Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Magnetoelectroelastic fields in rotating multiferroic composite cylindrical structures*

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Received Apr. 8, 2008; Revision accepted Aug. 10, 2008; Crosschecked Jan. 16, 2009

Abstract: An analytical solution is obtained for a rotating multiferroic composite hollow cylinder made of radially polarized piezoelectric and piezomagnetic materials. Both the number of layers and the stacking sequence of the composite cylinder can be arbitrary. General mechanical, electric and magnetic boundary conditions can be applied at both the inner and outer cylindrical surfaces. The state space method is employed so that only a 2×2 matrix is involved in the whole solving procedure. In the numerical experiments, the distributions of elastic, electric as well as magnetic fields in an internally pressurized rotating $BaTiO_3/CoFe_2O_4$ composite hollow cylinder subjected to different boundary conditions are presented graphically. The results clearly show that the stress fields in a multiferroic composite cylinder are controllable.

Key words: Analytical solution, Multiferroic composite, Rotating hollow cylinder, State space method
doi:10.1631/jzus.A0820517 **Document code:** A **CLC number:** O343.1

INTRODUCTION

The investigation of prediction, control and optimization of stress fields in rotating components has long been an important topic because of the broad applications of these basic structures in mechanical, electric and civil engineering, etc. It is well known that the centrifugal force increases remarkably in the rotating components with the increase in rotation angular velocity. Heavy deformation and excessive stress will be the key failure factors for high speed rotating components.

There have been some classic works devoted to rotating isotropic, orthotropic and anisotropic cylinders and discs made of elastic media (Timoshenko and Goodier, 1970; Chang, 1974; Genta and Gola, 1981). The investigations for rotating functionally graded cylinders and discs have also been reported (Durodola, 2000; Eraslan and Akis, 2006; You *et al.*, 2007; Zenkour, 2007). Stress analyses of rotating cylinders and discs in a thermal environment were further studied (El-Naggar *et al.*, 2002; Callioglu, 2004).

With the development of science and technology, many new materials have been fabricated and synthesized. Due to their advantages in mechanical, electric and magnetic behavior, the new materials have been widely applied in practical engineering. Consequently, theoretical research into the structures made of the new materials is of increasing interest to scientists and engineers. Due to the coupling effect between the mechanical and electric fields, piezoelectric materials have now been widely used in various devices and systems. On the topic of the rotation problem, Galic and Horgan (2003) studied the stress response of radially polarized rotating homogeneous piezoelectric cylinders. Wang and Ding (2007) analyzed the stress and electric fields in a

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* Project supported by the National Natural Science Foundation of China (Nos. 10872179 and 10725210), the Zhejiang Provincial Natural Science Foundation of China (No. Y7080298), and the Zijin Plan of Zhejiang University, China

rotating infinite hollow multilayered piezoelectric cylinder. Babaei and Chen (2008) presented the exact solutions for radially polarized rotating homogeneous magnetoelectroelastic cylinders.

More recent success was obtained with devices and systems where piezoelectric and piezomagnetic materials are involved (Spaldin and Fiebig, 2005). The multiferroic composites made of piezoelectric and piezomagnetic materials can exhibit a magneto-electro-elastic effect and have the ability to convert energy from one to the other among magnetic, electric and mechanical energies. Much effort has been devoted to the exact analysis of multiferroic composite structures with various structural forms in recent years. The contents include static analysis and free vibrations (Pan, 2001; Pan and Heyliger, 2003; Wang and Zhong, 2003; Chen *et al.*, 2005; Ramirez *et al*., 2006).

This investigation aims to derive the analytical solution for a rotating multiferroic composite hollow cylinder. To the best of our knowledge, such work is still unavailable in the literature. Compared with piezoelectric materials, the research into multiferroic materials is more complex due to the fully coupled magneto-electric-elastic effect. If the material properties of each layer are specified by those for magneto-electric-elastic or piezoelectric layers only, the present solution can directly degenerate into the solutions that have been considered before (Galic and Horgan, 2003; Wang and Ding, 2007; Babaei and Chen, 2008).

ANALYTICAL MODEL AND MATHEMATICAL EQUATIONS

Consider an infinite composite hollow cylinder of inside radius a and outside radius b rotating at a constant angular velocity ω about its central axis. The interior and exterior radii of the ith layer are denoted as r_{i-1} and r_i (i=1, 2, …, n), respectively. In particular, we have $r_0=a$ and $r_n=b$. At the internal and external surfaces, the hollow cylinder is subjected to the radial stresses P_a and P_b, electric potentials Φ_a and Φ_b as well as magnetic potentials Ψ_a and Ψ_b, respectively (Fig.1).

For magneto-electro-elastic media, the constitutive relations are described as (Pan 2001; Buchanan, 2003)

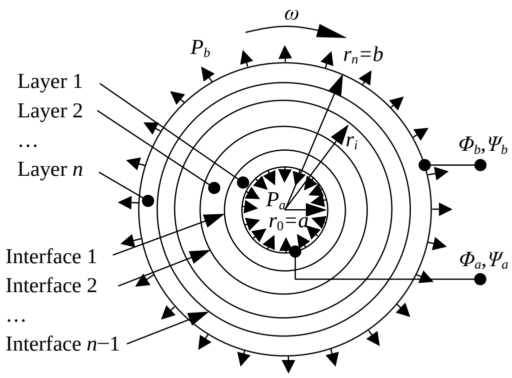

Fig.1 Model of rotating multiferroic composite hollow cylinder

$$\begin{aligned} \sigma_{ij}&=c_{ijkl}\gamma_{kl}-e_{kij}E_k-q_{kij}H_k,\\ D_i&=e_{ikl}\gamma_{kl}+\varepsilon_{ik}E_k+g_{ik}H_k,\\ B_i&=q_{ikl}\gamma_{kl}+g_{ik}E_k+\mu_{ik}H_k, \end{aligned} \tag{1}$$

where σ_{ij}, D_i and B_i are the components of stresses, electric displacement and magnetic induction, respectively. c_{ijkl}, e_{kij}, q_{kij}, ε_{ik}, g_{ik} and μ_{ik} are respectively the elastic, piezoelectric, piezomagnetic, dielectric, electromagnetic, and magnetic constants. The strain γ_{kl}, electric field E_k and magnetic field H_k are

$$\gamma_{ij}=(u_{i,j}+u_{j,i})/2,\ \ E_i=-\Phi_{,i},\ H_i=-\Psi_{,i}, \tag{2}$$

where u_i is the component of displacement; Φ and Ψ are electric and magnetic potentials, respectively. If each layer of the infinite composite hollow cylinder is characterized as cylindrical isotropy, the axisymmetric plane strain problem will be considered. In the cylindrical coordinate system (r, θ, z), the non-zero components of displacement, electric and magnetic potentials in the ith layer ($r_{i-1}\le r\le r_i$, i=1, 2, …, n) are denoted as $u_r^{(i)}=u_r^{(i)}(r)$, $\Phi^{(i)}=\Phi^{(i)}(r)$ and $\Psi^{(i)}=\Psi^{(i)}(r)$. Then the constitutive relations of the ith layer for magneto-electro-elastic media are written as

$$\begin{aligned} \sigma_{\theta\theta}^{(i)}&=c_{11}^{(i)}\frac{u_r^{(i)}}{r}+c_{13}^{(i)}\frac{\mathrm{d}u_r^{(i)}}{\mathrm{d}r}+e_{31}^{(i)}\frac{\mathrm{d}\Phi^{(i)}}{\mathrm{d}r}+q_{31}^{(i)}\frac{\mathrm{d}\Psi^{(i)}}{\mathrm{d}r},\\ \sigma_{rr}^{(i)}&=c_{13}^{(i)}\frac{u_r^{(i)}}{r}+c_{33}^{(i)}\frac{\mathrm{d}u_r^{(i)}}{\mathrm{d}r}+e_{33}^{(i)}\frac{\mathrm{d}\Phi^{(i)}}{\mathrm{d}r}+q_{33}^{(i)}\frac{\mathrm{d}\Psi^{(i)}}{\mathrm{d}r},\\ D_{rr}^{(i)}&=e_{31}^{(i)}\frac{u_r^{(i)}}{r}+e_{33}^{(i)}\frac{\mathrm{d}u_r^{(i)}}{\mathrm{d}r}-\varepsilon_{33}^{(i)}\frac{\mathrm{d}\Phi^{(i)}}{\mathrm{d}r}-g_{33}^{(i)}\frac{\mathrm{d}\Psi^{(i)}}{\mathrm{d}r},\\ B_{rr}^{(i)}&=q_{31}^{(i)}\frac{u_r^{(i)}}{r}+q_{33}^{(i)}\frac{\mathrm{d}u_r^{(i)}}{\mathrm{d}r}-g_{33}^{(i)}\frac{\mathrm{d}\Phi^{(i)}}{\mathrm{d}r}-\mu_{33}^{(i)}\frac{\mathrm{d}\Psi^{(i)}}{\mathrm{d}r}. \end{aligned} \tag{3}$$

In the absence of body force, electric charge density and electric current density, the equations of motion of the ith layer are expressed as

$$\frac{\mathrm{d}\sigma_{rr}^{(i)}}{\mathrm{d}r}+\frac{\sigma_{rr}^{(i)}-\sigma_{\theta\theta}^{(i)}}{r}+\rho^{(i)}\omega^2 r=0, \tag{4}$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[rD_{rr}^{(i)}\right]=0,\quad \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[rB_{rr}^{(i)}\right]=0, \tag{5}$$

where $\rho^{(i)}$ is the mass density of the ith layer. The boundary conditions are

$$\sigma_{rr}^{(1)}(a)=P_a,\quad \sigma_{rr}^{(n)}(b)=P_b, \tag{6}$$

$$\Phi^{(1)}(a)=\Phi_a,\quad \Phi^{(n)}(b)=\Phi_b, \tag{7}$$

$$\Psi^{(1)}(b)=\Psi_a,\quad \Psi^{(n)}(b)=\Psi_b. \tag{8}$$

For perfectly bonded interfaces, the continuity conditions are expressed as

$$u_r^{(i+1)}(r_i)=u_r^{(i)}(r_i),\quad \sigma_{rr}^{(i+1)}(r_i)=\sigma_{rr}^{(i)}(r_i), \tag{9}$$

$$D_{rr}^{(i+1)}(r_i)=D_{rr}^{(i)}(r_i),\quad B_{rr}^{(i+1)}(r_i)=B_{rr}^{(i)}(r_i), \tag{10}$$

$$\Phi^{(i+1)}(r_i)=\Phi^{(i)}(r_i),\quad \Psi^{(i+1)}(r_i)=\Psi^{(i)}(r_i). \tag{11}$$

In Eqs.(9)~(11), we have i=1, 2, …, n−1.

STATE EQUATION AND THE SOLUTION FOR MECHANICAL FIELD

The solutions of Eq.(5) can be derived as

$$D_r^{(i)}(r)=r^{-1}D^{(i)},\ B_r^{(i)}(r)=r^{-1}B^{(i)},\ i=1, 2, \ldots, n, \tag{12}$$

where $D^{(i)}$ and $B^{(i)}$ are unknown constants. The substitution of Eq.(12) into Eq.(10) yields

$$D^{(1)}=D^{(2)}=\ldots=D^{(n)}=D_0,\ B^{(1)}=B^{(2)}=\ldots=B^{(n)}=B_0, \tag{13}$$

where D_0 and B_0 are newly introduced unknown constants.

By means of Eq.(13), the last two equations of Eq.(3) can be rewritten as

$$\frac{\mathrm{d}\Phi^{(i)}}{\mathrm{d}r}=A_{11}^{(i)}\frac{u_r^{(i)}}{r}+A_{12}^{(i)}\frac{\mathrm{d}u_r^{(i)}}{\mathrm{d}r}+A_{13}^{(i)}\frac{D_0}{r}+A_{14}^{(i)}\frac{B_0}{r},$$

$$\frac{\mathrm{d}\Psi^{(i)}}{\mathrm{d}r}=A_{21}^{(i)}\frac{u_r^{(i)}}{r}+A_{22}^{(i)}\frac{\mathrm{d}u_r^{(i)}}{\mathrm{d}r}+A_{23}^{(i)}\frac{D_0}{r}+A_{24}^{(i)}\frac{B_0}{r}, \tag{14}$$

where

$$\begin{aligned}
&A_{11}^{(i)}=(e_{31}^{(i)}\mu_{33}^{(i)}-q_{31}^{(i)}g_{33}^{(i)})/R^{(i)},\quad A_{13}^{(i)}=-\mu_{33}^{(i)}/R^{(i)},\\
&A_{12}^{(i)}=(e_{33}^{(i)}\mu_{33}^{(i)}-q_{33}^{(i)}g_{33}^{(i)})/R^{(i)},\quad A_{14}^{(i)}=g_{33}^{(i)}/R^{(i)},\\
&A_{21}^{(i)}=(q_{31}^{(i)}\varepsilon_{33}^{(i)}-e_{31}^{(i)}g_{33}^{(i)})/R^{(i)},\quad A_{23}^{(i)}=g_{33}^{(i)}/R^{(i)},\\
&A_{22}^{(i)}=(q_{33}^{(i)}\varepsilon_{33}^{(i)}-e_{33}^{(i)}g_{33}^{(i)})/R^{(i)},\quad A_{24}^{(i)}=-\varepsilon_{33}^{(i)}/R^{(i)},\\
&R^{(i)}=\varepsilon_{33}^{(i)}\mu_{33}^{(i)}-g_{33}^{(i)}g_{33}^{(i)}.
\end{aligned} \tag{15}$$

The substitution of Eq.(14) into the first two equations of Eq.(3) derives

$$\begin{aligned}
\Sigma_{\theta\theta}^{(i)}&=c_{11D}^{(i)}u_r^{(i)}+c_{13D}^{(i)}\nabla u_r^{(i)}+e_{1D}^{(i)}D_0+q_{1D}^{(i)}B_0,\\
\Sigma_{rr}^{(i)}&=c_{13D}^{(i)}u_r^{(i)}+c_{33D}^{(i)}\nabla u_r^{(i)}+e_{3D}^{(i)}D_0+q_{3D}^{(i)}B_0,
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
&c_{11D}^{(i)}=c_{11}^{(i)}+(e_{31}^{(i)}A_{11}^{(i)}+q_{31}^{(i)}A_{21}^{(i)}),\\
&c_{13D}^{(i)}=c_{13}^{(i)}+(e_{31}^{(i)}A_{12}^{(i)}+q_{31}^{(i)}A_{22}^{(i)}),\\
&c_{33D}^{(i)}=c_{33}^{(i)}+(e_{33}^{(i)}A_{12}^{(i)}+q_{33}^{(i)}A_{22}^{(i)}),\\
&e_{1D}^{(i)}=e_{31}^{(i)}A_{13}^{(i)}+q_{31}^{(i)}A_{23}^{(i)},\quad e_{3D}^{(i)}=e_{33}^{(i)}A_{13}^{(i)}+q_{3}^{(i)}A_{23}^{(i)},\\
&q_{1D}^{(i)}=e_{31}^{(i)}A_{14}^{(i)}+q_{31}^{(i)}A_{24}^{(i)},\quad q_{3D}^{(i)}=e_{33}^{(i)}A_{14}^{(i)}+q_{33}^{(i)}A_{24}^{(i)}.
\end{aligned} \tag{17}$$

$$\Sigma_{\theta\theta}^{(i)}=r\sigma_{\theta\theta}^{(i)},\quad \Sigma_{rr}^{(i)}=r\sigma_{rr}^{(i)},\quad \nabla=r\frac{\mathrm{d}}{\mathrm{d}r}. \tag{18}$$

With the aid of newly introduced variables $\Sigma_{\theta\theta}^{(i)}$ and $\Sigma_{rr}^{(i)}$ in Eq.(18), Eqs.(4), (6) and (9) can be rewritten as

$$\nabla\Sigma_{rr}^{(i)}-\Sigma_{\theta\theta}^{(i)}+\rho^{(i)}\omega^2 r^3=0, \tag{19}$$

$$\Sigma_{rr}^{(1)}(a)=aP_a,\quad \Sigma_{rr}^{(n)}(b)=bP_b, \tag{20}$$

$$u_r^{(i)}(r_i)=u_r^{(i+1)}(r_i),\ \Sigma_{rr}^{(i)}(r_i)=\Sigma_{rr}^{(i+1)}(r_i),\ i=1, 2, \ldots, n-1. \tag{21}$$

Eqs.(16), (19)~(21) are the governing equations for the mechanical field of the rotating piezoelectric/piezomagnetic composite hollow cylinder.

The expression of $\nabla u_r^{(i)}$ can be derived from the second equation in Eq.(16). Then, substituting $\nabla u_r^{(i)}$ into the first equation of Eq.(16), we obtain the expression for $\Sigma_{\theta\theta}^{(i)}$. Subsequently, substituting $\Sigma_{\theta\theta}^{(i)}$ into Eq.(19), we derive the expression for $\nabla\Sigma_{rr}^{(i)}$.

Finally, the expressions of $\nabla u_r^{(i)}$ and $\nabla \Sigma_{rr}^{(i)}$ can be rewritten in a matrix form as

$$\nabla \boldsymbol{X}^{(i)} = \boldsymbol{N}^{(i)}\boldsymbol{X}^{(i)} + \boldsymbol{L}_1^{(i)}D_0 + \boldsymbol{L}_2^{(i)}B_0 + \boldsymbol{L}_3^{(i)}r^3, \quad (22)$$

where

$$\boldsymbol{X}^{(i)} = \begin{Bmatrix} u_r^{(i)}(r) \\ \Sigma_{rr}^{(i)}(r) \end{Bmatrix}, \quad \boldsymbol{N}^{(i)} = \begin{bmatrix} N_{11}^{(i)} & N_{12}^{(i)} \\ N_{21}^{(i)} & N_{22}^{(i)} \end{bmatrix},$$
$$\boldsymbol{L}_1^{(i)} = \begin{Bmatrix} L_{11}^{(i)} \\ L_{21}^{(i)} \end{Bmatrix}, \quad \boldsymbol{L}_2^{(i)} = \begin{Bmatrix} L_{12}^{(i)} \\ L_{22}^{(i)} \end{Bmatrix}, \quad \boldsymbol{L}_3^{(i)} = \begin{Bmatrix} 0 \\ -\rho^{(i)}\omega^2 \end{Bmatrix},$$
$$N_{11}^{(i)} = -c_{13D}^{(i)} \big/ c_{33D}^{(i)}, \quad N_{12}^{(i)} = 1\big/ c_{33D}^{(i)},$$
$$N_{21}^{(i)} = c_{11D}^{(i)} + c_{13D}^{(i)}N_{11}^{(i)}, \quad N_{22}^{(i)} = c_{13D}^{(i)}N_{12}^{(i)},$$
$$L_{11}^{(i)} = -e_{3D}^{(i)} \big/ c_{33D}^{(i)}, \quad L_{21}^{(i)} = e_{1D}^{(i)} + c_{13D}^{(i)}L_{11}^{(i)},$$
$$L_{12}^{(i)} = -q_{3D}^{(i)} \big/ c_{33D}^{(i)}, \quad L_{22}^{(i)} = q_{1D}^{(i)} + c_{13D}^{(i)}L_{12}^{(i)}. \quad (23)$$

The solution of Eq.(22) is

$$\boldsymbol{X}^{(i)}(r) = \boldsymbol{T}^{(i)}(r)(\boldsymbol{X}^{(i)}(r_{i-1}) + \boldsymbol{F}_1^{(i)}(r)D_0 + \boldsymbol{F}_2^{(i)}(r)B_0 + \boldsymbol{F}_3^{(i)}(r)), \quad (24)$$

where

$$\boldsymbol{T}^{(i)}(r) = (r/r_{i-1})^{N^{(i)}},$$
$$\boldsymbol{F}_1^{(i)}(r) = \int_{r_{i-1}}^{r} [\boldsymbol{T}^{(i)}(\zeta)]^{-1}\boldsymbol{L}_1^{(i)}\zeta^{-1}\mathrm{d}\zeta,$$
$$\boldsymbol{F}_2^{(i)}(r) = \int_{r_{i-1}}^{r} [\boldsymbol{T}^{(i)}(\zeta)]^{-1}\boldsymbol{L}_2^{(i)}\zeta^{-1}\mathrm{d}\zeta, \quad (25)$$
$$\boldsymbol{F}_3^{(i)}(r) = \int_{r_{i-1}}^{r} [\boldsymbol{T}^{(i)}(\zeta)]^{-1}\boldsymbol{L}_3^{(i)}\zeta^{2}\mathrm{d}\zeta.$$

The continuity conditions Eq.(21) can be rewritten as

$$\boldsymbol{X}^{(i)}(r_i) = \boldsymbol{X}^{(i+1)}(r_i), \ i=1, 2, \ldots, n-1. \quad (26)$$

Setting $r=r_i$ in Eq.(24) and repeatedly utilizing Eq.(26), we then obtain

$$\boldsymbol{X}^{(i)}(r_i) = \boldsymbol{H}^{(i)}\boldsymbol{X}^{(1)}(r_0) + \boldsymbol{M}_1^{(i)}D_0 + \boldsymbol{M}_2^{(i)}B_0 + \boldsymbol{M}_3^{(i)}, \quad (27)$$

where

$$\boldsymbol{H}^{(i)} = \tilde{\boldsymbol{T}}_1^{(i)}, \quad \tilde{\boldsymbol{T}}_m^{(i)} = \prod_{k=i}^{m} \boldsymbol{T}^{(k)}(r_k),$$

$$\boldsymbol{M}_j^{(i)} = \sum_{m=1}^{i} \tilde{\boldsymbol{T}}_m^{(i)}\boldsymbol{F}_j^{(m)}(r_m), \quad j=1,2,3. \quad (28)$$

Setting $i=n$ in Eq.(27) and utilizing the boundary conditions Eq.(20), we derive

$$\begin{Bmatrix} u_r^{(n)}(r_n) \\ bP_b \end{Bmatrix} = \begin{bmatrix} H_{11}^{(n)} & H_{12}^{(n)} \\ H_{21}^{(n)} & H_{22}^{(n)} \end{bmatrix} \begin{Bmatrix} u_r^{(1)}(r_0) \\ aP_a \end{Bmatrix} + \begin{Bmatrix} M_{11}^{(n)} \\ M_{12}^{(n)} \end{Bmatrix} D_0 + \begin{Bmatrix} M_{21}^{(n)} \\ M_{22}^{(n)} \end{Bmatrix} B_0 + \begin{Bmatrix} M_{31}^{(n)} \\ M_{32}^{(n)} \end{Bmatrix}. \quad (29)$$

From the second equation in Eq.(29), $u_r^{(1)}(r_0)$ can be obtained in the form

$$u_r^{(1)}(r_0) = (bP_b - H_{22}^{(n)}aP_a - M_{12}^{(n)}D_0 - M_{22}^{(n)}B_0 - M_{32}^{(n)})/H_{21}^{(n)}. \quad (30)$$

By means of Eqs.(26) and (27), Eq.(24) can be rewritten as

$$\boldsymbol{X}^{(i)}(r) = \boldsymbol{T}^{(i)}(r)(\boldsymbol{H}^{(i-1)}\boldsymbol{X}^{(1)}(r_0) + \boldsymbol{M}_1^{(i-1)}D_0 + \boldsymbol{M}_2^{(i-1)}B_0 + \boldsymbol{M}_3^{(i-1)} + \boldsymbol{F}_1^{(i)}(r)D_0 + \boldsymbol{F}_2^{(i)}(r)B_0 + \boldsymbol{F}_3^{(i)}(r)). \quad (31)$$

The substitution of Eq.(30) into Eq.(31), $u_r^{(i)}(r)$ can then be determined from the first equation of Eq.(31) as

$$u_r^{(i)}(r) = f_0^{(i)}(r) + f_1^{(i)}(r)D_0 + f_2^{(i)}(r)B_0, \quad (32)$$

where

$$f_0^{(i)}(r) = T_{11}^{(i)}(r)\left[aP_a\left(H_{12}^{(i-1)} - \frac{H_{22}^{(n)}}{H_{21}^{(n)}}H_{11}^{(i-1)}\right) + bP_b\frac{H_{11}^{(i-1)}}{H_{21}^{(n)}} + \left(M_{31}^{(i-1)} - \frac{M_{32}^{(n)}}{H_{21}^{(n)}}H_{11}^{(i-1)} + F_{31}^{(i)}(r)\right)\right]$$
$$+ T_{12}^{(i)}(r)\left[aP_a\left(H_{22}^{(i-1)} - \frac{H_{22}^{(n)}}{H_{21}^{(n)}}H_{21}^{(i-1)}\right) + bP_b\frac{H_{21}^{(i-1)}}{H_{21}^{(n)}} + \left(M_{32}^{(i-1)} - \frac{M_{32}^{(n)}}{H_{21}^{(n)}}H_{21}^{(i-1)} + F_{32}^{(i)}(r)\right)\right],$$

$$f_1^{(i)}(r)=\left(M_{11}^{(i-1)}-\frac{M_{12}^{(n)}}{H_{21}^{(n)}}H_{11}^{(i-1)}+F_{11}^{(i)}(r)\right)T_{11}^{(i)}(r)+\left(M_{12}^{(i-1)}-\frac{M_{12}^{(n)}}{H_{21}^{(n)}}H_{21}^{(i-1)}+F_{12}^{(i)}(r)\right)T_{12}^{(i)}(r),$$
$$f_2^{(i)}(r)=\left(M_{21}^{(i-1)}-\frac{M_{22}^{(n)}}{H_{21}^{(n)}}H_{11}^{(i-1)}+F_{21}^{(i)}(r)\right)T_{11}^{(i)}(r)+\left(M_{22}^{(i-1)}-\frac{M_{22}^{(n)}}{H_{21}^{(n)}}H_{21}^{(i-1)}+F_{22}^{(i)}(r)\right)T_{12}^{(i)}(r). \quad (33)$$

DETERMINATION OF D_0 AND B_0

The unknown constants D_0 and B_0 will be determined in this section. Integrating Eq.(14) at each interval $[r_{i-1},r]$ ($r_{i-1}\le r\le r_i$, i=1, 2, …, n) and utilizing Eq.(32), we obtain

$$\Phi^{(i)}(r)=\Phi^{(i)}(r_{i-1})+K_{10}^{(i)}(r)+K_{11}^{(i)}(r)D_0+K_{12}^{(i)}(r)B_0,$$
$$\Psi^{(i)}(r)=\Psi^{(i)}(r_{i-1})+K_{20}^{(i)}(r)+K_{21}^{(i)}(r)D_0+K_{22}^{(i)}(r)B_0,$$
$$i=1, 2, \ldots, n, \quad (34)$$

where

$$K_{j0}^{(i)}(r)=A_{j1}^{(i)}\int_{r_{i-1}}^{r}\frac{f_0^{(i)}(\zeta)}{\zeta}\mathrm{d}\zeta+A_{j2}^{(i)}[f_0^{(i)}(r)-f_0^{(i)}(r_{i-1})],$$
$$K_{j1}^{(i)}(r)=A_{j1}^{(i)}\int_{r_{i-1}}^{r}\frac{f_1^{(i)}(\zeta)}{\zeta}\mathrm{d}\zeta+A_{j2}^{(i)}[f_1^{(i)}(r)-f_1^{(i)}(r_{i-1})]+A_{j3}^{(i)}\ln\left(\frac{r}{r_{i-1}}\right),$$
$$K_{j2}^{(i)}(r)=A_{j1}^{(i)}\int_{r_{i-1}}^{r}\frac{f_2^{(i)}(\zeta)}{\zeta}\mathrm{d}\zeta+A_{j2}^{(i)}[f_2^{(i)}(r)-f_2^{(i)}(r_{i-1})]+A_{j4}^{(i)}\ln\left(\frac{r}{r_{i-1}}\right),$$
$$j=1, 2. \quad (35)$$

Setting $r=r_i$ (i=1, 2, …, n) in Eq.(34), the relation between the electric potential at the interior and exterior surfaces for each layer is then constructed. By means of the continuity conditions of electric and magnetic potentials Eq.(11) and the boundary conditions of electric and magnetic potentials Eqs.(7) and (8), the summation of the n equations denoted in Eq.(34) derives

$$G_{11}D_0+G_{12}B_0=Q_1,\ \ G_{21}D_0+G_{22}B_0=Q_2, \quad (36)$$

where

$$Q_1=\Phi_b-\Phi_a-G_{10},\quad Q_2=\Psi_b-\Psi_a-G_{20},$$
$$G_{jm}=\sum_{i=1}^{n}K_{jm}^{(i)}(r_i),\quad j=1, 2,\quad m=0, 1, 2. \quad (37)$$

D_0 and B_0 are then determined from Eq.(36) as

$$D_0=\frac{Q_1G_{22}-Q_2G_{12}}{G_{11}G_{22}-G_{12}G_{21}},\quad B_0=\frac{Q_2G_{11}-Q_1G_{21}}{G_{11}G_{22}-G_{12}G_{21}}. \quad (38)$$

Substituting Eq.(38) into Eq.(32), the non-zero displacement component $u_r^{(i)}(r)$ is then totally determined. Therefore, the magneto-electro-elastic fields can be completely performed by means of Eqs.(16) and (3).

SPECIAL CASES

If the material constants of each layer are treated as the same, the solution then becomes that for a rotating homogeneous magneto-electro-elastic hollow cylinder (Babaei and Chen, 2008).

If the material constants of piezomagnetic layers are replaced by those of piezoelectric layers, the solution is just that for a multilayered piezoelectric hollow cylinder (Wang and Ding, 2007). If we further set the material constants of each layer with the same values, then the present solution degenerates to that for a homogeneous piezoelectric hollow cylinder (Galic and Horgan, 2003).

NUMERICAL EXAMPLES

Numerical results for a rotating homogeneous multiferroic hollow cylinder are first performed. In order to employ the present solution, the homogeneous hollow cylinder is divided into five virtual sub-cylinders with equal thickness in the radial direction. The results obtained are completely identical to those that have been reported (Babaei and Chen, 2008). The validity of the present solution is thus clarified.

As an illustrative example, a five-layered (n=5) rotating multiferroic composite hollow cylinder constructed of $BaTiO_3$ and $CoFe_2O_4$ layers will be considered here. The material constants can be reached from (Buchanan, 2003). For the sake of convenience, in the following we denote $BaTiO_3$ as "B" and $CoFe_2O_4$ as "F". The stacking sequence is taken as B/F/B/F/B from inner to outer. In the numerical results, the following non-dimensional parameters are introduced as

$$u^{(i)}=\frac{u_r^{(i)}}{b},\ \sigma_r^{(i)}=\frac{\sigma_{rr}^{(i)}}{c_{33}^{(1)}},\ \sigma_\theta^{(i)}=\frac{\sigma_{\theta\theta}^{(i)}}{c_{33}^{(1)}},\ \phi^{(i)}=\frac{\Phi^{(i)}}{\Phi_0},$$
$$\psi^{(i)}=\frac{\Psi^{(i)}}{\Psi_0},\quad \xi=\frac{r}{b},\quad \xi_i=\frac{r_i}{b},\ i=0,1,\cdots,5,$$
$$\phi_a=\frac{\Phi_a}{\Phi_0},\quad \phi_b=\frac{\Phi_b}{\Phi_0},\quad \psi_a=\frac{\Psi_a}{\Psi_0},\quad \psi_b=\frac{\Psi_b}{\Psi_0},$$
$$p_a=\frac{P_a}{c_{33}^{(1)}},\ p_b=\frac{P_b}{c_{33}^{(1)}},\ \Omega=\frac{\omega b}{c_v},\ c_v=\sqrt{c_{33}^{(1)}/\rho^{(1)}},$$
$$\Phi_0=b\sqrt{c_{33}^{(1)}/\varepsilon_{33}^{(1)}},\quad \Psi_0=b\sqrt{c_{33}^{(1)}/\mu_{33}^{(1)}}. \tag{39}$$

In the calculation, the non-dimensional geometric parameters of the composite cylinder are taken as ξ_0=0.2, ξ_1=0.4, ξ_2=0.5, ξ_3=0.7, ξ_4=0.8 and ξ_5=1.0, respectively. Suppose that the cylinder is subjected to constant pressure at the inner cylindrical surface. The mechanical boundary conditions at the inner and outer cylindrical surfaces are prescribed as

$$p_a=-1,\ p_b=0. \tag{40}$$

Figs.2a and 2b show the distributions of radial and tangential stresses of the internally pressurized five-layered $BaTiO_3/CoFe_2O_4$ composite hollow cylinder rotating about its central axis at the constant angular velocities Ω=1. The magnetic potential on both the inner and outer surfaces is zero (ψ_a=0, ψ_b=0). The cylinder is electrically shorted at the outer surface and is subjected to different electric potentials at the inner surface. Clearly, the distributions of radial and tangential stresses can be adjusted by applying different electric potentials. In Fig.2b, for ϕ_a=0, the maximum value of tensile tangential stress appears at the inner surface; while for ϕ_a=50 and 100, the maximum value is greatly suppressed.

Figs.3a~3d show the effect of rotation on the distributions of magneto-electro-elastic fields for the electric and magnetic boundary conditions ϕ_a=1,ϕ_b=0 and ψ_a=1, ψ_b=0. For Ω=0 (static case), the radial stress in the whole cross section is compressive. For Ω=1, the tension radial stress appears at the outer region. With increase of angular velocity Ω, the region with tensile radial stress becomes larger and larger (Fig.3a). Under three different angular velocities Ω=0, 1 and 2, the tangential stress in the whole cross section is tensile and the magnitude of the tangential stress grows rapidly with the increase in Ω (Fig.3b). In Figs.3c and 3d, it is found that the angular velocity has little effect on the electric and magnetic potentials.

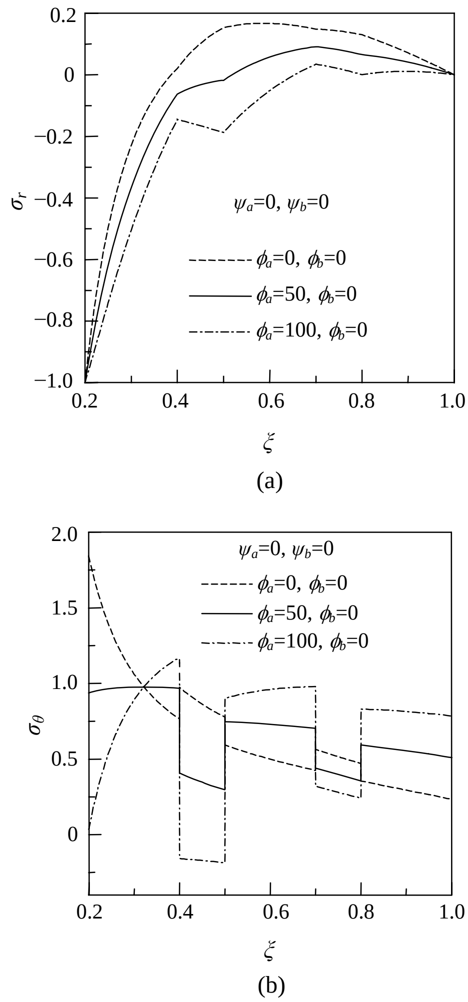

Fig.2 Distributions of (a) radial stress and (b) tangential stress for different electric boundary conditions (Ω=1)

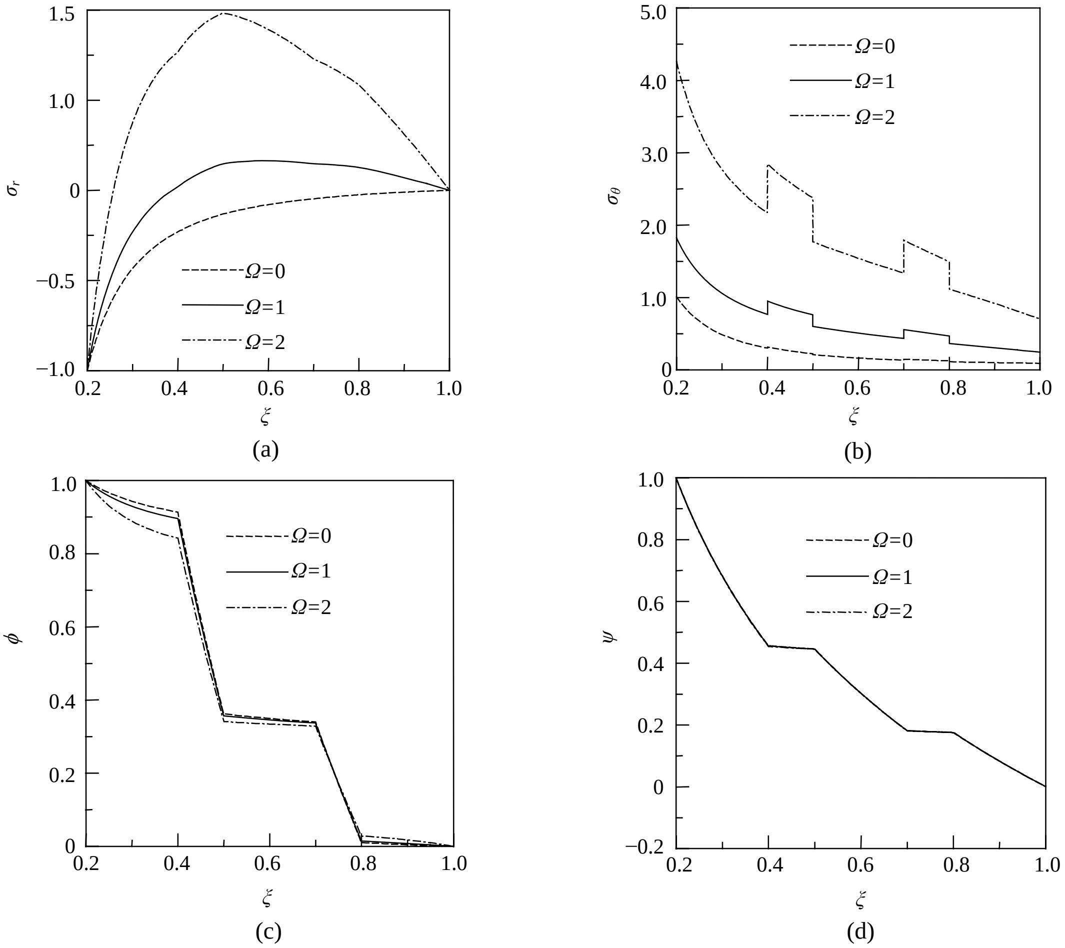

Fig.3 Distributions of (a) radial stress; (b) tangential stress; (c) electric potential and (d) magnetic potential for different values of *Ω*

CONCLUSION

Based on the elasticity theory for magneto-electro-elastic media, an analytical solution is obtained for rotating a radially polarized multiferroic composite hollow cylinder constructed of piezoelectric and piezomagnetic layers by means of the state space method. Regardless of the number of layers, the analysis is completed only by operating 2×2 matrices. With the aid of the presented solution, numerical analysis can be easily carried out.

Due to the coupling effect among the mechanical, electric and magnetic fields, the distribution form of the stresses in a rotating multiferroic composite hollow cylinder can be adjusted and controlled by applying proper electric and magnetic potentials on the correct surfaces.

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