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Seismic response analysis of damper-connected adjacent structures with stochastic parameters*

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Abstract: Dynamic response analysis of damper connected adjacent multi-story structures with uncertain parameters is carried out. A formula of the multi degree of freedom (MDOF) for the structure-damper system with stochastic parameters is derived. The uncertainties of mass and stiffness are taken into consideration firstly. The ground acceleration is represented by Kanai-Tajimi filtered non-stationary process. The mean square random responses of structural displacement and story drift are chosen as the optimization objective. The variations of mean square responses of top floor displacements and bottom story drifts in neighboring structures with the damper stiffness and damping coefficient are analyzed in detail. Through the parametric study, the acquiring optimum parameters of damper are regarded as numerical results. Then, a reducing order model of the MDOF system for adjacent structures with mean parameters is presented. The explicit expressions for determining optimal parameters of Kelvin model-defined damper which is used to connect adjacent single degree of freedom (SDOF) structures subjected to a white-noise excitation are employed to achieve the appropriate damper parameters, which are called theory results. Through a comparative study, it can be found that the theory values of damper parameters are consistent with the results based on extensive parametric studies. The analytical results can be obtained by using the first natural frequencies and the total mass of the adjacent deterministic structures with mean parameters. The analytical formulas can be used to find appropriate parameters of damper between adjacent structures for engineering applications. The performance of damper is investigated on the basis of mitigations of mean square random responses of inter-story drifts, displacements and accelerations in adjacent structures. The numerical results demonstrate the robustness of coupled building control strategies.

Key words: Adjacent structures, Passive control, Stochastic parameters, Seismic response
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1 Introduction

In modern cities, buildings are often built closely to each other because of increasing population but limited land available. It is possible that pounding between adjacent structures in close distance may occur, when they are subjected to an intensive ground motion. In recent decades, numerous researchers have proposed various actively, passively, and semi-actively controlled devices to connect neighboring structures to make use of the interaction between them, which has been proved to be an effective approach to avoid collision between adjacent structures and to enhance the earthquake resistant ability of adjacent structures.

Xu *et al.* (1999) carried out a theoretical investigation on seismic resistant behavior of damper linked adjacent buildings under earthquake ground motion. The optimum parameters of the dampers were obtained through extensive numerical parametric studies. Zhang and Xu (1999; 2000) achieved the

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optimum parameters of Kelvin and Maxwell model defined dampers between adjacent structures through both parametric and sensitivity studies for mitigating the maximum seismic response of adjacent buildings. The passive devices between 2-single degree of freedom (SDOF) structures also have been studied in detail, that is, the connecting viscoelastic damper was characterized by the Kelvin model (Zhu and Iemura, 2000) and the connecting viscous fluid damper was represented by the Maxwell model (Zhu and Xu, 2005). Ni *et al.* (2001) proposed a method to analyze the random seismic responses of two adjacent structures linked by non-linear hysteretic dampers. Aida *et al.* (2001) performed a study on increasing the damping ratios of the first vibration modes of adjacent structures using a damping device to connect them. The connecting member consists of a spring and a damper. Bhaskararao and Jangid (2006a; 2006b) investigated the dynamic performances of two adjacent structures connected by friction dampers and excited by harmonic ground motion and various earthquakes. Kim *et al.* (2006) studied the performance of viscoelastic dampers installed between a building and a sky-bridge and across seismic joints. Basili and Angelis (2007a; 2007b) performed a research on passive control performance of adjacent structures linked by nonlinear hysteretic damping devices. Hwang *et al.* (2007) employed viscous dampers at the connection between the exterior and interior structures to enhance earthquake resistant performance of the factory structures.

In the reported studies of coupling structures control strategies, all the linked adjacent structures were regarded as deterministic structural systems. In practical engineering problems, not only do the external excitations, such as wind loading, seismic waves, demonstrate uncertainty, but also the structural parameters exhibit uncertainly. It was reported that the uncertainty of structural parameters may greatly influence the dynamic characteristics of structures (Ghanem and Spanos, 1991; Jensen and Iwan, 1992). Therefore, it is more reasonable to consider the variability of structural parameters in dynamic response analysis of structures.

In this paper, a research on optimal passive control of damper connected adjacent structures with random parameters was performed. At first, the optimum damper parameters were achieved by means of a parametric study, while the structure subjected to ground acceleration excitation represented by filtered white noise. While taking into account the uncertainties of mass and stiffness which have great influence on the dynamic characteristics of structure, the mean square responses of structure displacement and story drift were calculated over the stiffness and damping coefficient of the dampers. The analytical formula for the Kelvin model-defined damper connecting 2-SDOF structures was adopted to obtain the damper parameters by using a reducing order model of the adjacent structures with mean parameters. In other words, the uncertainties of mass and stiffness were no longer considered in the theoretical analysis. Subsequently, a detailed comparison of the optimum damper parameters obtained through analytical formulas and parametric study was carried out. Results illustrated that the theoretical and numerical values coincided with each other. At last, the uncertainties of damper damping coefficients were also taken into consideration, and the control effectiveness was studied based on the seismic response reductions of displacements, story drifts and accelerations for both structures.

2 Modeling of the 2-multi degree of freedom system

2.1 Fundamental assumptions

As shown in Fig. 1, it is supposed that two adjacent structures are symmetric with their symmetric planes coincided with each other. Like the assumptions in (Xu *et al.*, 1999; Zhang and Xu, 1999; 2000), the earthquake acceleration is considered to input in the direction of the symmetric planes, therefore, the problem can be regarded as a 1D problem. Both structures are assumed to be excited by the same seismic acceleration and the spatial effect of ground motion is neglected. The adjacent structures are supposed to remain at the stage of linear elasticity, because the seismic intensity is considered to be moderate and the connected dampers strengthen the seismic resistant and energy absorption capacities of adjacent structures. The story heights of both structures are uniform, but the total height of each structure can be taken different values. Each structure is simplified as a linear multi degree of freedom (MDOF) flexible shear type structure with lumped mass at the floor levels.

2.2 Basic equations

Fig. 1 displays a structure-damper system composed of two adjacent high-rise structures connected by dampers at some stories. In civil engineering, different force-deformation models are applied to capture the characteristics of various damping devices. Viscoelastic damper is proposed in this study. This damper produces damping force to dissipate energy depending on relative shear deformation in the viscoelastic polymer material. This damping device adds damping and stiffness to the structure. Each damper can be represented by the classical Kelvin-Voigt viscoelastic model which consists of a spring in parallel with the series combination of a dashpot.

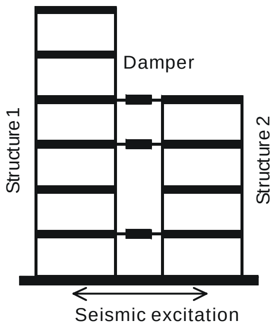

Fig. 1 Adjacent structures linked by dampers

The uncertainties of mass and stiffness have great influences on the dynamic characteristics of structure, their variability need to be considered first. The viscoelastic dampers enhance the energy dissipation ability of the coupled structures by increasing the modal damping of the adjacent structures. The modal damping ratios of the neighboring structures themselves are low, compared with the increasing modal damping ratios induced by damping devices (Xu *et al.*, 1999; Zhang and Xu, 1999; 2000). The damping coefficients greatly influence the effects of dissipating energy and vibration reduction of dampers compared to the damper stiffness which can only deliver energy. So the variability of damper damping coefficient also has to be taken into account. While the uncertainties of damper parameters and structural parameters are considered, the dynamic equilibrium equation of structural system with stochastic parameters can be written as

$$\left(\bar{\boldsymbol{m}}+\sum_{j=1}^{n_m}\boldsymbol{m}_j\xi_j\right)\ddot{\boldsymbol{y}}(t)+\left(\boldsymbol{c}+\bar{\boldsymbol{c}}_{\mathrm{D}}+\sum_{j=1}^{n_{c\mathrm{D}}}\boldsymbol{c}_{\mathrm{D}j}\xi_j\right)\dot{\boldsymbol{y}}(t)+\left(\bar{\boldsymbol{k}}+\sum_{j=1}^{n_k}\boldsymbol{k}_j\xi_j+\boldsymbol{k}_{\mathrm{D}}\right)\boldsymbol{y}(t)=-\left(\bar{\boldsymbol{m}}+\sum_{j=1}^{n_m}\boldsymbol{m}_j\xi_j\right)\boldsymbol{I}\ddot{u}(t), \tag{1}$$

where $\bar{\boldsymbol{m}}$ and $\boldsymbol{m}_j$ are the mean value and deviator components of mass matrix for structure, respectively; while $\bar{\boldsymbol{k}}$ and $\boldsymbol{k}_j$ are the expected value and deviator components of stiffness matrix for structure, respectively; $\bar{\boldsymbol{c}}_{\mathrm{D}}$ and $\boldsymbol{c}_{\mathrm{D}j}$ are the mean value and deviator components of damping matrix of damper, respectively; $\boldsymbol{k}_{\mathrm{D}}$ is the stiffness matrix of damper; n_m and n_k denote the numbers of stochastic variables in mass and stiffness matrices of adjacent structures, respectively; $n_{c\mathrm{D}}$ is the amount of uncertain variables in damping matrices of damper; ξ_j represent the spatial random variables; $\ddot{\boldsymbol{y}}$, $\dot{\boldsymbol{y}}$ and $\boldsymbol{y}$ are the random acceleration, velocity and displacement vectors, respectively; $\boldsymbol{I}$ is the load index vector with all its elements equal to 1; $\ddot{u}(t)$ is the applied acceleration of the ground.

To highlight the priorities and simplify the problem, it is supposed that the damping matrix of structure $\boldsymbol{c}$ and stiffness matrix of damper $\boldsymbol{k}_{\mathrm{d}}$ are the deterministic matrices. To make the problem manageable, it is assumed that all the elements in $\boldsymbol{m}$, $\boldsymbol{k}$ and $\boldsymbol{c}_{\mathrm{d}}$ are related to three stochastic coefficients, ξ_1, ξ_2 and ξ_3, respectively. Three stochastic coefficients are independent from each other and satisfy uniform distribution. Subsequently, the matrix $\boldsymbol{m}$, $\boldsymbol{k}$ and $\boldsymbol{c}_{\mathrm{d}}$ can be regarded as a sum of its mean matrix and another random matrix, respectively, namely

$$\boldsymbol{m}=\bar{\boldsymbol{m}}+\xi_1\boldsymbol{m}_{\mathrm{r}},\quad \boldsymbol{k}=\bar{\boldsymbol{k}}+\xi_2\boldsymbol{k}_{\mathrm{r}},\quad \boldsymbol{c}_{\mathrm{D}}=\bar{\boldsymbol{c}}_{\mathrm{D}}+\xi_3\boldsymbol{c}_{\mathrm{Dr}}, \tag{2}$$

where $\bar{\boldsymbol{m}}$, $\bar{\boldsymbol{k}}$, $\bar{\boldsymbol{c}}_{\mathrm{D}}$, $\boldsymbol{m}_{\mathrm{r}}$, $\boldsymbol{k}_{\mathrm{r}}$, and $\boldsymbol{c}_{\mathrm{Dr}}$ are all deterministic matrices.

3 Dynamic analysis of structure with random parameters

For the dynamic response analysis of random systems, there are three kinds of mathematical algorithms available. The first one is the Monte-Carlo simulation method (Astill *et al*., 1972), which is robust but very time-consuming. The second one is the stochastic perturbation method, owing to the notorious secular term, which is only applied to systems with random variables varied within a small range (Zhu and Wu, 1992). The third one is the orthogonal polynomial expansion method with high precision. In this algorithm, the determination of response of the original random system is converted to the determination of response of an equivalent order-expanded deterministic system (Li, 1996; Li and Liao, 2001), but the dimension number of the problem increased with the variable number dramatically. The dimensional explosion limits the engineering application of the orthogonal polynomial expansion method. For random variables with different distributions, the selection of the corresponding orthogonal basis relies on the probability density function. The orthogonal polynomial expansion method is chosen for the dynamic response analysis of the structure-damper system with random parameters in this investigation.

Generally, the random variables in stochastic system can be regarded as uncertain parameters with deterministic statistical characteristics (Zhu and Wu, 1992; Li, 1996). The random variables of engineering structures are bounded factually. Taking the normal probability distribution assumption for uncertain variables deviates from reality. When the uncertain variables take sufficiently small negative values, the sample system become unstable. The uniform distribution model for uncertain parameters can avoid the instability of system. The fluctuation range of uncertain parameters is from –1 and +1. In accordance with this probability density function, the Legendre polynomials can be selected as the corresponding orthogonal basis (Li, 1996).

3.1 Legendre polynomials

The general mathematic formula of Legendre polynomials can be expressed as

$$P_n(\xi)=\sum_{k=0}^{n/2}(-1)^k\frac{(n-k)!}{k!(n-2k)!}(2\xi)^{n-2k}. \quad (3)$$

The most important properties of the orthogonal polynomials are their recurrence relationship. The recurrent formula for normalized Legendre polynomials is

$$\xi P_n(\xi)=\alpha_n P_{n+1}(\xi)+\beta_n P_n(\xi)+\gamma_n P_{n-1}(\xi), \quad (4)$$

where α_n, β_n and γ_n are deterministic coefficients and can be respectively expressed as

$$\alpha_n=\frac{n}{\sqrt{4n^2-1}},\ \beta_n=0,\ \gamma_n=\frac{n+1}{\sqrt{4n^2+8n+3}}. \quad (5)$$

3.2 Orthogonal polynomial expansion method

In this algorithm, the structural response is expanded as a series of orthogonal polynomials. The structure response vectors in Eq. (1) can be decomposed in probabilistic subspaces expanded by random variables ξ_1, ξ_2 and ξ_3. Supposing the truncation number of polynomials for random variables ξ_1, ξ_2 and ξ_3 are n_1, n_2 and n_3, respectively, $\boldsymbol{y}(t)$ can be expressed as

$$\boldsymbol{y}(t,\xi)=\sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\sum_{l_3=0}^{n_3}\boldsymbol{y}_{l_1l_2l_3}(t)P_{l_1}(\xi_1)P_{l_2}(\xi_2)P_{l_3}(\xi_3), \quad (6)$$

where $\boldsymbol{y}_{l_1l_2l_3}(t)$ are spatial random variables and also unknown generalized response vectors with randomness. The subscripts l_1, l_2 and l_3 are the sequence numbers for ransacking of n_1, n_2 and n_3, respectively. Substituting Eqs. (6) and (2) into Eq. (1), and then utilizing the above recurrent formula and the orthogonal relationship of normalized Legendre polynomials, Eq. (1) can be rewritten as

$$\begin{aligned}&\left(\bar{\boldsymbol{m}}\frac{\mathrm{d}^2}{\mathrm{d}t^2}+(\boldsymbol{c}+\bar{\boldsymbol{c}}_{\mathrm{D}})\frac{\mathrm{d}}{\mathrm{d}t}+\bar{\boldsymbol{k}}\right)\sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\sum_{l_3=0}^{n_3}\boldsymbol{y}_{l_1l_2l_3}(t)P_{l_1}(\xi_1)P_{l_2}(\xi_2)P_{l_3}(\xi_3)\\&+\boldsymbol{m}_{\mathrm{r}}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\sum_{l_3=0}^{n_3}\boldsymbol{y}_{l_1l_2l_3}(t)[\alpha_{l_1}P_{l_1-1}(\xi_1)+\gamma_{l_1}P_{l_1+1}(\xi_1)]P_{l_2}(\xi_2)P_{l_3}(\xi_3)\\&+\boldsymbol{c}_{\mathrm{Dr}}\frac{\mathrm{d}}{\mathrm{d}t}\sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\sum_{l_3=0}^{n_3}\boldsymbol{y}_{l_1l_2l_3}(t)[\alpha_{l_3}P_{l_3-1}(\xi_2)+\gamma_{l_3}P_{l_3+1}(\xi_2)]P_{l_1}(\xi_1)P_{l_2}(\xi_3)\\&+\boldsymbol{k}_{\mathrm{r}}\sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\sum_{l_3=0}^{n_3}\boldsymbol{y}_{l_1l_2l_3}(t)[\alpha_{l_2}P_{l_2-1}(\xi_2)+\gamma_{l_2}P_{l_2+1}(\xi_2)]P_{l_1}(\xi_1)P_{l_3}(\xi_3)\\&\qquad=-(\bar{\boldsymbol{m}}+\boldsymbol{m}_{\mathrm{r}}\xi_1)\boldsymbol{I}\ddot{u}(t).\end{aligned} \quad (7)$$

Multiply both sides of the equation by $P_{r_1}(\xi_1)P_{r_2}(\xi_2)P_{r_3}(\xi_3)$ in sequence. Then take the expectations of ξ_1, ξ_2 and ξ_3. Note that $\xi_1=P_1(\xi_1)/\sqrt{3}$.

Making use of the orthogonal relationships of the normalized Legendre Polynomials, the equivalent certainty system of Eq. (7) can be derived as follows:

$$\begin{aligned}&\left(\bar{\boldsymbol{m}}\frac{\mathrm{d}^2}{\mathrm{d}t^2}+(\boldsymbol{c}+\bar{\boldsymbol{c}}_{\mathrm{D}})\frac{\mathrm{d}}{\mathrm{d}t}+\bar{\boldsymbol{k}}\right)\boldsymbol{y}_{r_1r_2r_3}(t)\\&\quad+\boldsymbol{m}_{\mathrm{r}}\frac{\mathrm{d}^2}{\mathrm{d}t^2}[\alpha_{r_1}\boldsymbol{y}_{r_1-1,r_2,r_3}(t)+\gamma_{r_1}\boldsymbol{y}_{r_1+1,r_2,r_3}(t)]\\&\quad+\boldsymbol{c}_{\mathrm{Dr}}\frac{\mathrm{d}}{\mathrm{d}t}[\alpha_{r_3}\boldsymbol{y}_{r_1-1,r_2,r_3}(t)+\gamma_{r_3}\boldsymbol{y}_{r_1,r_2,r_3+1}(t)]\\&\quad+\boldsymbol{k}_{\mathrm{r}}[\alpha_{r_2}\boldsymbol{y}_{r_1,r_2-1,r_3}(t)+\gamma_{r_2}\boldsymbol{y}_{r_1,r_2+1,r_3}(t)]\\&=-\bar{\boldsymbol{m}}\boldsymbol{I}\ddot{u}(t)\delta_{0r_1}\delta_{0r_2}\delta_{0r_3}-\frac{1}{\sqrt{3}}\delta_{1r_1}\delta_{0r_2}\delta_{0r_3}\boldsymbol{m}_{\mathrm{r}}\boldsymbol{I}\ddot{u}(t),\end{aligned}\tag{8}$$

where the subscripts r_1=0, 1, …, n_1, r_2=0, 1,…, n_2, r_3=0, 1, …, n_3, and δ is the Kronecker symbol.

As Eq. (8) represents a known order-expanded system, any efficient method for the solution of a deterministic system can be employed to determine the covariance matrices of $\boldsymbol{y}_{r_1r_2r_3}(t)$. The total amount of the equations is n_{123}=(n_1+1)(n_2+1)(n_3+1). Combining the n_{123} sets of equations, the global order-expanded equation can be expressed as

$$\boldsymbol{B}_m\ddot{\boldsymbol{y}}_{r_1r_2r_3}(t)+\boldsymbol{B}_c\dot{\boldsymbol{y}}_{r_1r_2r_3}(t)+\boldsymbol{B}_k\boldsymbol{y}_{r_1r_2r_3}(t)=-\boldsymbol{B}_g\boldsymbol{I}\ddot{u}(t),\tag{9}$$

where $\boldsymbol{B}_m$, $\boldsymbol{B}_c$ and $\boldsymbol{B}_k$ denote the mass, damping and stiffness matrices for the equivalent order-expanded system, respectively, and $\boldsymbol{B}_g$ is an order-expanded load matrix.

Now all the order-expanded matrices are known, and the solution of Eq. (9) can be obtained using any classic vibration analysis methods. Once the dynamic responses of order-expanded systems are achieved, the mean value, variance and mean square value can be obtained through the orthogonal relationship of Legendre Polynomials. For example, the mean value of displacement $\boldsymbol{y}(t,\xi)$ can be evaluated as follows:

$$\begin{aligned}E(\boldsymbol{y}(t,\xi))&=\sum_{r_1=0}^{n_1}\sum_{r_2=0}^{n_2}\sum_{r_3=0}^{n_3}E(\boldsymbol{y}_{r_1r_2r_3}(t)P_{r_1}(\xi_1)P_{r_2}(\xi_2)P_{r_3}(\xi_3))\\&=\boldsymbol{y}_{000}(t),\end{aligned}\tag{10}$$

where $E(\ )$ means the expectation operation.

The covariance matrix of the mean square random responses of $\boldsymbol{y}(t,\xi)$ can be expressed as

$$\begin{aligned}\mathrm{cov}(\boldsymbol{y}(t,\xi),\boldsymbol{y}(t,\xi))&=E(\boldsymbol{y}(t)\boldsymbol{y}(t)^{\mathrm{T}})-E(\boldsymbol{y}(t))E(\boldsymbol{y}(t)^{\mathrm{T}})\\&=\sum_{r_1=1}^{n_1}\sum_{r_2=1}^{n_2}E(\boldsymbol{y}_{r_1r_2r_3}(t)\boldsymbol{y}_{r_1r_2r_3}(t)^{\mathrm{T}}).\end{aligned}\tag{11}$$

Note that the formula above based on matrix descriptions, the diagonal elements of the matrix are just the variances of the random response $\boldsymbol{y}(t,\xi)$.

3.3 Response of system under filtered white noise excitation

In earthquake engineering, the earthquake excitation usually characterized by the well-known Kanai-Tajimi filtered white noise spectrum (Housner, 1955), which is given as

$$S_g(\omega)=\frac{\omega_g^4+4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\zeta_g^2\omega_g^2\omega^2}S_0,\tag{12}$$

where ω_g and ζ_g represent the characteristic parameters of the soil surrounding the adjacent structures; S_0 denotes the intensity of the seismic ground motion.

Lin *et al*. (1994) put forward a pseudo-excitation algorithm, which provides a useful method for dynamic response analysis of complex engineering structures under random excitations. The earthquake random excitations can be converted to a series of harmonic excitations. The pseudo-excitation is constituted as follows:

$$\ddot{u}_g(t)=\sqrt{S_g(\omega)}\mathrm{e}^{\mathrm{i}_0\omega t}.\tag{13}$$

The displacement response vector for the deterministic system of Eq. (9) subjected to the pseudo-excitation is

$$\boldsymbol{y}_{r_1r_2r_3}(\omega)=-\boldsymbol{B}_g\boldsymbol{I}\sqrt{S_g(\omega)}[-\omega^2\boldsymbol{B}_m+\mathrm{i}_0\omega\boldsymbol{B}_c+\boldsymbol{B}_k]^{-1}.\tag{14}$$

While the adjacent structures excited by stochastic seismic ground motion, mean square random response is an effective index to evaluate the statistical characteristics of the stochastic earthquake responses for the structures (Fang *et al*., 2003). The mean square random response of $y(\omega,\xi)$ can be expressed as

$$S_{yy}(\omega)=\sum_{r_1=0}^{n_1}\sum_{r_2=0}^{n_2}\sum_{r_3=0}^{n_3}\boldsymbol{y}^{*}_{r_1r_2r_3}(\omega)\boldsymbol{y}^{\mathrm{T}}_{r_1r_2r_3}(\omega), \tag{15}$$

where the superscript * denotes the conjugate operation. For each floor of the adjacent structures, the mean square deviation of displacement response is achieved by means of the numerical integration of its mean square value. For example, the mean square deviation of displacement response for the first floor of Structure 1 can be expressed as

$$\sigma_{y_1}=\left[\int_{-\infty}^{+\infty}S_{y_1y_1}(\omega)\mathrm{d}\omega\right]^{\frac{1}{2}}. \tag{16}$$

4 Reduced order model of adjacent structure with mean parameters

4.1 Equations of motion of 2-single degree of freedom system

In practical engineering application, the mass, stiffness and damping of structures are usually regarded as deterministic quantities, which are the mean values of them. In other words, in the elastic design method of civil engineering structures, the mass, stiffness and damping matrices are deterministic. Some simplified approaches are usually used to deal with practical structures. For example, to approximate the motion of Structure 1 or Structure 2 (Fig. 1) with an SDOF, it is necessary to assume that it will deform only in a single shape (Clough and Joseph, 2004). The shape function of Structure 1 and Structure 2 will be designated as $\boldsymbol{\psi}_i$ (i=1,2) which must satisfy the displacement boundary conditions, and the amplitude of the motion relative to the moving base will be represented by the generalized coordinate $z_i(t)$, which are the generalized displacements. The response of each mass point can be given in terms of the generalized displacement as

$$\boldsymbol{u}_i(t)=\boldsymbol{\psi}_i z_i(t). \tag{17}$$

Basili and Angelis (2007a; 2007b) determined the optimal design of 2-MDOF structures coupled by hysteretic devices. By using the principle of virtual displacement, the original 2-MDOF model of adjacent structures was substituted with a generalized 2-SDOF reduced order model. The design procedure proved that optimal parameter of damper shows little dependence on the supposed deformed shape.

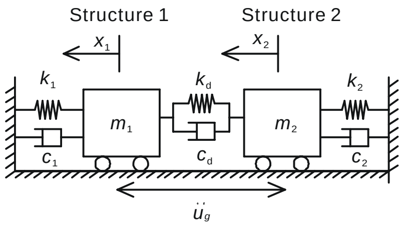

Fig. 2 Structural model of 2-SDOF structures linked by damper

The generalized SDOF system of Structure 1 or Structure 2 is obtained through the principle of virtual displacement. The virtual-work principle requires that the external virtual work, which is performed by the external loadings acting through their corresponding virtual displacements, is equated to the internal virtual work, and the generalized equation of motion is expressed as follows:

$$m_i^*\ddot{z}(t)+c_i^*\dot{z}(t)+k_i^*z(t)=-L_i\ddot{u}(t), \tag{18}$$

where $m_i^*=\boldsymbol{\psi}_i^{\mathrm{T}}\overline{\boldsymbol{m}}_i\boldsymbol{\psi}_i$, $c_i^*=\boldsymbol{\psi}_i^{\mathrm{T}}\overline{\boldsymbol{c}}_i\boldsymbol{\psi}_i$, $k_i^*=\boldsymbol{\psi}_i^{\mathrm{T}}\overline{\boldsymbol{k}}_i\boldsymbol{\psi}_i$, $L_i=\boldsymbol{\psi}_i^{\mathrm{T}}\overline{\boldsymbol{m}}_i\boldsymbol{I}_i$ (i=1,2). $\boldsymbol{\psi}_i$ is chosen as the first-order mode shape to represent the shape function of structure i; $\overline{\boldsymbol{m}}_i$, $\overline{k}_i$ and $\overline{\boldsymbol{c}}_i$ are the mass, stiffness, and damping matrices of structure i, respectively; L_i is the load vector. By considering a reduced order model of damper connected 2-SDOF structures (Fig. 2), and excited by the base acceleration $\ddot{u}_g(t)$, the equations of the system are given as

$$m_1\ddot{x}_1(t)+(c_1+c_\mathrm{d})\dot{x}_1(t)+(k_1+k_\mathrm{d})x_1(t) \\ -k_\mathrm{d}x_2(t)-c_\mathrm{d}\dot{x}_2(t)=-m_1\alpha_1\ddot{u}_g(t), \tag{19a}$$

$$m_2\ddot{x}_2(t)+(c_2+c_\mathrm{d})\dot{x}_2(t)+(k_2+k_\mathrm{d})x_2(t) \\ -k_\mathrm{d}x_1(t)-c_\mathrm{d}\dot{x}_1(t)=-m_2\alpha_2\ddot{u}_g(t), \tag{19b}$$

where $c_i=2\zeta_i\sqrt{k_im_i}$, $k_i=m_ik_i^*/m_i^*$, $\alpha_i=L_i/m_i^*$, m_i^* and k_i^* are the generalized mass and stiffness of Structure i, respectively, and m_i and ζ_i are the total mass and the first modal damping ratio of Structure i, respectively. c_d and k_d are the damping coefficient and stiffness coefficient of Kelvin model-defined damper, respectively.

4.2 Analytical formulas for optimum damper parameters

Zhu and Iemura (2000) regarded two adjacent structures as 2-SDOF damper systems, and the connecting damper was defined by the Kelvin model. Using the statistical energy analysis method, and not considering the damping coefficients of the two adjacent structures, the optimum parameters of damper are determined under the objective that minimizes the total vibration energy of the adjacent structures excited by stationary white noise ground motion. The explicit expressions of the optimal parameters of dampers are expressed as a formula of the mass ratio and the natural frequency ratio of 2-SDOF structures.

In this study, the frequency ratio is restricted to $\beta\le 1$. The expressions also can be applied to the case of $\beta>1$ only requiring reciprocal exchange of the two structures. Correspondingly, the optimal stiffness coefficient $\beta_{\rm opt}$ and optimal damping coefficient $\varDelta_{\rm opt}$ connecting Kelvin model-defined damper can be expressed as follows:

1. When $\mu\ge 1$:

$$\beta_{\rm opt}=\frac{(\mu-1)(1-\beta^2)}{(1+\mu)^2}, \tag{20a}$$

$$\varDelta_{\rm opt}=\frac{(1-\beta^2)}{2(1+\mu)}\sqrt{\frac{2\mu}{(1+\mu)(\mu+\beta^2)}}; \tag{20b}$$

2. When $\mu<1$:

$$\beta_{\rm opt}=0, \tag{20c}$$

$$\varDelta_{\rm opt}=\frac{1-\beta^2}{2(1+\mu)}\sqrt{\frac{(1+\mu^2)}{(1+\mu)(\mu+\beta^2)}}, \tag{20d}$$

where $\mu=m_1/m_2$ is the total mass ratio of Structure 1 to Structure 2, and $\beta=\omega_2/\omega_1$ is the modal frequency ratio of Structure 2 to Structure 1. In fact, ω_1 and ω_2 are the best approximate results of the lowest frequencies of Structure 1 and Structure 2, since they employ the generalized-coordinate concept to reduce the adjacent structures to 2-SDOF systems, and the first vibration mode is the most dominant mode for the horizontal movement of structure. The optimum parameters of stiffness and damping are expressed as

$$k_{\rm d\ opt}=\beta_{\rm opt}\omega_1^2 m_1, \tag{21a}$$

$$c_{\rm d\ opt}=2\varDelta_{\rm opt}m_1\omega_1. \tag{21b}$$

In 2-SDOF damper systems, the explicit formulas of Kelvin model-defined damper are just expressed by the frequency ratio and mass ratio. Considering only the horizontal translation in *X* direction, the first mode shape is the dominant mode shape in MDOF system. It is natural to suppose that the total masses and first frequencies of 2-MDOF adjacent structures are closely related to the parameters of linking dampers. Ge *et al.* (2008) verified that the optimal damper parameters can be obtained using the total mass and first natural frequencies of the adjacent deterministic structures, which are regarded as 2-MDOF system. But if the uncertainties of parameters in adjacent structures are taken into consideration, whether the analytical formulas can be used to obtain the proper damper properties remains a problem. It is necessary to discuss the assumption that the total masses and first frequencies of 2-MDOF adjacent structures with mean parameters are used to obtain the proper damper properties.

5 Numerical example

Two adjacent 20-story high-rise steel structures with the same story heights are considered in this investigation. The dampers are installed between two neighboring stories. The mass and shear stiffness of each floor are identical for Structure 1. The mean values of the mass and shear stiffness are 1.29×10^6 kg and 4.0×10^9 N/m, respectively. For Structure 2, the expected value of mass of each story are the same as Structure 1, however, the shear stiffness is taken as 2.0×10^9 N/m only on average. Therefore, the dynamic behavior of Structure 1 is different from Structure 2 obviously. The damping matrices of the two structures are deterministic and achieved on the basis of the mean value of the mass and shear stiffness. The Rayleigh proportional damping is adopted in this study and it is assumed that both the first and the second modal damping ratios of the two structures are 0.02. In the subsequent analysis, the uncertainty of the mass of the adjacent structures is described as uniform random field with a variability coefficient δ_m.

Both the shear stiffness of Structure 1 and Structure 2 satisfy the same uniform distribution. The variability coefficients δ_m and δ_k are assumed to be 0.2. In the Kanai-Tajimi model defined spectrum of ground motion, the earthquake intensity S_0 is selected as 4.65×10^{-4} m^2/(rad·s^3), and the characteristic parameters in spectral density functions are taken as ζ_g=0.6 and ω_g=15.0 rad/s (Housner, 1955; Xu *et al.*, 1999).

5.1 Parametric study

In this study, the total number of dampers with uniform property is selected to be 20. Ten of the coupling dampers are installed at the top floor level. Another 10 dampers locate in 10th floor level.

To minimize the random seismic responses of two adjacent stochastic structures, parametric studies are carried out to find appropriate stiffness and damping coefficient of damper. Taking several different representative values of damper damping coefficient, the influence of damper stiffness on seismic response reduction is studied, and the optimal damper stiffness is achieved. Using the Kanai-Tajimi excitation spectrum, the variations of the mean square deviations of inter-story drift of the bottom floor with damper stiffness for Structures 1 and 2 are obtained and shown in Figs. 3a and 4a, respectively. The mean square deviation of top floor displacement changes with damper stiffness for Structure 1 and Structure 2 are also obtained and depicted in Figs. 3b and 4b, respectively.

Figs. 3a and 3b show that both inter-story drift and displacement of Structure 1 are greatly mitigated while the stiffness of damper is less than 1.0×10^6 N/m. Both the figures demonstrate that the control performance deteriorates rapidly if the damper stiffness is larger than 6.0×10^6 N/m, and the appropriate damper stiffness ranges from 1.0×10^6 to 3.0×10^6 N/m. Considering the dynamic responses of Structure 2, it

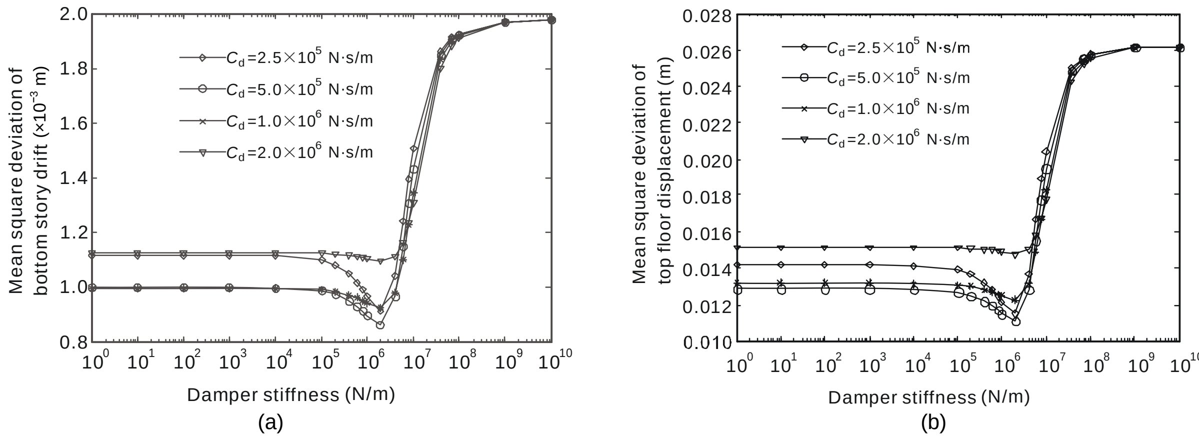

Fig. 3 Variations of mean square random response of the (a) first floor and (b) top floor of Structure 1 with the damper stiffness

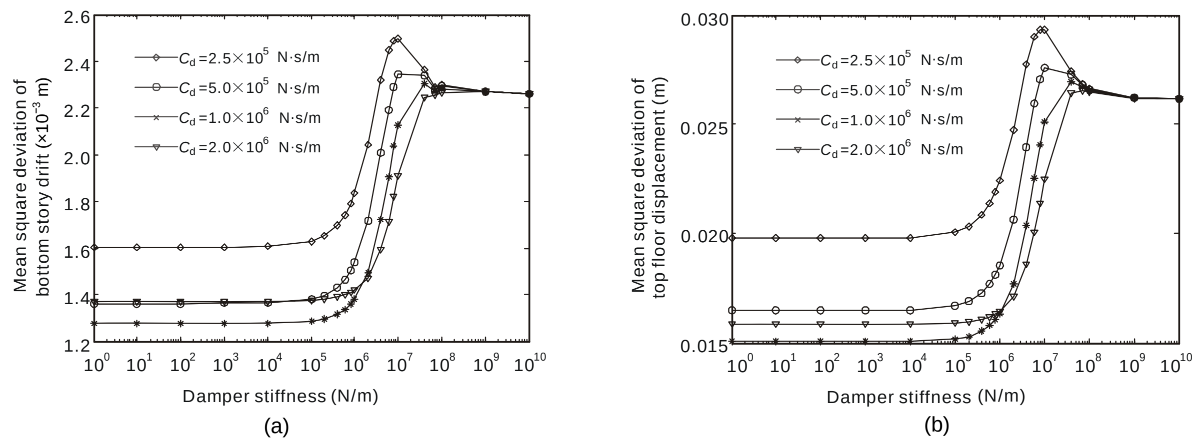

Fig. 4 Variations of mean square random response of the (a) first floor and (b) top floor of Structure 2 with the stiffness of each damper

can be clearly seen that the optimal damper stiffness is less than 1.0×10^5 N/m from Figs. 4a and 4b, and the control effectiveness of dampers reduces significantly when damper stiffness begins to increase from 1.0×10^5 N/m. It also can be found that the mean square random response are almost not influenced by damper over a wide range, while the value of damper stiffness is not more than 1.0×10^5 N/m. If the random seismic responses of Structure 1 and Structure 2 are taken into account comprehensively, the proper damper stiffness should be not larger than 1.0×10^5 N/m.

The optimal damper stiffness is selected as 0. To achieve the optimum damping coefficient of damper, the mean square deviations of bottom story drift and top floor displacement random responses are calculated against a large range of damping coefficients. Figs. 5 and 6 reveal the results in terms of random responses of bottom inter-story drift and top floor displacement, respectively. All the curves demonstrate that the mean square deviations of top floor displacements and bottom inter-story drifts of both structures are reduced to the minimum while the optimum value of damping coefficient is taken as 1.0×10^6 N·s/m, and if damping coefficient increases or decreases from the optimal value, the control effectiveness becomes worse gradually.

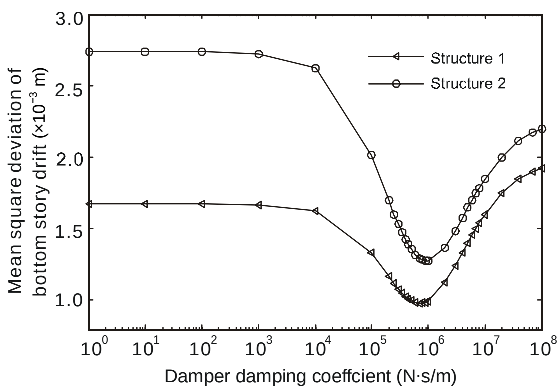

Fig. 5 Random responses of bottom story drifts of adjacent structures vary with damper damping coefficient

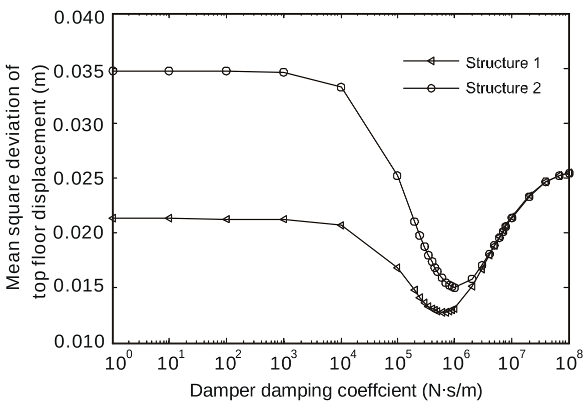

Fig. 6 Displacement responses of the top floors change with the damper damping coefficient for adjacent structures

In summary, for all the 20 dampers, the sum of stiffness is no more than 2.0×10^6 N/m and the sum of damping coefficients is about 2.3×10^7 N·s/m. The damping coefficient exerts more tremendous influence on the performances of dampers compared to the damper stiffness. Damper stiffness can deliver energy between the adjacent structures, but have no effect on the passive energy dissipation. However, the control performance shows low sensitivity to damper stiffness in a certain range. This conclusion would benefit the application of viscoelastic dampers in coupling structure control strategies. Therefore, the variability of damper stiffness is not considered in this study.

5.2 Theory results

If the 20-story adjacent structures are considered as deterministic structures here, the first natural frequencies of the Structure 1 and Structure 2 are 4.27 and 3.02 rad/s, respectively. Both the total mass of the two structures are 2.58×10^7 kg. Using the optimal parameter expression of the Kelvin model-defined element which is used to interconnect 2-SDOF deterministic systems, the optimal damping coefficient is 2.25×10^7 N·s/m and the optimal stiffness is 0. This means that the sum of damping coefficient of all the dampers between Structure 1 and Structure 2 is about 2.25×10^7 N·s/m. As mentioned above, the sum of all the damper damping coefficients is 2.2×10^7 N·s/m based on parametric study, which is consistent with the theory result. In fact, when the total damper stiffness is less than 2.0×10^6 N/m, the seismic response mitigation of adjacent structures is not sensitive to damper stiffness as mentioned above.

Xu *et al.* (1999) performed a research on earthquake resistant behavior of adjacent deterministic structures coupled by Kelvin model-defined viscoelastic dampers, the mass and stiffness coefficients of adjacent structures are the same as those used in this study. In Xu *et al.* (1999), all the identical dampers distributed uniformly along the floors. The optimization target was selected to maximize the modal damping of adjacent structures. Through considerable parametric studies, the optimum value of damping coefficient and stiffness are identified to be 1.0×10^6 N·s/m and 1.0×10^5 N/m for each damper, respectively. Consequently, the sum of all the damping coefficients is 2.0×10^7 N·s/m and the sum of total stiffness is 2.0×10^6 N/m. Obviously, the theory result of 2.25×10^7 N·s/m is very close to the result through parametric studies for the sum of total damping coefficients.

In a word, the optimum stiffness and damping coefficient of damper obtained through deterministic reducing order model are basically in accordance with those based on complicated parametric analysis. All the results and comparative analyses verify the practicability of using the first natural frequencies and the total mass of the original structures directly to achieve the optimal value of connecting damper for 2-MDOF systems in the practical engineering, even if the stochastic parameters in the 2-MDOF systems are considered. In the subsequent analysis, the damping coefficient of each damper is taken as 1.125×10^6 N·s/m without consideration of damper stiffness on the basis of theory results.

As compared with the stiffness of damper, the control effectiveness of damper is much more sensitive to the damping coefficient. The damping coefficient of damper is also a random parameter actually. Further research about the performance of damper with variable damping coefficient is necessary. In this investigation, we suppose that the variability coefficient δ_{cd} of damping coefficient is 0.2 and satisfies the same uniform distribution for each damper.

Using the Kanai-Tajimi spectral density function, Fig. 7a shows the mean square deviation of inter-story drift at each floor of 20-story adjacent structures with and without dampers. The mean square deviation of bottom story drift of the unlinked Structure 1 is 1.68 mm, but it is decreased to 1.01 mm with the percentage reduction of 66%, with the installation of dampers. For Structure 2, the mean square deviations of bottom story drift are 2.73 mm and 1.28 mm for the unlinked and linked structures, respectively, leading to a 53% reduction. It also can be found that the mean square deviations of inter-story drifts in all the stories are mitigated significantly after both structures are controlled. The variations of mean square deviation of acceleration response with the structure height are depicted in Fig. 7b. The mean square deviations of top floor accelerations are 0.43 and 0.51 m/s^2 for unconnected Structure 1 and Structure 2, respectively. After controlling, the random response of top floor acceleration is decreased to 0.26 m/s^2 for Structure 1 and 0.29 m/s^2 for Structure 2, reductions up to 40% and 43% are achieved, respectively. For all the other stories of both connected structures, the mean square deviations of acceleration responses are mitigated obviously.

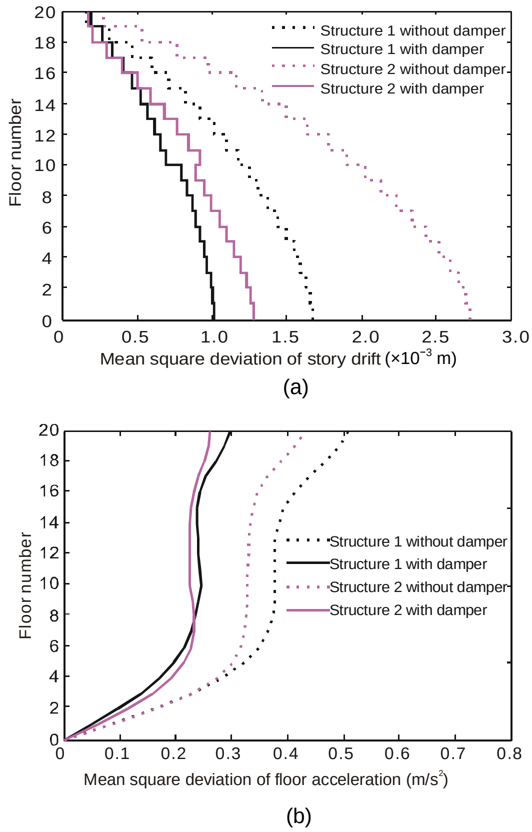

Fig. 7 Mean square deviation of (a) story drift and (b) acceleration response of 20-story adjacent structures

5.3 Results in time domain

The further work is to study the seismic resistant behavior of damper coupled adjacent structures in the time domain. Three earthquake records are used in this research, the NS components of the 1940 El Centro record, the N21E components of the 1952 Taft accelerogram and the NS components of the 1995 Kobe earthquake, with the maximum ground acceleration scaled to 0.2*g*.

The numerical results for standard deviation of top floor displacement of Structure 1 without and with control are described in Fig. 8, which shows the curves derived from the Monte-Carlo method coincide with that from the Orthogonal expansion method well, especially for the adjacent structures with dampers. In the simulation method, the sample number is selected as 20000, and all the samples for ξ_1, ξ_2 and ξ_3 satisfy uniform distribution on the interval [–1, 1]. In the Orthogonal expansion method, the truncation number of polynomials for every random variable is taken as 5. The comparison curves show the high calculation accuracy of the analysis program for solving dynamic response of random structures.

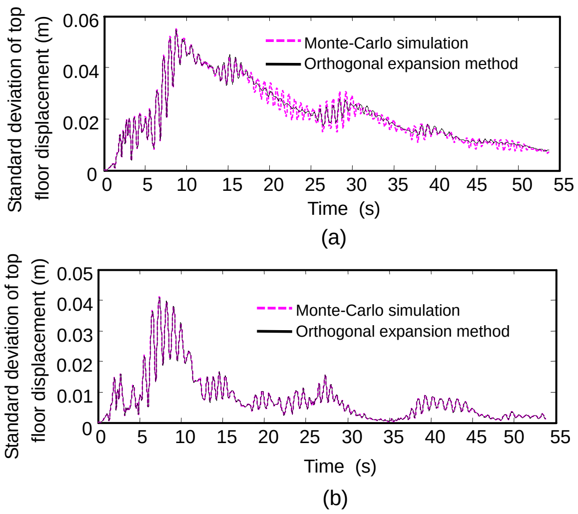

Fig. 8 Time histories of top floor displacement of Structure 1 excited by El Centro acceleration (a) without damper and (b) with damper

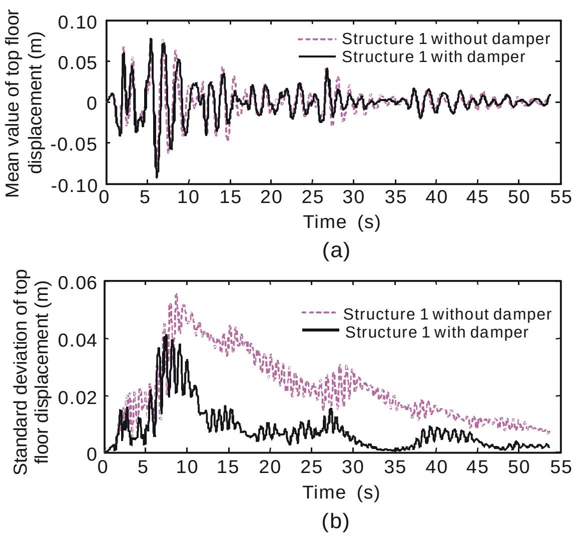

Fig. 9 Mean value (a) and standard deviation (b) of top floor displacement versus time for Structure 1 under El Centro earthquake

The top floor displacements of Structure 1 and Structure 2 change with time, which are revealed in Figs. 9 and 10, respectively. It can be observed that there is a certain decrease of the mean value of top floor displacement in the connected structures, comparing the time-history curves with uncontrolled structures. But the standard deviations of top floor displacement in both structures are significantly reduced. The standard deviations of bottom story drifts and top floor accelerations of two neighboring structures exhibit the same trend, which are plotted in Figs. 11 and 12.

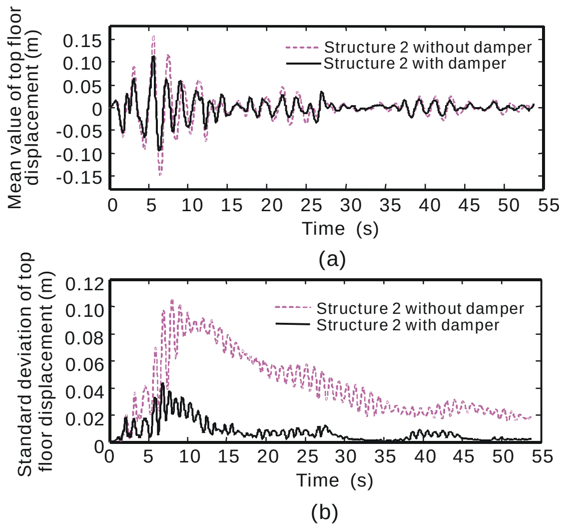

Fig. 10 Mean value (a) and standard deviation (b) of top floor displacement versus time for Structure 2 under El Centro earthquake

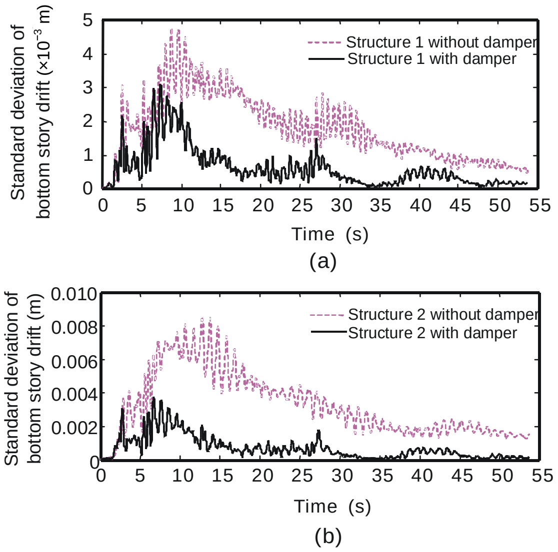

Fig. 11 Bottom story drift versus time for (a) Structure 1 and (b) Structure 2 under El Centro earthquake without damper and with damper

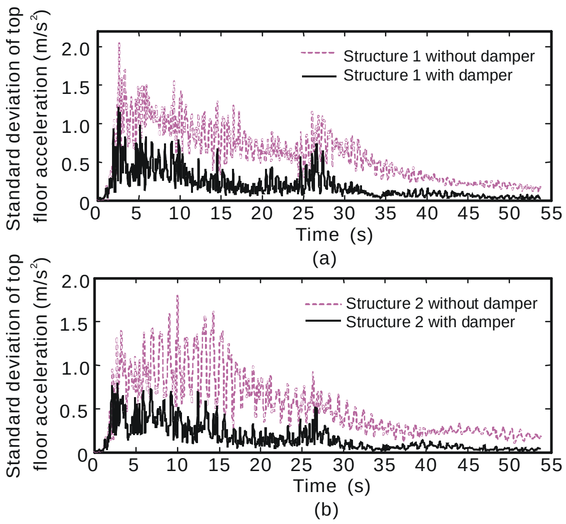

Fig. 12 Top floor acceleration versus time for (a) Structure 1 and (b) Structure 2 under El Centro earthquake without damper and with damper

Table 1 lists the percentage reduction of standard deviation peaks of random responses in Structure 1 and Structure 2 under various earthquake excitations. In general, the control effectiveness of dampers change with earthquakes, it can be found that while the input acceleration record is El Centro wave, the control performance of Structure 1 is a little below the other two. Contrarily, the control effect in Structure 2 is slightly higher than those under Taft N21E and Kobe earthquake records.

Table 1 Percentage reduction of standard deviation peaks of random response in adjacent structures

Random seismic response	Percentage reduction (%)					
	Structure 1			Structure 2		
	El Centro	Taft	Kobe	El Centro	Taft	Kobe
Top floor displacement	26	44	49	59	44	28
Top floor velocity	37	46	52	55	39	32
Top floor acceleration	42	45	37	56	44	31
Bottom story drift	35	43	55	56	49	36

6 Conclusions

Dynamic response analysis of damper connected adjacent random structures under seismic excitation is performed in this study. If the variability of structural parameters is not taken into account, the two adjacent multi-story shear type structures with mean parameters can be represented by two 2-SDOF systems, which is a reducing order model of original adjacent structures. The general expressions for the determination of optimal damper parameters to link two 2-SDOF systems are adopted to achieve the proper damper parameters. Using the Kanai-Tajimi spectrum for ground acceleration, the appropriate parameters of damper linking adjacent structures with random parameters are obtained through careful parametric studies. A comparative study proves that the optimal theoretical values of damper parameters are very close to those through extensive numerical parametric studies. The theory results can be calculated using the first natural frequencies and the total mass of the adjacent deterministic structures with mean parameters. In terms of the mitigations of the mean square random responses of displacement, acceleration and inter-story drift in adjacent structures, the performance of connecting dampers is investigated. The numerical results demonstrate that coupling adjacent structures is an effective means of protection for flexible building structures.

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