, 2008). However, as potting soil forms a porous medium with large pores, the poromechanic effects on the heat transfer behavior are relatively insignificant and complete freezing occurs at a single freezing point. Therefore, the poromechanic effects are ignored in the present modelling study of freezing and thawing of porous potting soil. This assumption is not valid for soil where capillary effects are significant (e.g., fine soils).

The moving phase-change interface heat transfer problem can be formulated in spherical coordinates.

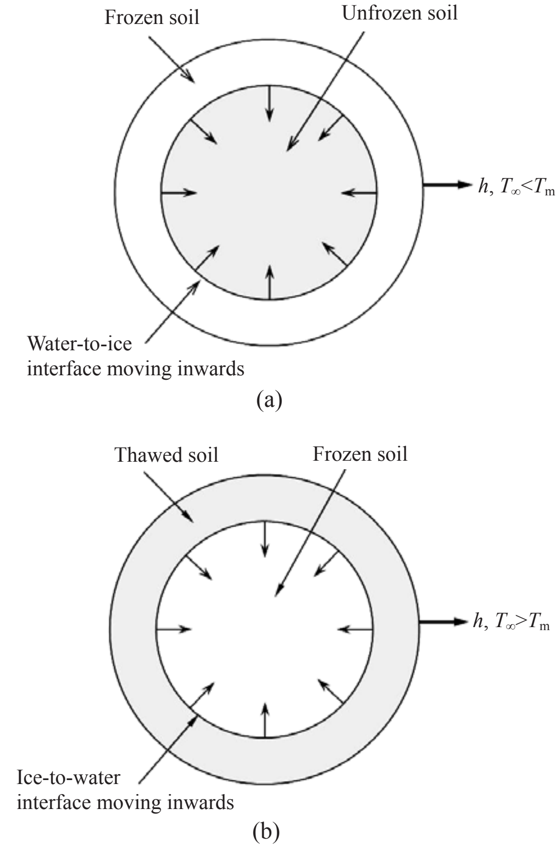

Fig.1 Moving phase-change interface heat transfer of potting soil. (a) Freezing; (b) Thawing.

It is assumed that the body temperature, T, varies only in the radial direction, r, and with time, t. It is also assumed that the body has constant physical and thermal properties. Thus, the heat conduction equation including the latent heat gain (or loss) of the moving interface can be written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T(r,t)}{\partial r}\right)+\frac{\rho L}{k}\delta(r-r_\text{I}(t))\frac{\partial r_\text{I}(t)}{\partial t}=\frac{1}{\alpha}\frac{\partial T(r,t)}{\partial t}, \tag{1}$$

where ρ is density of body; k is thermal conductivity; α is thermal diffusivity; L is the latent heat. The latent heat effect is expressed in terms of the movement of the phase-change interface, $r_\text{I}(t)$, and Dirac delta function, δ. The two boundary conditions, interfacial condition, and initial condition are respectively,

$$\partial T(r,t)/\partial r=0,\ \text{for } r=0, \tag{2}$$
$$k\partial T(r,t)/\partial r+hT(r,t)=0,\ \text{for } r=R_\text{o}, \tag{3}$$
$$T(r,t)=T_\text{m},\ \text{for } r=r_\text{I}(t), \tag{4}$$

$$T(r,t)=T_i, \text{ for } t=0, \tag{5}$$

where R_o is the radius of the spherical body, h is the convective heat transfer coefficient, T_m represents the phase-change temperature (freezing or melting), and T_i represents the initial body temperature. Note that all temperatures are measured from a common reference point equal to the ambient temperature (i.e., $T_\infty=0$).

The theoretical solution to the above partial differential equation problem was previously derived for analysis of thawing of frozen meat (Leung *et al.*, 2005). Using the method of Green's function, the solution can be obtained as a sum of a homogeneous part, $T_h(r,t)$, and a non-homogeneous part, $T_n(r,t)$,

$$T(r,t)=T_h(r,t)+T_n(r,t)=\int_{r'=0}^{R_o} r'^2 G(r,t\,|\,r',\tau)|_{\tau=0}\, T_i \mathrm{d}r' \\ +\frac{\alpha}{k}\int_{\tau=0}^{t}\mathrm{d}\tau\int_{r'=0}^{R_o} r'^2 G(r,t\,|\,r',\tau)\rho L\delta(r'-r_I(\tau))\frac{\mathrm{d}r_I(\tau)}{\mathrm{d}\tau}\mathrm{d}r', \tag{6}$$

where $G(r,t|r',\tau)$ is Green's function. The homogeneous part is derived from a model without moving interface and the solution can be written as

$$T_h(r,t)=\frac{2T_i}{r}\sum_{m=1}^{\infty}\left[\mathrm{e}^{-\alpha\beta_m^2 t}\sin(\beta_m r)\frac{\beta_m^2+K^2}{R_o(\beta_m^2+K^2)+K} \cdot\frac{\sin(\beta_m R_o)-\beta_m R_o\cos(\beta_m R_o)}{\beta_m^2}\right], \tag{7}$$

where the eigenvalues β_m are the positive roots of the eigenfunction,

$$\beta_m R_o\cot(\beta_m R_o)+R_o K=0, \tag{8}$$

and the coefficient K is

$$K=h/k-1/R_o. \tag{9}$$

Comparing Eqs.(6) and (9), yields the corresponding non-homogenous solution

$$T_n(r,t)=\frac{2\alpha\rho L}{k}\sum_{m=1}^{\infty}\left[\frac{\beta_m^2+K^2}{R_o(\beta_m^2+K^2)+K}\frac{\sin(\beta_m r)}{r} \cdot\int_{\tau=0}^{t}\mathrm{e}^{-\alpha\beta_m^2(t-\tau)}\sin(\beta_m r_I(\tau))r_I(\tau)\frac{\mathrm{d}r_I(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau\right]. \tag{10}$$

The moving phase-change interface is determined by solving the interfacial condition,

$$T(r_I(t),t)=T_h(r_I(t),t)+T_n(r_I(t),t)=T_m. \tag{11}$$

The one-step Newton-Raphson method can be used to effectively obtain the numerical values of $r_I(t)$ for the whole period from the formation of the phase-change interface to the complete phase change of the body. The theoretical solution expressed by Eqs.(6)~(11) are applicable to both freezing and thawing processes of a porous spherical body.

EXPERIMENTS

Experiments on freezing and thawing of potting soil were conducted for validation of the theoretical solution. The test setup is illustrated in Fig.2. Moist soil was carefully packed layer by layer into a 100 mm-diameter spherical shell with a uniform density. The sphere was made of thin copper sheet for its good thermal conductance. The spherical shell was sealed to maintain the water content in the soil specimen. The two potting soil samples tested in this study had low density (550 and 660 kg/m^3, compared with 1000 kg/m^3 for water). The low soil density indicated that the porous media had many air voids.

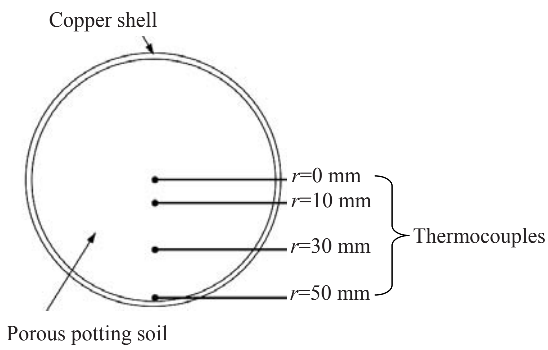

Fig.2 Experimental setup for freezing and thawing tests of potting soil

In the freezing test, the soil sample originally at a uniform room temperature was suddenly placed in a freezer. The temporal body temperature changes were measured by four K-type thermocouples fixed at different radial coordinates. The experiment was stopped after the sample was completely frozen and had reached a state of thermal equilibrium.

The thawing test was conducted in a similar manner. At the beginning, the soil sample was placed in a freezer long enough to be frozen at a uniform temperature. Suddenly, the sample was moved to a place at a temperature above the melting point. The experiment was stopped after the sample was completely thawed and had reached a state of thermal equilibrium.

Experiments were also carried out to measure the thermal properties of the potting soil. The specific heat capacity, c, and latent heat, L, of the moist soil were measured by means of calorimetry. The thermal conductivity, k, was obtained by matching the calculated temperature from the homogeneous solution (Eq.(7)) with the measured temperature from the freezing and thawing tests before the formation of phase-change interface.

RESULTS AND DISCUSSION

Experimental results

To avoid confusion with different temperature reference points, all the temperatures mentioned from now on in degree Celsius (°C) are measured from the absolute temperature of 273 K. The conditions of the four different tests conducted and the measured properties of the soil samples are summarized in Table 1. The convective heat transfer coefficient, h, was determined by the Nusselt number for natural convection and the value of h was found to be 23 W/(m^2·K).

From the experimental results of the freezing test (Test 1, Fig.3a), the temperature of the surface (r=50 mm) first dropped from the initial temperature of 25 °C to the freezing point of 0 °C in about 990 s. At this time, the cooling process began to remove latent heat to cause freezing of the soil. The water-to-ice phase-change interface was first formed on the surface of the potting soil and progressively moved inwards. At the center (r=0 mm), the temperature dropped to the freezing point in about 6000 s. The temperature remained unchanged and the water content remained in the liquid phase for an extended period of time. At t=9400 s, the water-to-ice phase-change interface traveled to the center and thus, the spherical body of potting soil was completely frozen. Subsequently, the center temperature dropped below the freezing point. Eventually, at about t=13 000 s, the whole body reached thermal equilibrium at a temperature equal to the surrounding temperature of −17 °C. The temperatures measured at r=10 mm and r=30 mm, not shown in Fig.3a, fell proportionately between the two measurements at r=0 mm and r=50 mm. Opposite phenomena were observed for the thawing test (Test 2, Fig.3b). The freezing and thawing tests were repeated with soil packed to higher density and the similar results were obtained (Figs.4a and 4b, respectively).

No obvious expansion and contraction of potting soil were observed during the freezing and thawing processes.

Experimental validation of the theoretical model

Corresponding theoretical simulation results are plotted in Figs.3~4 for comparison. In general, the theoretical results agreed reasonably well with the experimental measurements. The surface temperature determined by the theoretical model agreed very well with the experimental measurements (Fig.3a). At the center, the theoretical prediction of the time taken for complete freezing of the soil (i.e., until the center dropped below melting point) also agreed very well

Table 1 Parameters involved in both experiments and theoretical models

	Test condition			Measured property of moist soil					
	T_i	T_∞	h	ρ	k	c	α	L	T_m
Test 1 (freezing)	25	−17	23	550	0.28	2.394	0.21	144.8	0
Test 2 (thawing)	−18	23	23	550	0.50	2.394	0.38	−144.8	0
Test 3 (freezing)	19	−16	23	660	0.45	2.394	0.28	144.8	0
Test 4 (thawing)	−17	22	23	660	1.00	2.394	0.63	−144.8	0

Initial body temperature, T_i (°C); surrounding temperature, T_∞ (°C); convective heat transfer coefficient, h (W/(m^2·K)); density, ρ (kg/m^3); thermal conductivity, k (W/(m·K)); specific heat, c (kJ/(kg·K)); thermal diffusivity, α (mm^2/s); latent heat, L (kJ/kg); phase-change temperature, T_m (°C) with the measurements. The theoretical model overpredicted the cooling rate at the beginning of the freezing process. The similar results were found for freezing of a soil sample of higher density (Fig.4a). The thawing test results are plotted in Figs.3b and 4b. The model underpredicted the thawing rate and overpredicted the heat gain at the center (r=0 mm) after the sample was completely thawed. These discrepancies between the theoretical and experimental results could be explained by the assumptions made in the theoretical model that all the thermal properties of soil were constant throughout the phase-change heat transfer process. In fact, moist soil would have lower specific heat capacity and higher thermal conductivity in its frozen state. The discrepancies could also be caused by some experimental variation in the porosity of the soil medium as the soil was packed in the spherical shell manually. Despite the discrepancies, the present theoretical model performed well in predicting the heat transfer in freezing and thawing cycles of potting soil and thus the model was experimentally validated.

Effect of soil density

Comparisons between Tests 1 and 3 and between Tests 2 and 4 revealed that as the porous soil was packed to a higher density, the heat transfer rate increased, resulting in a shorter time period for complete phase change in freezing or thawing of potting soil. The phenomenon could be explained that when the porous soil density increased, the porosity of the medium decreased. The decreases in both number and size of the tiny voids caused less filling of thermal-insulating air. As a result, the thermal conductivity of the porous soil became higher and so was the heat transfer rate.

Practical use of theoretical solution

Using the theoretical solution, both temporal and spatial temperature variations of porous potting soil under various freezing/thawing conditions can be easily predicted. One may apply these findings to quantify the phase-change heat transfer rates and accordingly design effective measures to prevent freeze damage of potting plants in cold region. For

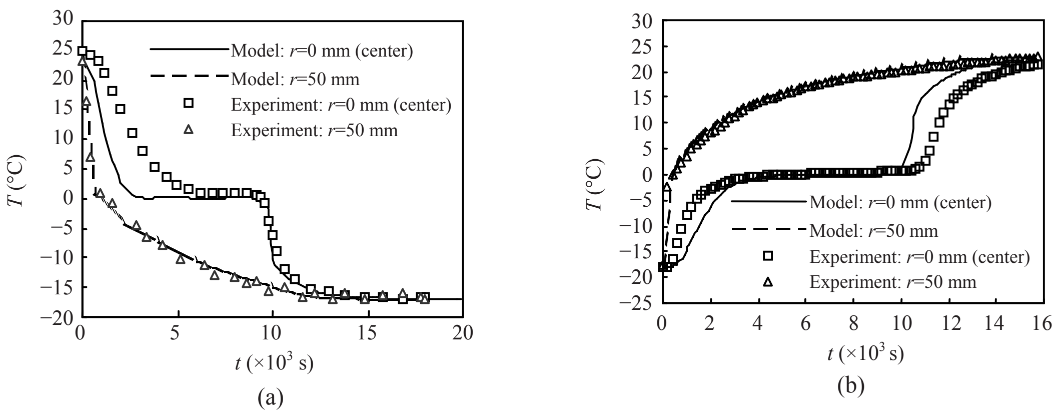

Fig.3 Experimental results of (a) freezing test and (b) thawing test of potting soil (ρ=550 kg/m^3)

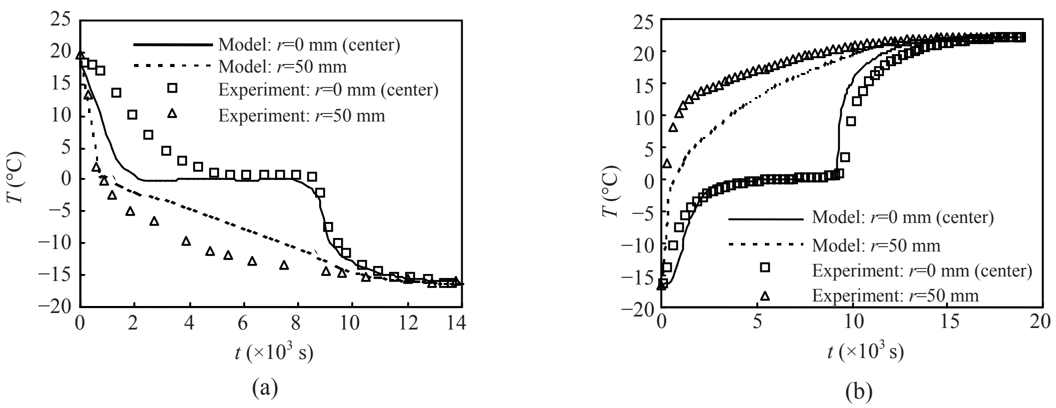

Fig.4 Experimental results of (a) freezing test and (b) thawing test of potting soil (ρ=660 kg/m^3)

instance, the following features can be characterized quantitatively: (1) Amount of frozen potting soil formed in periodic freezing/thawing cycles; (2) Extended time taken for complete freezing due to use of bigger pot; (3) Extended time taken for complete freezing due to use of loosely packed potting soil to reduce thermal conductivity; (4) Maximum time of exposure to a low-temperature environment without freeze damage.

It is noted that the presence of plant root in the potting soil is omitted in the modeling analysis. Plant root normally has a higher water content than the potting soil and thus, the local latent heat will be slightly higher, resulting in longer freezing and thawing time periods. Nevertheless, the plant root normally has a small mass fraction in potting soil. Therefore, the expected effect of the presence of root is insignificant.

General soil freezing and freeze damage of plant roots can be studied by a coupled thermo-hydro-mechanical model (Lai *et al.*, 1998; Neaupane *et al.*, 1999; Boukpeti, 2008). The theoretical solution presented here can perform as the heat transfer part of the coupled thermo-hydro-mechanical model. The theoretical solution could be coupled with a fluid transfer and a poromechanical model to form a complete model to assess freeze damage and its prevention.

CONCLUSION

A theoretical solution was derived for prediction of the heat transfer in freezing and thawing of porous potting soil. Assumptions, such as constant physical and thermal properties and negligible microscopic poromechanics effects, were made to facilitate the derivation. These assumptions were justified by the reasonable agreement between the theoretically predicted temperatures and the experimental measurements. Despite some discrepancies found in the theoretical predictions, the important time periods for complete freezing and complete thawing of porous potting soil consistently agreed very well with the measurements. Besides freezing and thawing of porous potting soil, the theoretical model developed is applicable to study other similar processes, such as frozen food processing, cryopreservation, and phase-change-material energy storage.

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