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Unified analytical solutions for a circular opening based on non-linear unified failure criterion*

Chang-guang ZHANG[†1,2], Qing-he ZHANG[1,2], Jun-hai ZHAO[3], Fei XU[4], Chuang-zhou WU[1,2]

([1]*MOE Key Laboratory of Geotechnical and Underground Engineering, Tongji University, Shanghai 200092, China*)

([2]*Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China*)

([3]*School of Civil Engineering, Chang'an University, Xi'an 710061, China*)

([4]*Shanghai No. 1 Municipal Engineering Co. Ltd., Shanghai 200083, China*)

[†]E-mail: zcg1016@163.com

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Abstract: Unified analytical solutions are presented for the predictions of the stresses and displacements around a circular opening based on non-linear unified failure criterion and the elastic-brittle-plastic softening model. Unified analytical solutions not only involve generally traditional solutions which are based on the Hoek-Brown (H-B) failure criterion or the non-linear twin-shear failure criterion, but also involve other new results. The results of the radius of plastic zone, radial displacements and stresses are obviously different using three rock masses when different values of the unified failure criterion parameter or different material behavior models are used. For a given condition, the radius of plastic zone and radial displacements are reduced by increasing the unified failure criterion parameter. The latent potentialities of rock mass result from considering the effect of intermediate principal stress. It is shown that proper choices of the failure criterion and the material behavior model for rock mass are significant in the tunnel design.

Key words: Analytical solution, Elastic-brittle-plastic rock, Non-linear unified failure criterion, Intermediate principal stress, Dilatancy

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1 Introduction

Strength theory is very important for strength calculation and engineering structure design. For metallic materials, strength theory is usually called yield criterion. For granular materials, such as concrete, soil and rock, a failure criterion is often used (Yu *et al.*, 1999). The properties of rock are different for various rock types. The tensile strength of rock varies greatly with its compressive strength. The rock often reacts under complex multiaxial stress state, and sometimes yields or fails in extreme circumstances. The elastic-plastic analytical solutions give an opportunity of validating numerical methods and computer software (Sharan, 2008). Also, some significant factors can be found from analytical solutions. In mining, tunneling and other geotechnical engineering, simple analytical solutions are usually feasible for an axisymmetric circular opening in an infinite medium with a hydrostatic in-situ stress. Many researchers have obtained closed-form solutions for this problem using linear Mohr-Coulomb (M-C) or non-linear Hoek-Brown (H-B) failure criteria (Wang, 1996; Sharan, 2003; 2005; 2008; Carranza-Torres, 2004; Park and Kim, 2006; Luo *et al.*, 2007; Tu *et al.*, 2008). However, the M-C and H-B failure criteria do not contain intermediate principal stress, so they cannot take into account the effect of intermediate principal stress. There is evidence that the intermediate principal stress does have a pronounced influence on rock strength (Xu and Geng, 1985; Haimson and Chang,

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2000; Colmenares and Zoback, 2002; Zhang, 2008). The non-linear unified failure criterion takes into account the influences of all principal stress components on the failure of materials, which fully reflects the different effects of intermediate principal stress for various materials (Yu *et al.*, 2002). The H-B failure criterion is a special case of the non-linear unified failure criterion.

Based on a non-linear unified failure criterion and the elastic-brittle-plastic softening model, the elastic-plastic analysis for a circular opening is advanced by using a non-associated linear flow rule. The unified analytical solutions for the stresses and displacements are deduced. To investigate the effects of the intermediate principal stress and the material behavior model, the results of radius of plastic zone, radial displacements and stresses are compared for three different rock masses, whose qualities range from very good to very poor. The effect of shear dilation on the radial displacements is discussed. In this paper, as a rule, compressive stress and inward radial displacement are considered positive.

2 Non-linear unified failure criterion

The H-B failure criterion can be classified as a multi-parameter single-shear failure criterion (Yu, 2002). The latest generalized H-B failure criterion is written as (Hoek and Brown, 1997; Hoek *et al.*, 2002)

$$\sigma_1=\sigma_3+\sigma_c\left(m_b\frac{\sigma_3}{\sigma_c}+s\right)^a, \tag{1}$$

where σ_1 is major principal stress and σ_3 is minor principal stress. σ_c is uniaxial compressive strength of intact rock; m_b, s and a are constants of rock mass before failure, given by

$$m_b=m_i\exp\left(\frac{\mathrm{GSI}-100}{28-14D}\right),\quad s=\exp\left(\frac{\mathrm{GSI}-100}{9-3D}\right),$$
$$a=\frac{1}{2}+\frac{1}{6}\left[\exp\left(\frac{-\mathrm{GSI}}{15}\right)-\exp\left(\frac{-20}{3}\right)\right], \tag{2}$$

where m_i is a material constant of intact rock, GSI is the Geological Strength Index for rock mass, and D is a disturbance factor of rock mass. The ranges of GSI and D are normally from 10 to 100 and from 0 to 1, respectively. When GSI=100 in Eq. (2), a=0.5. Thus the original H-B failure criterion is a special case of the latest generalized H-B failure criterion with GSI=100, which represents an extremely good quality of rock mass.

Based on true triaxial compression test data on rock and taking the effect of intermediate principal stress into account, a new non-linear unified failure criterion was proposed by Yu *et al.* (2002). In terms of principal stresses, the non-linear unified failure criterion is expressed as

$$F=\sigma_1-\frac{1}{1+b}(b\sigma_2+\sigma_3)-\sigma_c\left(\frac{m_b}{1+b}(b\sigma_2+\sigma_3)+s\right)^a=0,\quad \text{when } F\geq F', \tag{3a}$$

and

$$F'=\frac{1}{1+b}(\sigma_1+b\sigma_2)-\sigma_3-\sigma_c\left(m_b\frac{\sigma_3}{\sigma_c}+s\right)^a=0,\quad \text{when } F'>F, \tag{3b}$$

where σ_c, m_b, s and a are constants as the same as those in the latest generalized H-B failure criterion. b is a material parameter which reflects the effect of intermediate principal shear stress and its related principal stresses acting on the same plane. b is also a parameter of the failure criterion, with $0\leq b\leq 1$. The ratio of tensile strength to compressive strength α, an index of material strength difference, is given as

$$\alpha=\begin{cases}\dfrac{\sqrt{m_b^2+4s^2}-m_b}{2}, & \text{for rock mass,}\\ \dfrac{\sqrt{m_b^2+4}-m_b}{2}, & \text{for intact rock.}\end{cases} \tag{4}$$

It is evident that the intermediate principal stress σ_2 has been taken into account in the non-linear unified failure criterion. When the parameter b=0, the non-linear unified failure criterion is equivalent to the latest generalized H-B failure criterion, i.e., Eq. (1). This is one of the main advantages of employing the non-linear unified failure criterion. The non-linear twin-shear failure criterion can be obtained using the non-linear unified failure criterion with the parameter b=1 as follows (Yu *et al.*, 2002):

$$F=\sigma_1-\frac{1}{2}(\sigma_2+\sigma_3)-\sigma_c\left(\frac{m_b}{2}(\sigma_2+\sigma_3)+s\right)^a=0,$$

when $F \geq F'$, (5a)

and

$$F' = \frac{1}{2}(\sigma_1 + \sigma_2) - \sigma_3 - \sigma_c \left(m_b \frac{\sigma_3}{\sigma_c} + s \right)^a = 0,$$

when $F' > F$. (5b)

The parameter b plays an important role in the non-linear unified failure criterion and shows the effect of intermediate principal stress (Xu and Yu, 2005). On the other hand, b is also a parameter for the choice of failure criteria. Therefore, it is called the unified failure criterion parameter. The value of b can be determined by rock material mechanical tests. If uniaxial tensile strength σ_t, uniaxial compressive strength σ_c and shear strength τ_o are used, b can be expressed as (Yu, 1994; 1998; Zhao *et al.*, 2007; Zhang *et al.*, 2008; Li *et al.*, 2008)

$$b = \frac{(\sigma_c + \sigma_t)\tau_o - \sigma_t \sigma_c}{(\sigma_t - \tau_o)\sigma_c} = \frac{(1+\alpha)\tau_o - \sigma_t}{\sigma_t - \tau_o}. \quad (6)$$

The non-linear unified failure criterion is especially versatile in reflecting the intermediate principal stress effect to different extents for different materials. The limit loci of the non-linear unified failure criterion on the deviatoric plane with the variation of b are shown in Fig. 1 (Yu *et al.*, 2002).

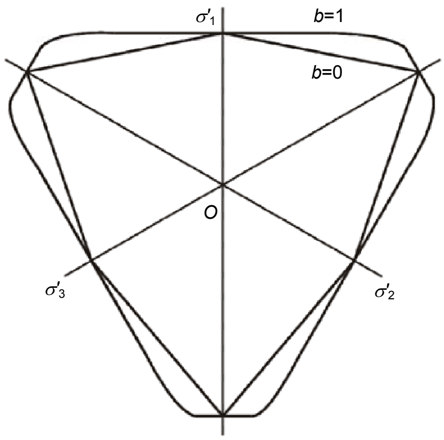

Fig. 1 Limit loci on the deviatoric plane

It can be seen that the limit locus of the H-B failure criterion (single-shear failure criterion) is the lower bound on the deviatoric plane; the upper bound is the non-linear twin-shear failure criterion. The effect of intermediate principal stress becomes clear with b changing from 0 to 1. The non-linear unified failure criterion considers the difference of tensile and compressive strengths of rocks (through m_b), the effect of intermediate principal stress on rock strength (through b), cracking degree of rock mass (though s), and the behavior of failure with a parabola formula.

3 Problem definition

Fig. 2 shows a sketch map of a mechanical model subjected to a uniformly supported pressure p_i at the opening surface, and a hydrostatic stress p_o at infinity. The circular opening is assumed in a continuous, homogeneous, isotropic, initially elastic rock mass. In underground engineering, the uniformly supported pressure p_i is usually less than the in-situ stress p_o. With p_i decreasing gradually, the radial displacement develops. When p_i is decreased to less than the critical yielding stress, a plastic zone occurs around the opening.

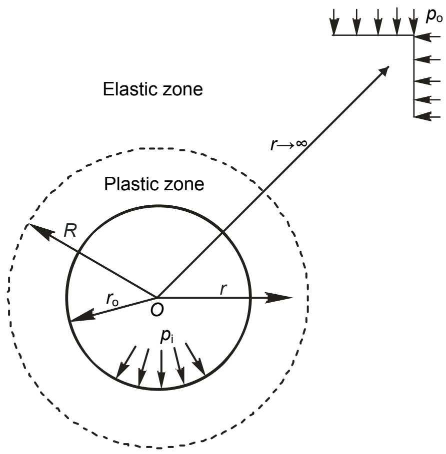

Fig. 2 Mechanical model in an infinite medium

After yielding, the elastic-brittle-plastic softening model is adopted in the plastic zone. The strength of rock mass suddenly drops from peak strength and follows the post-yield softening behavior. The material behaviors of the elastic-brittle-plastic softening model are shown in Fig. 3. It can be seen from Fig. 3 that the effect of shear dilation is also considered in the plastic zone.

In the plastic zone, the radial and circumferential stresses, σ_r and σ_θ, will be principal stresses σ_1 and σ_3, respectively. In the plane strain state, $\sigma_2 = k(\sigma_\theta + \sigma_r)/2$ (Yu, 1994; 1998; Lee and Ghosh, 1996; Zhao *et al.*,

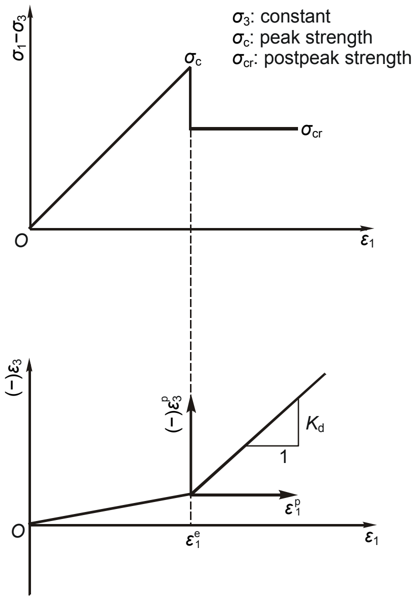

Fig. 3 Elastic-brittle-plastic softening model
ε_1 is the major principal strain and ε_3 is the minor principal strain. $\varepsilon_1^{\mathrm{e}}$, $\varepsilon_1^{\mathrm{p}}$ and $\varepsilon_3^{\mathrm{p}}$ are elastic and plastic parts of ε_1 and ε_3 in the plastic zone, respectively

2007; Li *et al.*, 2008), where the parameter k is in the rang of $2\nu \leq k \leq 1$, ν is Poisson's ratio. It is assumed that $k=2\nu$ in the elastic zone and $k\rightarrow 1$ in the plastic zone. Therefore, $F<F'$, Eq. (3b) should be adopted. With $k=1$, Eq. (3b) can be expressed as

$$\sigma_\theta = \sigma_r + \frac{2(1+b)}{2+b}\sigma_{\mathrm{c}}\left(m_{\mathrm{b}}\frac{\sigma_r}{\sigma_{\mathrm{c}}}+s\right)^a. \quad (7)$$

For the plastic zone $r_{\mathrm{o}} \leq r \leq R$ (Fig. 2), Eq. (7) may be transmuted as

$$\sigma_\theta = \sigma_r + \frac{2(1+b)}{2+b}\sigma_{\mathrm{c}}\left(m_{\mathrm{br}}\frac{\sigma_r}{\sigma_{\mathrm{c}}}+s_{\mathrm{r}}\right)^{a_{\mathrm{r}}}, \quad (8)$$

where m_{br}, s_{r} and a_{r} are the residual values of m_{b}, s and a, respectively.

As σ_{c} is the uniaxial compressive strength of intact rock, its value should not be altered after the rock mass yielded (Sharan, 2003).

4 Unified analytical solutions

The differential equation of stress equilibrium for the axisymmetric problem in polar coordinates can be written as

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \quad (9)$$

By introducing Eq. (8) into Eq. (9) and with the stress boundary condition of the internal opening surface, $\sigma_r = p_{\mathrm{i}}$ at $r=r_{\mathrm{o}}$, the stresses in the plastic zone can be deduced as

$$\sigma_r = \frac{\sigma_{\mathrm{c}}}{m_{\mathrm{br}}}\left(C_1 + C_2\ln\left(\frac{r}{r_{\mathrm{o}}}\right)\right)^{\frac{1}{1-a_{\mathrm{r}}}} - C_3, \quad (10)$$

$$\sigma_\theta = C_0\left(C_1 + C_2\ln\left(\frac{r}{r_{\mathrm{o}}}\right)\right)^{\frac{a_{\mathrm{r}}}{1-a_{\mathrm{r}}}} + \frac{\sigma_{\mathrm{c}}}{m_{\mathrm{br}}}\left(C_1 + C_2\ln\left(\frac{r}{r_{\mathrm{o}}}\right)\right)^{\frac{1}{1-a_{\mathrm{r}}}} - C_3, \quad (11)$$

where

$$C_0 = \frac{2(1+b)}{2+b}\sigma_{\mathrm{c}},\ C_1 = \left(m_{\mathrm{br}}\frac{p_{\mathrm{i}}}{\sigma_{\mathrm{c}}}+s_{\mathrm{r}}\right)^{1-a_{\mathrm{r}}},$$
$$C_2 = \frac{2(1+b)(1-a_{\mathrm{r}})m_{\mathrm{br}}}{2+b},\ C_3 = \frac{s_{\mathrm{r}}\sigma_{\mathrm{c}}}{m_{\mathrm{br}}}.$$

The radial and circumferential stresses in the elastic zone can be given as (Park and Kim, 2006)

$$\sigma_r = p_{\mathrm{o}} - \left(\frac{R}{r}\right)^2(p_{\mathrm{o}} - \sigma_R), \quad (12)$$

$$\sigma_\theta = p_{\mathrm{o}} + \left(\frac{R}{r}\right)^2(p_{\mathrm{o}} - \sigma_R). \quad (13)$$

By using the stress condition at the elastic-plastic interface, i.e., $r=R$ in the elastic zone where the rock material constants do not decrease, the unified critical yielding stress σ_R at the elastic-plastic interface may be obtained by solving the following non-linear equation:

$$2(p_{\mathrm{o}} - \sigma_R) = C_0\left(m_{\mathrm{b}}\frac{\sigma_R}{\sigma_{\mathrm{c}}}+s\right)^a. \quad (14)$$

For the non-linear unified failure criterion, the approximate solution of σ_R in Eq. (14) could be achieved by performing iterations of a numerical method such as the Newton-Raphson method with σ_{R0} as the initial estimated value.

The σ_{R0} is used for the original H-B failure criterion, i.e., for a=0.5, and can be written as

$$\sigma_{R0}=p_o-C_4\sigma_c, \quad (15)$$

where

$$C_4=\frac{1+b}{2+b}\left[\sqrt{\left(\frac{(1+b)m_b}{2(2+b)}\right)^2+m_b\frac{p_o}{\sigma_c}+s}-\frac{(1+b)m_b}{2(2+b)}\right].$$

Substituting $r=R$ and $\sigma_r=\sigma_R$ into Eq. (10), the radius of plastic zone R can be given by

$$R=r_o\exp\left[\left((m_{br}\frac{\sigma_R}{\sigma_c}+s_r)^{1-a_r}-C_1\right)\Big/C_2\right]. \quad (16)$$

Using a non-associated linear flow rule and small strain theory, the radial displacement in the plastic zone can be written as (Park and Kim, 2006)

$$u=\frac{1}{r^{K_d}}\int_R^r r^{K_d}\left(\varepsilon_r^e+K_d\varepsilon_\theta^e\right)dr+u_R\left(\frac{R}{r}\right)^{K_d}, \quad (17)$$

where ε_r^e and ε_θ^e are the elastic parts of radial and circumferential strains in the plastic zone, respectively; shear dilation parameter $K_d=(1+\sin\psi)/(1-\sin\psi)$, where ψ is the dilation angle of rock mass. $\psi=0°$ represents no plastic volume change and $\psi>0°$ corresponds to plastic volume increase.

The radial displacement u_R at the elastic-plastic interface ($r=R$) is expressed as

$$u_R=\left(\frac{1+\nu}{E}\right)(p_o-\sigma_R)R, \quad (18)$$

where E and ν are Young's modulus and Poisson's ratio of rock mass, respectively.

Expressions for ε_r^e and ε_θ^e are deduced from Hooke's law in consideration of initial hydrostatic stress and can be written as

$$\varepsilon_r^e=\frac{1+\nu_r}{E_r}[(1-\nu_r)(\sigma_r-p_o)-\nu_r(\sigma_\theta-p_o)], \quad (19)$$

$$\varepsilon_\theta^e=\frac{1+\nu_r}{E_r}[(1-\nu_r)(\sigma_\theta-p_o)-\nu_r(\sigma_r-p_o)], \quad (20)$$

where E_r and ν_r are Young's modulus and Poisson's ratio of rock mass after failure, respectively.

By Eqs. (10), (11), (19) and (20), the integration of Eq. (17) can be made analytically. The closed-form solution for the radial displacement u can be simplified as

$$\begin{aligned}\frac{u}{r}=\frac{1}{r^{K_d+1}}\Bigg[&\frac{(1+\nu)(p_o-\sigma_R)R^{K_d+1}}{E}\\&+[D_1f_1(r)+D_2f_2(r)+D_3f_3(r),\\&-D_1f_1(R)-D_2f_2(R)-D_3f_3(R)]\frac{(1+\nu_r)}{E_r}\Bigg],\end{aligned} \quad (21)$$

where

$$C_5=-\frac{C_2}{(K_d+1)(1-a_r)},\quad C_6=\frac{(C_2)^2a_r}{(K_d+1)^2(1-a_r)^2},$$

$$C_7=-\frac{(C_2)^3a_r(2a_r-1)}{(K_d+1)^3(1-a_r)^3},$$

$$D_1=-(1-2\nu_r)(1+K_d)(p_o+C_3),$$

$$D_2=(1-2\nu_r)(1+K_d)\frac{\sigma_c}{m_{br}},$$

$$D_3=(K_d-K_d\nu_r-\nu_r)C_0,$$

$$D_4=-\frac{C_2a_r}{(K_d+1)(1-a_r)},\quad D_5=\frac{(C_2)^2a_r(2a_r-1)}{(K_d+1)^2(1-a_r)^2},$$

$$f_1(r)=\int r^{K_d}dr=\frac{r^{K_d+1}}{K_d+1},$$

$$\begin{aligned}f_2(r)&=\int r^{K_d}(C_1+C_2\ln(r/r_o))^{\frac{1}{1-a_r}}dr\\&=\frac{r^{K_d+1}}{K_d+1}\Bigg[(C_1+C_2\ln(r/r_o))^{\frac{1}{1-a_r}}+C_5(C_1+C_2\ln(r/r_o))^{\frac{a_r}{1-a_r}}\\&+C_6(C_1+C_2\ln(r/r_o))^{\frac{2a_r-1}{1-a_r}}+C_7(C_1+C_2\ln(r/r_o))^{\frac{3a_r-2}{1-a_r}}+\ldots\Bigg],\end{aligned}$$

$$\begin{aligned}f_3(r)&=\int r^{K_d}(C_1+C_2\ln(r/r_o))^{\frac{a_r}{1-a_r}}dr\\&=\frac{r^{K_d+1}}{K_d+1}\Bigg[\left(C_1+C_2\ln(r/r_o)\right)^{\frac{a_r}{1-a_r}}\\&+D_4\left(C_1+C_2\ln(r/r_o)\right)^{\frac{2a_r-1}{1-a_r}}\\&+D_5(C_1+C_2\ln(r/r_o))^{\frac{3a_r-2}{1-a_r}}+\ldots\Bigg].\end{aligned}$$

The analytical displacement u_o at the opening surface could be obtained by substituting $r=r_o$ into Eq. (21). For simplicity and practical engineering demands, choice can be made only from the first three parts in $f_2(r)$, and the first two parts in $f_3(r)$, respectively.

The unified analytical solutions are a series of results, not a generally special result, and are suitable for a wide range of rock masses and engineering structures. The stresses and radial displacements in the plastic zone based on the H-B failure criterion and the non-linear twin-shear failure criterion can be obtained from the unified solutions, i.e., Eqs. (10), (11) and (21), with b=0 and b=1, respectively. Thus the unified analytical solutions not only involve generally traditional results in the literature, but also involve more new results which are comparable to the traditional ones.

5 Discussion

To investigate the effects of intermediate principal stress and the material behavior model used in the derivation of the unified analytical solutions, the radius of plastic zone, radial displacements and stresses are computed for three cases. Examples A and B represent very good and average qualities of rock masses, respectively, whereas example C corresponds to very poor quality. Table 1 shows the material mechanical properties of these three rock masses (Sharan, 2008). Examples A and B are elastic-brittle-plastic softening materials and example C is assumed to be a perfectly plastic rock mass. The radius of the opening r_o is assumed to be 2 m. For each rock mass, the value of the initial hydrostatic stress p_o is considered to be $\sigma_c/2$.

The dimensionless radius of plastic zone (R/r_o) is plotted in Fig. 4. The different values of the unified failure criterion parameter b reflect the effects of different extents of intermediate principal stress and different failure criteria. Fig. 5 shows the dimensionless radial displacements at the opening (u_o/r_o) for example A of ψ=0° and ψ=30°. The adaptation of the high dilation angle in this study is used to illustrate the potential maximum effect of shear dilation on the radial displacements. The dimensionless radial displacements for examples B and C of ψ=0° are shown in Fig. 6.

Table 1 Mechanical parameters of rock masses (Sharan, 2008)

Rock mass	Mechanical parameter		
	A	B	C
Radius of opening, r_o (m)	2	2	2
Initial stress, p_o (MPa)	75	40	10
Internal pressure, p_i (MPa)	0	0	0
GSI	75	50	30
Uniaxial compressive strength, σ_c (MPa)	150	80	20
Young's modulus, E (GPa)	42	9	1.4
Poisson's ratio, ν	0.2	0.25	0.3
m_b	10.2	2.01	0.657
s	0.062	0.0039	0.0004
a	0.5	0.51	0.52
Young's modulus after failure, E_r (GPa)	10	5	1.4
Poisson's ratio after failure, ν_r	0.2	0.25	0.3
m_{br}	1.27	0.34	0.657
s_r	0.0002	0	0.0004
a_r	0.51	0.53	0.52

A, B and C represent very good, average and very poor qualities of rock masses, respectively

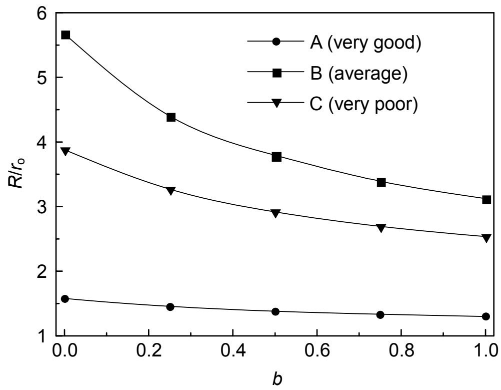

Fig. 4 Effect of b on the radius of the plastic zone

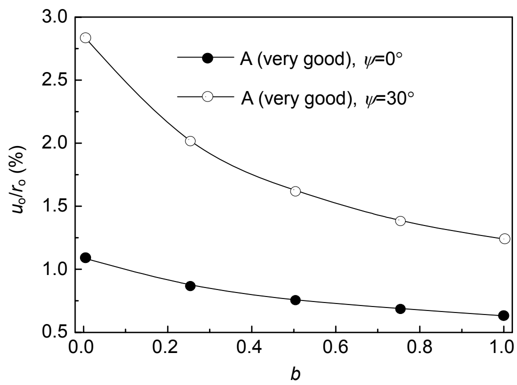

Fig. 5 Effect of b on the radial displacement (example A)

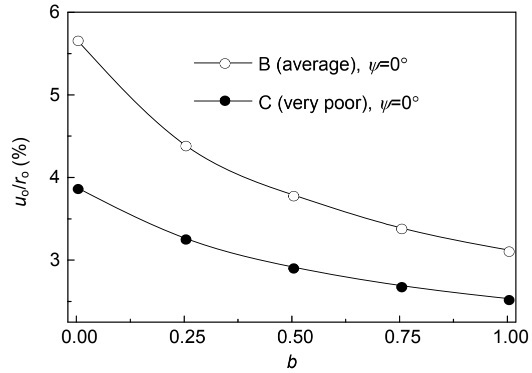

Fig. 6 Effect of *b* on the radial displacement (examples B and C)

R and u_o are reduced with the increasing unified failure criterion parameter b for all the cases analyzed (Figs. 4–6). R is decreased by 17.6%, 44.9% and 34.6% when b=1 compared with b=0 for examples A, B, and C, respectively. For example A, the case b=1, ψ=0° gives the minimal radial displacement, while the case b=0, ψ=30° gives the maximal one; i.e., the solutions for radial displacements based on the H-B failure criterion and considering the plastic volume increase, are found to be on the conservative side. It can be concluded that by proper use of the non-linear unified failure criterion, the latent potentialities of rock mass are achieved by considering the effect of intermediate principal stress. The u_o increases with a higher dilation angle that has no influence of R. Comparison of Figs. 5 and 6 shows that the variation of u_o with b for examples B and C is similar to that for example A; however, the predicted values are significantly higher.

The effects of parameter b are more significant on examples B and C than on A (Fig. 4). This may be attributed to the fact that different rock masses have different intermediate principal stress influences. Average and very poor rock masses have a pronounced effect of intermediate principal stress, whereas the intermediate principal stress plays a less significant role on very good rock mass. The results of example C are smaller than those of example B. This might be due to different material models between examples B and C.

Figs. 7 and 8 show the dimensionless stresses for examples A and C, respectively. For each rock example, three different values of b (0, 0.5 and 1) are considered.

From Figs. 7 and 8, the same tendency can be found with a different unified failure criterion parameter b. However the radial and circumferential stresses are significantly influenced by the parameter b. Therefore, the value of b should be chosen reasonably according to rock material parameters in tunnel design. Comparison of Figs. 7 and 8 shows that the material behavior model plays an important role in calculating stresses. With respect to the elastic-brittle-plastic softening model, the perfectly plastic model underestimates the radius of plastic zone and radial displacements (Figs. 4 and 6). So the tunnel design based on the perfectly plastic model is insecure. The material behavior model should be adopted with discretion and according to the rock material mechanical experiments.

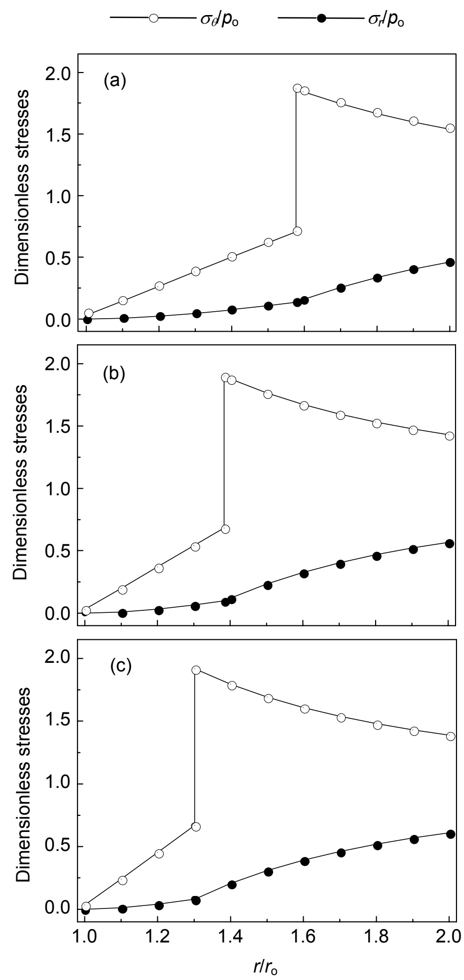

Fig. 7 The stresses for example A, elastic-brittle-plastic

(a) b=0; (b) b=0.5; (c) b=1

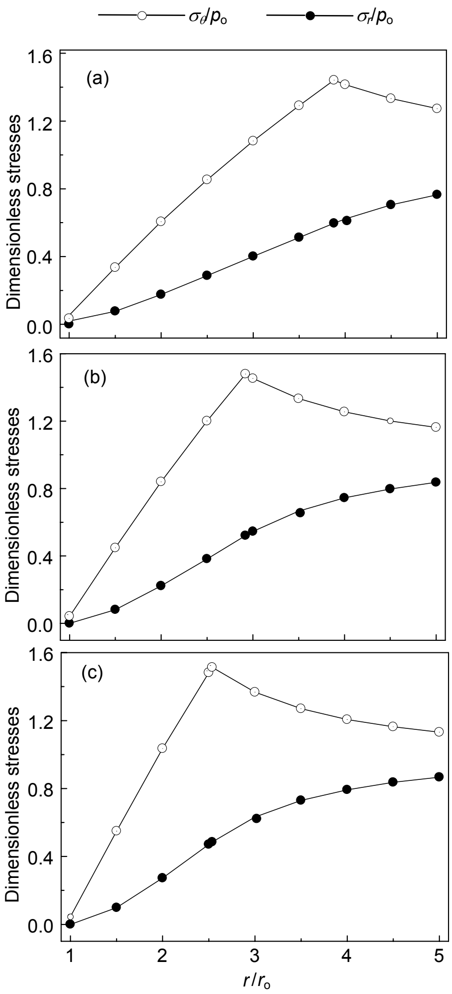

Fig. 8 The stresses for example C, perfectly plastic
(a) b=0; (b) b=0.5; (c) b=1

6 Conclusion

The unified analytical solutions of the stresses and displacements of a circular opening are presented based on a non-linear unified failure criterion and the elastic-brittle-plastic softening model. A non-associated linear flow rule is adopted in obtaining the radial displacements in the plastic zone.

The generally traditional solutions, based on the H-B failure criterion and the non-linear twin-shear failure criterion, are special cases of the unified solutions with the unified failure criterion parameter b=0 and b=1, respectively. The unified analytical solutions can compare with the generally traditional results and have a very important academic value.

The results of the radius of plastic zone, radial displacements and stresses are compared for three different rock masses, whose qualities range from very good to very poor, to reflect the effects of intermediate principal stress and material behavior model. For a given condition, the radius of plastic zone and radial displacements are higher for the unified failure criterion parameter b=0 than for b=1. The latent potentialities of rock mass are achieved due to considering the effect of intermediate principal stress. Radial displacements and stresses are significantly influenced by the failure criterion and material behavior model. It is concluded that proper choices of the failure criterion and material behavior model for rock mass are significant in tunnel design.

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