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Continuum damage mechanics based modeling progressive failure of woven-fabric composite laminate under low velocity impact

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Abstract: A continuum damage mechanics (CDM) meso-model was derived for both intraply and interply progressive failure behaviors of a 2D woven-fabric composite laminate under a transversely low velocity impact. An in-plane anisotropic damage constitutive model of a 2D woven composite ply was derived based on CDM within a thermodynamic framework, an elastic constitutive model with damage for the fibre directions and an elastic-plastic constitutive model with damage for the shear direction. The progressive failure behavior of a 2D woven composite ply is determined by the damage internal variables in different directions with appropriate damage evolution equations. The interface between two adjacent 2D woven composite plies with different ply orientations was modeled by a traction-separation law based interface element. An isotropic damage constitutive law with CDM properties was used for the interface element, and a damage surface which combines stress and fracture mechanics failure criteria was employed to derive the damage initiation and evolution for the mixed-mode delamination of the interface elements. Numerical analysis and experiments were both carried out on a 2D woven glass fibre/epoxy laminate. The simulation results are in agreement with the experimental counterparts, verifying the progressive failure model of a woven composite laminate. The proposed model will enhance the understanding of dynamic deformation and progressive failure behavior of composite laminate structures in the low velocity impact process.

Key words: Continuum damage mechanics (CDM), Woven composite laminate, Low velocity impact, Interface element, Cohesive zone

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1 Introduction

The application of fibre reinforced composite is steadily increasing because of high specific stiffness, specific strength, and fatigue strength. However, one of the most important problems in designing composite structures is their vulnerability to the internal damage induced by a transversely low velocity impact in manufacture, service, and maintenance processes. As a low velocity impact proceeds, part of the impact energy is transformed into recoverable elastic strain energy, and the rest is absorbed mainly by composite, inducing irretrievable damage in composite structures. Major damage failure modes of laminated composite material subjected to impact loading can be classified as intraply failure and interply failure. The intraply failure mode can be subdivided into matrix micro cracking, fibre rupture, fibre/matrix interface debonding, fibre pull-out, etc.; the interply failure mode is mainly delamination. The internal damage can significantly degrade the residual strength and stiffness of composite structures, without any visible damage indication on the impacted surface (de Moura and Goncalves, 2004). Hence, understanding the deformation and damage mechanisms in the impact process is important for the effective design of composite structures.

The failure of laminated composite structures may be treated with three methods from the viewpoint of theory: the stress method, the fracture mechanics method, and the continuum damage mechanics (CDM) method.

The stress method can handle the initiation of damage, but cannot describe the progressive failure process at a material point. Choi and Chang (1992) proposed two stress-based failure criteria to study the interaction between matrix cracking and delamination resulted from an impact in unidirectional ply laminated composites. However, Choi and Chang (1992)'s model needs empirical parameters in the failure criteria. Hou *et al*. (2000) suggested three stress-based failure criteria for the intraply failure and one stress-based failure criterion for the interply delamination based on the Chang-Chang failure criteria to study the damage initiation and propagation in impacted unidirectional ply laminated composite plates. Hou *et al*. (2001) developed the stress-based quadratic delamination initiation criterion for laminated composites under a low-velocity impact to account for the delaying effect of compressive stresses on delamination. Zhao and Cho (2007) employed the Tsai-Wu failure criteria for the intraply damage and the delamination criterion of Choi and Chang (1992) to simulate the damage of curved laminates under a low-velocity impact. Cui *et al*. (2009) adopted the stress failure criteria of Hou *et al*. (2000) and Tserpes *et al*. (2002) in a whole-process analysis method to study the whole damage initiation and development process of composite laminates under an impact, as well as tensile loading after the impact. Sevkat *et al*. (2009) proposed stress-based orthotropic failure criteria for a composite ply and a stress-based delamination failure criterion for the interface to simulate the drop-weight tests of hybrid woven composite panels. Bouvet *et al*. (2009) simulated the matrix cracking and delamination with spring elements based on a stress criterion, and a strain criterion was adopted for the fibre failure.

The fracture mechanics method (Zheng and Sun, 1995) cannot directly deal with a progressive damage failure process alone without an initial crack or defect. Meanwhile, difficulties occur when fracture mechanics techniques are implemented in finite element software, especially for progressive random crack growth (Zou *et al*., 2003). The compliance derivative technique (CDT) and the energy derivative technique (EDT) are inherently impossible to predict the crack growth, because the crack growth path and rate are the prerequisites for applying these two techniques. The virtual crack closure technique (VCCT) and the *J*-integral are hindered by the lack of topological information of the crack profile that is needed to calculate the energy release rate for the crack growth (Fan *et al*., 2008). Additional difficulties arise from the sophisticated grid movement required for the crack tip growth in the finite element model.

The CDM method has been employed successfully to analyze the progressive failure of a composite laminate. This method can predict the initiation, propagation, and final rupture of composite structures effectively with finite element codes, avoiding a subjective definition of the initial flaw. Based on Ladeveze and LeDantec (1992)'s model, Johnson (2001) put forward intraply damage evolution equations of a 2D woven composite ply under impact loading, assuming that the damage modes in the fibre and shear directions were decoupled. Hochard *et al*. (2001; 2006) introduced an equivalent thermodynamic force to take into account the traction-shear coupling in the intraply shear damage evolution. Iannucci (2006a) and Donadon *et al*. (2008) detailed a 2D and 3D numerical modeling methodology for the intraply damage of a woven composite ply based on the CDM approach in DYNA, respectively. The cohesive zone model with CDM properties utilizing fracture mechanics indirectly is becoming more and more popular for the impact-induced delamination simulation in laminated composite materials. Mi *et al*. (1998) described a progressive failure criterion for the mixed-mode delamination of fibre reinforced composites, based on the cohesive interface element in conjunction with softening relationships between stresses and relative displacements. Fracture mechanics is indirectly introduced by relating the areas under the stress-relative displacement curves to the critical fracture energies. Johnson *et al*. (2001) and Johnson and Holzapfel (2006) predicted the response of 2D woven fabric-reinforced composite laminates subjected to different impact energies with an interply delamination failure model. Laminated composites are modeled as a stack of shell elements tied by contact interfaces with a cohesive zone based softening traction-displacement law. Borg *et al*. (2002) postulated that there was a maximum load surface or damage surface to limit the adhesive force magnitude in the delamination crack process zone. The damage surface unifies the initiation and propagation of the delamination progress by combining the conventional stress based and fracture mechanics based failure criteria. This model was employed by Zou *et al*. (2003) and Li *et al*. (2006). de Moura and Goncalves (2004), Nishikawa *et al*. (2007) and Aymerich *et al*. (2008) proposed cohesive zone based interface elements to simulate the interaction between matrix cracking and delamination in unidirectional laminates under a low velocity impact. The interface elements are placed at all potential cracking regions, where delamination could occur between plies and where matrix cracking was experimentally observed inside plies. Iannucci (2006b) and Iannucci and Willows (2006) presented an interface delamination modeling technique, intraply damage and interply delamination model for woven composites in DYNA, respectively. Hu *et al*. (2008) and Elmarakbi *et al*. (2009) proposed a modified adaptive cohesive model for the delamination stability and accuracy of composite laminates under transverse loads, with the introduction of a pre-softening zone and viscosity parameter. In the pre-softening zone, with the increase of effective relative displacement, the initial stiffness and interface strength at the integration points of cohesive elements are gradually reduced. However, the onset displacement for starting the real softening process is not changed, and the critical energy release rate for determining the final displacement of complete decohesion is kept constant.

To the authors' knowledge, existing prediction methods for the low velocity impact damage of composite structures employ either simple stress failure criteria or complicated functions with many material variables which are difficult to implement in finite element codes. To date few integrated damage models have been established for the impact event of a 2D woven laminated composite, in which the intraply and interply damages are simultaneously developed with the CDM theory. Hence, the aim of this study is to establish an integrated progressive failure model with CDM properties for a 2D woven fabric-reinforced composite laminate under a transversely low velocity impact. Four main types of failure modes from the viewpoint of engineering applications, i.e., matrix micro cracking, fibre rupture, fibre/matrix interface debonding, and interply delamination, are considered to understand the dynamic deformation and progressive failure mechanisms of a 2D woven fabric-reinforced composite laminate in the impact events.

A laminated composite can be defined by means of two constituents based on the 'meso-model' proposed by Allix (2001) according to the preceding damage failure characters:

1. Single ply, which is assumed to be homogeneous and orthotropic, can be modeled by general shell elements. The intraply damage modes can occur and grow in the ply layer.

2. Interface, which is a mechanical surface connecting two adjacent composite plies with different fibre orientations, can be modeled by interface elements. The interply damage mode can appear and propagate in the interface layer.

2 Intraply constitutive model with damage for 2D woven-fabric composite ply

CDM attempts to determine the effects of internal morphological changes in material on its mechanical response through internal state variables, which are referred to as damage variables, in the sense of irreversible thermodynamics. The damage variables derived from the change of elastic characteristics have been applied to a wide variety of materials, such as metal (Andrade Pires *et al*., 2004), geomaterial (Salari *et al*., 2004), and fibre-reinforced composite (Johnson, 2001; Johnson *et al*., 2001; Hochard *et al*., 2001; Iannucci, 2006a). In this method, the damage variable d is defined as $d=(E^0-E)/E^0$, where E^0 and E are the elastic moduli of undamaged and damaged materials, respectively. Scalar variable d is bounded between 0 and 1, i.e., $0\le d\le 1$, where d=0 represents the undamaged state, and d=1 represents the final failure.

The intraply failure modes can result in the degradation of elastic modulus; therefore, three intraply damage variables, d_1, d_2, and d_{12}, are introduced within a phenomenological framework, which are defined as follows:

1. $d_1=1-E_1/E_1^0$ represents Young's modulus degradation in the warp yarn direction due to fibre rupture.

2. $d_2=1-E_2/E_2^0$ represents Young's modulus degradation in the fill yarn direction due to fibre rupture.

3. $d_{12}=1-G_{12}/G_{12}^0$ represents shear modulus degradation in the in-plane shear direction due to fibre/matrix debonding and matrix micro cracking.

2D woven composite ply is brittle-elastic in the warp and fill yarn directions, and thus an elastic constitutive model with damage is adequate for the mechanical behavior of a single ply in these two directions. However, inelastic strain occurs when a shear loading is imposed on the composite due to matrix permanent deformation, slipping/friction between fibre and matrix as a consequence of the damage. Hence, an elastic-plastic constitutive model with damage is required in the shear direction. Accordingly, the total strain tensor $\boldsymbol{\varepsilon}$ needs to be decomposed into elastic component $\boldsymbol{\varepsilon}^{\mathrm{e}}$ and inelastic component $\boldsymbol{\varepsilon}^{\mathrm{in}}$. Strain additive decomposition is applicable to a fibre-reinforced composite ply because this kind of ply is brittle, and the overall strain magnitude is small before final failure. Thus the total strain tensor $\boldsymbol{\varepsilon}$ is decomposed as

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{\mathrm{e}}+\boldsymbol{\varepsilon}^{\mathrm{in}}. \tag{1}$$

2.1 Constitutive model with damage for 2D woven-fabric ply

The Gibbs free energy Ψ should have the following form, to introduce both damage and 'inelastic' flow process of a 2D woven composite ply under isothermal and small deformation condition:

$$\Psi(\boldsymbol{\sigma},p,d)=\Psi^{\mathrm{e}}(\boldsymbol{\sigma},d)+\Psi^{\mathrm{p}}(p), \tag{2}$$

where $\boldsymbol{\sigma}$ is stress, Ψ^{e} is the elastic strain complementary energy, p is the accumulated inelastic strain, and Ψ^{p} is the dissipation energy of the inelastic strain.

The elastic strain complementary energy Ψ^{e} in Eq. (2) has the following form:

$$\Psi^{\mathrm{e}}(\boldsymbol{\sigma},d)=\frac{1}{2}\boldsymbol{\sigma}:\boldsymbol{S}(d):\boldsymbol{\sigma}, \tag{3}$$

where $\boldsymbol{S}(d)$ is the material compliance matrix with damage. The following assumptions are made for engineering simplicity:

1. Stresses that do not lie in the ply plane are usually small for an orthotropic composite ply; thus, it is reasonable to set σ_3, σ_{23}, and σ_{13} as zero.

2. Although strains ε_3, ε_{31}, and ε_{32} still occur, these are not of particular interest.

3. Poisson's ratios degrade in a similar manner to Young's modulus to maintain the positive-definiteness of the material stress-strain law, i.e., $\nu_{12}=\nu_{12}^0(1-d_1)$, $\nu_{21}=\nu_{21}^0(1-d_2)$.

Thus, $\boldsymbol{S}(d)$ takes the following form:

$$\boldsymbol{S}(d)=\begin{bmatrix}\dfrac{1}{E_1^0(1-d_1)} & \dfrac{-\nu_{21}^0}{E_2^0} & 0\\ \dfrac{-\nu_{12}^0}{E_1^0} & \dfrac{1}{E_2^0(1-d_2)} & 0\\ 0 & 0 & \dfrac{1}{G_{12}^0(1-d_{12})}\end{bmatrix}.$$

$\dfrac{\nu_{12}^0}{E_1^0}=\dfrac{\nu_{21}^0}{E_2^0}$, hence,

$$\boldsymbol{S}(d)=\begin{bmatrix}\dfrac{1}{E_1^0(1-d_1)} & \dfrac{-\nu_{12}^0}{E_1^0} & 0\\ \dfrac{-\nu_{12}^0}{E_1^0} & \dfrac{1}{E_2^0(1-d_2)} & 0\\ 0 & 0 & \dfrac{1}{G_{12}^0(1-d_{12})}\end{bmatrix}. \tag{4}$$

Substituting Eq. (4) into Eq. (3), the elastic strain complementary energy with damage can be expressed as

$$\Psi^{\mathrm{e}}=\frac{1}{2}\left[\frac{\sigma_1^2}{E_1^0(1-d_1)}+\frac{\sigma_2^2}{E_2^0(1-d_2)}-2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2+\frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})}\right]. \tag{5}$$

Impact can be regarded as an instantaneous and adiabatic process if attention is confined to a mechanical system. Thus the Clausius-Duhem inequality takes the following form:

$$\boldsymbol{\sigma}:\dot{\varepsilon}-\dot{\Phi}\geq 0, \tag{6}$$

where Φ is the Helmholtz free energy. The following equations which fulfill the inequality can be obtained, according to the Second Principle of Thermodynamics and the Legendre-Fenchel transformation of stress-strain relationship:

$$\boldsymbol{\varepsilon}^{\mathrm{e}} = \frac{\partial \Psi}{\partial \boldsymbol{\sigma}} = \frac{\partial \Psi^{\mathrm{e}}}{\partial \boldsymbol{\sigma}} = \boldsymbol{S}(d) : \boldsymbol{\sigma}, \tag{7a}$$

$$R = \frac{\partial \Psi}{\partial p}, \tag{7b}$$

$$\boldsymbol{Y} = \frac{\partial \Psi}{\partial d} = \frac{\partial \Psi^{\mathrm{e}}}{\partial d}, \tag{7c}$$

where R is the yield surface radius, and the damage energy release rate $\boldsymbol{Y}$ is the generalized thermodynamic force, which may be understood as the resistance of material against the interior micro flaw. R and $\boldsymbol{Y}$ work conjugate to the accumulated inelastic strain p and damage variable d, respectively.

Fibre can prevent traction inelastic strains, and thus the behavior of a 2D woven composite ply is brittle-elastic in the fibre directions. But the intraply shear inelastic strain is significant due to matrix permanent deformation, slipping/friction between fibre and matrix as the consequence of damage. Plasticity theory with an elastic domain function and hardening law, coupled with damage, can deal with the observed inelastic behavior of a composite ply in the in-plane shear direction.

Hill's anisotropic yield criterion has been employed with reasonable success for orthotropic composite materials:

$$\begin{aligned} f(\boldsymbol{\sigma}_{ij}) = {} & a_{11}(\sigma_{22}-\sigma_{33})^2 + a_{22}(\sigma_{33}-\sigma_{11})^2 \\ & + a_{33}(\sigma_{11}-\sigma_{22})^2 + a_{23}\sigma_{23}^2 + a_{31}\sigma_{31}^2 + a_{12}\sigma_{12}^2. \end{aligned} \tag{8}$$

The increment of the plastic strain can be described in the following form according to the associated plastic flow rule:

$$\mathrm{d}\boldsymbol{\varepsilon}_{ij}^{\mathrm{p}} = \frac{\partial f}{\partial \boldsymbol{\sigma}_{ij}} \mathrm{d}\lambda, \tag{9}$$

where $\mathrm{d}\lambda$ is a proportional factor. It is reasonable to assume that no plastic strain occurred in the warp and fill yarn directions from an engineering viewpoint, i.e., $\mathrm{d}\varepsilon_{11}^{\mathrm{p}} = \mathrm{d}\varepsilon_{22}^{\mathrm{p}} = 0$, and thus a_{11}, a_{22}, and a_{33} all equal zero. Therefore, Hill's yield criterion can be simplified as the following for the plane-stress state:

$$f(\boldsymbol{\sigma}_{ij}) = a_{12}\sigma_{12}^2. \tag{10}$$

It can be further assumed that a_{12} equaled unity without loss of generality. Accordingly the final yield criterion of a 2D woven fabric composite ply can be described as

$$f(\boldsymbol{\sigma}_{ij}) = \sigma_{12}^2. \tag{11}$$

The coupling between the damage and plasticity can be treated with effective stress $\bar{\boldsymbol{\sigma}}$ and effective inelastic strain rate $\dot{\bar{\boldsymbol{\varepsilon}}}^{\mathrm{in}}$. They satisfy the following relationship (Hochard *et al*., 2001):

$$\bar{\boldsymbol{\sigma}} : \dot{\bar{\boldsymbol{\varepsilon}}}^{\mathrm{in}} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{\mathrm{in}}. \tag{12}$$

The effective stress and effective inelastic strain rate in the in-plane shear direction are

$$\bar{\sigma}_{12} = \frac{\sigma_{12}}{1-d_{12}}, \tag{13a}$$

$$\dot{\bar{\varepsilon}}_{12}^{\mathrm{in}} = \dot{\varepsilon}_{12}^{\mathrm{in}}(1-d_{12}). \tag{13b}$$

The elastic field can be defined by introducing a yield space function

$$f = |\bar{\sigma}_{12}| - R(p) - R_0 \le 0, \tag{14}$$

where R_0 represents the initial size of the elastic field, and $R(p)$ is the hardening function of the accumulated inelastic strain, which can be supposed to follow a power law hardening rule

$$R(p) = Ap^n, \tag{15}$$

where A and n are material constants. By introducing a multiplier λ, the following form can be derived:

$$\dot{\bar{\boldsymbol{\varepsilon}}}^{\mathrm{in}} = \dot{\lambda}\frac{\partial f}{\partial \bar{\boldsymbol{\sigma}}}, \tag{16a}$$

$$\dot{p} = -\dot{\lambda}\frac{\partial f}{\partial R}. \tag{16b}$$

They must satisfy the following Kuhn-Tucker condition under the loading/unloading state:

$$f \le 0,\ \dot{\lambda} \ge 0,\ f\dot{\lambda} = 0. \tag{17}$$

2.2 Damage evolution modeling for 2D woven-fabric ply

The constitutive model with damage of a single 2D woven ply has been derived in Subsection 2.1. The evolution rules of damage variable d are developed in this subsection.

Damage variable in the unloading stage is assumed to be fixed until further damage accumulation caused by the next positive loading in Johnson (2001) and Johnson *et al*. (2001). Although the tension energy and compression energy are split in Hochard *et al*. (2001), it is supposed that Young's moduli in two fibre directions were not affected by the compressive stress. Only the tensile stress is adopted in Hochard *et al*. (2001)'s damage evolution equation. In fact, the state of stress has a great influence on the damage growth rate. Voids and micro-cracks in materials that grow under tension will partially close under a compressive stress state, reducing the damage growth rate. This phenomenon may be accounted for by modifying the damage energy release rate $\boldsymbol{Y}$. In the case of crack closure, tension/compression additive decomposition of the stress tensor $\boldsymbol{\sigma}_i$ is used:

$$\boldsymbol{\sigma}_i = \langle \boldsymbol{\sigma}_i \rangle - \langle -\boldsymbol{\sigma}_i \rangle, \tag{18}$$

where $\langle\ \rangle$ is the MacCauley operator. The decomposition has the following properties:

$$\boldsymbol{\sigma}_i^2 = \langle \boldsymbol{\sigma}_i \rangle^2 + \langle -\boldsymbol{\sigma}_i \rangle^2, \tag{19a}$$

$$\langle \boldsymbol{\sigma}_i \rangle \langle -\boldsymbol{\sigma}_i \rangle = 0. \tag{19b}$$

The elastic strain complementary energy function Ψ^{e} including crack partial closure effect under a compressive stress state for a 2D woven composite ply can be described as follows:

$$\Psi^{\mathrm{e}} = \frac{1}{2}\left[\frac{\langle \sigma_1 \rangle^2}{E_1^0(1-d_1)} + \frac{\langle -\sigma_1 \rangle^2}{E_1^0(1-h_1 d_1)} + \frac{\langle \sigma_2 \rangle^2}{E_2^0(1-d_2)} + \frac{\langle -\sigma_2 \rangle^2}{E_2^0(1-h_2 d_2)} - 2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2 + \frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})} \right], \tag{20}$$

where a coefficient h_i ($0 \le h_i \le 1$) is introduced to account for the extent of damage deactivated by the crack closure effect. $h_i=0$ indicates the damage is fully deactivated; $h_i=1$ represents the damage is fully activated. Then detailed forms of the damage energy release rates with crack closure effect are

$$Y_i = \frac{\langle \sigma_i \rangle^2}{2E_i^0(1-d_i)^2} + \frac{h_i \langle -\sigma_i \rangle^2}{2E_i^0(1-h_i d_i)^2} = Y_i^+ + Y_i^-, \tag{21a}$$

$$Y_{12} = \frac{\sigma_{12}^2}{2G_{12}^0(1-d_{12})^2}, \tag{21b}$$

where $Y_i^+ = \dfrac{\langle \sigma_i \rangle^2}{2E_i^0(1-d_i)^2}$ and $Y_i^- = \dfrac{h_i \langle -\sigma_i \rangle^2}{2E_i^0(1-h_i d_i)^2}$ (i=1,2) are the damage energy release rates corresponding to a tensile and compressive stress state in the fibre directions, respectively. Value of Y_i^- is less than Y_i^+ at the same stress magnitude, which indicates that the compressive stress state reduces the damage growth rate.

The specific forms of damage evolution equations required for a 2D woven fabric composite ply are constituted with experimental data. The general evolution equations of damage variable d can be described as follows by the damage energy release rates $\boldsymbol{Y}$ within a mechanical framework, without temperature effect:

$$\begin{cases} d_1 = f_1(\underline{Y}_1, \underline{Y}_2, \underline{Y}_{12}), \\ d_2 = f_2(\underline{Y}_1, \underline{Y}_2, \underline{Y}_{12}), \\ d_{12} = f_{12}(\underline{Y}_1, \underline{Y}_2, \underline{Y}_{12}), \end{cases} \tag{22}$$

where $\underline{Y}_i = \sqrt{Y_i}$. Since the damage growth is an irreversible process under unilateral loading, the damage variables cannot be reduced, and $\underline{Y}_i$ should be the maximum value during the loading history, i.e., $\underline{Y}_i(t) = \max\limits_{\tau \le t} \left\{ \sqrt{Y_i(t)} \right\}$.

It is assumed that the damage modes in the fibre directions and the in-plane shear direction were decoupled, that the damage evolutions of d_1 and d_2 took linear forms, and that d_{12} took a nonlinear form according to the subsequent experimental data in Subsection 4.1. Eq. (22) can be simplified to Eq. (23) based on the assumption

$$d_1=\begin{cases}0, & \underline{Y}_1\le \underline{Y}_{10},\\ \alpha_1(\underline{Y}_1-\underline{Y}_{10}), & \underline{Y}_{10}<\underline{Y}_1<\underline{Y}_{1c},\\ d_{1c}, & \underline{Y}_1\ge \underline{Y}_{1c},\end{cases} \quad (23a)$$

$$d_2=\begin{cases}0, & \underline{Y}_2\le \underline{Y}_{20},\\ \alpha_2(\underline{Y}_2-\underline{Y}_{20}), & \underline{Y}_{20}<\underline{Y}_2<\underline{Y}_{2c},\\ d_{2c}, & \underline{Y}_2\ge \underline{Y}_{2c},\end{cases} \quad (23b)$$

$$d_{12}=\begin{cases}0, & \underline{Y}_{12}\le \underline{Y}_{120},\\ \alpha_{12}(\ln\underline{Y}_{12}-\ln\underline{Y}_{120}), & \underline{Y}_{120}<\underline{Y}_{12}<\underline{Y}_{12c},\\ d_{12c}, & \underline{Y}_{12}\ge \underline{Y}_{12c},\end{cases} \quad (23c)$$

where $\underline{Y}_{10}$, $\underline{Y}_{20}$, $\underline{Y}_{120}$, $\underline{Y}_{1c}$, $\underline{Y}_{2c}$, and $\underline{Y}_{12c}$ are threshold parameters that characterize the initiation and limit of damage in different directions, respectively, and α_1, α_2, and α_{12} are material parameters.

3 Interply delamination damage model

For the laminated composite structures comprising plies with different fibre orientations, delamination always occurs between two adjacent plies with different fibre orientations, and separates the laminate into sublaminates along the thickness direction. A cohesive zone model with CDM properties for the interface delamination based on Borg *et al*. (2002) is established.

The interface elements are located between two adjacent 2D woven composite plies with different ply orientations where delamination could occur. A local coordinate system of the interface element (Fig. 1) is constituted: normal vector $\boldsymbol{e}_1$ indicates the thickness direction, corresponding to the mode I fracture, i.e., opening fracture; $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ are two orthogonal directions in the interface plane, corresponding to the modes II and III fracture, respectively.

The displacement of one interface node can be expressed in the local coordinate system as

$$\begin{Bmatrix}u_1'\\u_2'\\u_3'\end{Bmatrix}=\boldsymbol{\theta}^{\mathrm{T}}\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}, \quad (24)$$

where $\boldsymbol{\theta}$ is the transformation matrix between the local element coordinate system and the global

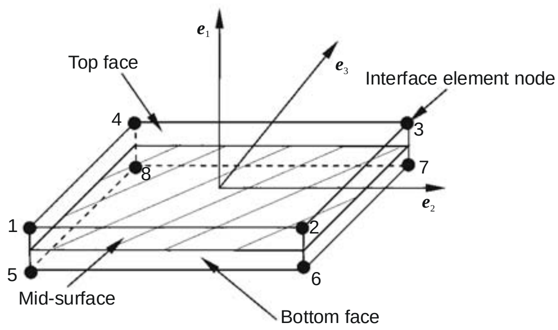

Fig. 1 Interface element for delamination damage

coordinate system. Shell element is employed to describe the dynamic response of a single ply due to high calculation efficiency. Hence, the nodal displacement of the top and bottom faces of the interface element must satisfy the following continuity relationship in the undelaminated region:

$$\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}_{\mathrm{top}}=\begin{Bmatrix}u_i+t_i\psi_i/2\\v_i+t_i\phi_i/2\\w_i\end{Bmatrix}, \quad (25a)$$

$$\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}_{\mathrm{bottom}}=\begin{Bmatrix}u_j-t_j\psi_j/2\\v_j-t_j\phi_j/2\\w_j\end{Bmatrix}, \quad (25b)$$

where i and j indicate the upper and lower composite plies, respectively; t is the thickness of the ply reference surface; u, v and w are the nodal displacements of the ply reference surfaces; ψ and ϕ are the rotation angles of the ply reference surfaces.

Thus separations (relative displacements) in the normal and the in-plane orthogonal directions between two corresponding interface nodes can be described as

$$\begin{Bmatrix}[u_1]\\ [u_2]\\ [u_3]\end{Bmatrix}=\begin{Bmatrix}u_1'\\u_2'\\u_3'\end{Bmatrix}_{\mathrm{top}}-\begin{Bmatrix}u_1'\\u_2'\\u_3'\end{Bmatrix}_{\mathrm{bottom}}. \quad (26)$$

The mechanical behavior of the interface in the undamaged region should be linear-elastic, and can be expressed as

$$P_i=k_i^0[u_i] \quad (i=1,2,3), \quad (27)$$

where k_i^0 is the original interface stiffness. As the loading level gradually increases, the damage process zone containing micro defects (e.g., micro cracks and micro voids) occurs at the interface, indicating the initiation of delamination. Then the micro defects grow and coalesce continuously, indicating the gradual evolution of delamination at the interface. One single scalar damage parameter ω is introduced to indicate the extent of the micro-cracks and micro-voids in a representative volume of the interface. A zero value of ω corresponds to an undamaged state, i.e., no micro-crack or micro-void; and ω equaling unity indicates a fully delaminated state. It is assumed that the existing micro-cracks and micro-voids could not be healed any more, i.e., $\mathrm{d}\omega\geq 0$. With the micro-crack or micro-void extent ω, the effective stiffness of the interface is expressed as $(1-\omega)k_i^0$, and the damage constitutive equations of the interface are described as

$$P_1 = k_1^0(1-\omega)\langle[u_1]\rangle - k_1^0\langle-[u_1]\rangle, \quad (28a)$$

$$P_2 = k_2^0(1-\omega)[u_2], \quad (28b)$$

$$P_3 = k_3^0(1-\omega)[u_3]. \quad (28c)$$

A stress-based failure criterion is suitable for predicting the initiation of delamination in an intact interface, i.e.,

$$f_{\mathrm{s}}(P_i) - 1 = 0 \quad (i = 1,2,3), \quad (29)$$

where f_{s} is a failure function according to an appropriate criterion. The coupled quadratic failure criterion is frequently adopted (Zou *et al*., 2003):

$$f_{\mathrm{s}} = \left(\frac{\max(P_1,0)}{P_{1\mathrm{c}}}\right)^2 + \left(\frac{P_2}{P_{2\mathrm{c}}}\right)^2 + \left(\frac{P_3}{P_{3\mathrm{c}}}\right)^2, \quad (30)$$

where $P_{i\mathrm{c}}$ (i=1,2,3) are the normal and shear strengths of the interface.

A fracture mechanics approach can deal with subsequent delamination propagation for an existing delamination. The criterion for the delamination propagation can be expressed in the general form:

$$f_{\mathrm{g}}(G_i) - 1 = 0 \quad (i = 1,2,3), \quad (31)$$

where G_i (i=1,2,3) are the energy dissipation rates produced by the traction in fracture modes I, II, and III, which are the applied work minus currently stored elastic energy:

$$G_i = \int_0^{[u_i]} P_i \mathrm{d}[u_i] - \frac{1}{2}P_i[u_i]. \quad (32)$$

The following coupled linear and quadratic criteria are often employed:

$$f_{\mathrm{g}} = \left(\frac{G_1}{G_{1\mathrm{c}}}\right) + \left(\frac{G_2}{G_{2\mathrm{c}}}\right) + \left(\frac{G_3}{G_{3\mathrm{c}}}\right), \quad (33a)$$

$$f_{\mathrm{g}} = \left(\frac{G_1}{G_{1\mathrm{c}}}\right)^2 + \left(\frac{G_2}{G_{2\mathrm{c}}}\right)^2 + \left(\frac{G_3}{G_{3\mathrm{c}}}\right)^2, \quad (33b)$$

where $G_{i\mathrm{c}}$ (i=1,2,3) are the individual critical energy release rates in modes I, II, III for delamination propagation.

Treated as a damage process in the context of CDM, delamination includes two aspects, the initiation and the growth of the damage. Borg *et al*. (2002) postulated that there was a damage surface to limit the adhesive force magnitude in the crack process zone, and that the size of the damage surface was dependent on the amount of dissipated crack opening work such that the damage surface shrank to zero as a predefined amount of work was consumed. The softening interfacial constitutive law with damage evolution can be derived by the damage surface shrinkage as damage develops. Like the concept of yield surface in classical plasticity theory, the damage surface is represented as the following form in the P_i-G_i space:

$$F(P_i, G_i) = f_{\mathrm{s}}(P_i) - [1 - f_{\mathrm{g}}(G_i)] = 0. \quad (34)$$

When F is less than zero, there is no damage growth at the interface; thus, the increment of damage is zero, i.e., $\Delta\omega=0$. Damage is allowed to grow only when F equals zero; it is not admissible for F to be greater than zero. These requirements satisfy the Kuhn-Tucker conditions by the following expression:

$$F \leq 0, \ \dot{\omega} \geq 0, \ F\dot{\omega} = 0. \quad (35)$$

4 Experiments and numerical results

The preceding constitutive model has been implemented into the commercial finite element software ABAQUS/Explicit, via its user subroutine VUMAT, to analyze the dynamic response and damage in a 2D woven glass fibre/epoxy laminate $[0_3/45_3]_s$ under a low velocity impact. The specimen is balanced reinforced in the warp and fill yarn directions in a single ply, and stacked in a quasi-isotropic layout. This kind of laminated composite can provide more balanced and equal properties.

4.1 Parameter determination procedure

Standard tests on $[0]_8$ and $[\pm 45]_{3s}$ 2D balanced woven laminated composite specimens were performed with an Instron testing machine to obtain the material parameters needed in the constitutive model.

The stress-strain curves of $[0]_8$ 2D woven composite laminate specimens under monotonic tension along the fibre direction are shown in Fig. 2. The mechanical behaviors of this kind of material in the warp and fill yarn directions are thought to be identical. This material is brittle-elastic in the fibre direction. Damage occurs when the tensile strain reaches about 0.005, and the specimens begin to rupture when the tensile strain reaches about 0.016. The initial Young's modulus E_1^0 is 29 GPa, and Poisson's ratio ν_{12}^0 is 0.17.

Some data points are selected from the stress-strain curves in Fig. 2 to calculate the secant modulus $E_{1,i}$ and the damage energy release rate $\underline{Y}_1$ for the damage evolution equation in the fibre direction:

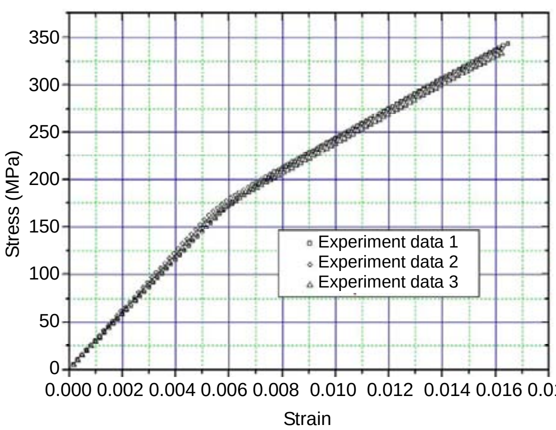

Fig. 2 Stress-strain curves in the warp direction under monotonic tension

$$d_{1,i} = 1 - \frac{E_{1,i}}{E_1^0}, \tag{36a}$$

$$\begin{aligned} \underline{Y}_1(t_i) &= \max\left\{\sqrt{Y_i(t_i)}\right\} = \max\left\{\sqrt{\frac{1}{2}\frac{\sigma_{1,i}^2}{E_1^0(1-d_{1,i})^2}}\right\} \\ &= \max\left\{\sqrt{\frac{1}{2}\frac{\left[E_1^0(1-d_{1,i})\varepsilon_{1,i}^{\mathrm{e}}\right]^2}{E_1^0(1-d_{1,i})^2}}\right\} \\ &= \max\left\{\sqrt{\frac{1}{2}E_1^0(\varepsilon_{1,i}^{\mathrm{e}})^2}\right\}. \end{aligned} \tag{36b}$$

The damage evolution relationship of d_1-$\underline{Y}_1$ from experiments according to Eq. (36) is shown in Fig. 3. The result seems to be well approximated by a straight line. The linear fitted function of Eq. (23a) is also shown in Fig. 3. The fitting constant α_1 is 1.72×10^{-4}, and $\underline{Y}_{10}$ is 286.58 $\mathrm{Pa}^{1/2}$. It is assumed that d_2 has the same evolution function as d_1 for a 2D balanced woven composite ply.

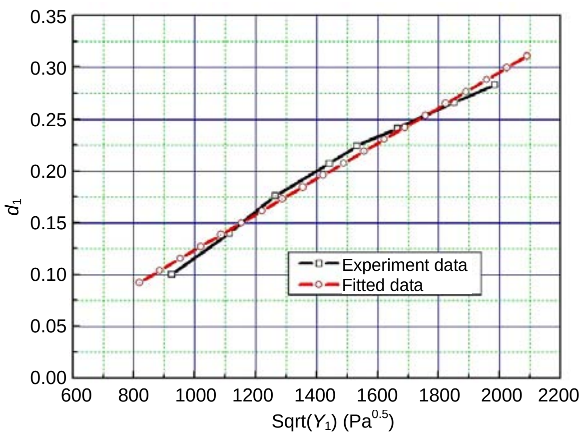

Fig. 3 Elastic damage evolution in the warp direction

The in-plane shear stress-strain curves of $[\pm 45]_{3s}$ 2D woven composite laminate specimens under monotonic and cyclic tension are shown in Figs. 4 and 5, respectively. Test data of specimens 2 and 3 are very close in Fig. 4. The initial shear modulus G_{12}^0 is 6.8 GPa.

The shear modulus decreases gradually in the 'loading/unloading' cycles of Fig. 5, and the inelastic strain also occurs. The decayed shear modulus $G_{12,i}$ and the elastic strain $2\varepsilon_{12,i}^{\mathrm{e}}$ of the ith cyclic stress-strain curve are used to establish the damage evolution equation in the in-plane shear direction:

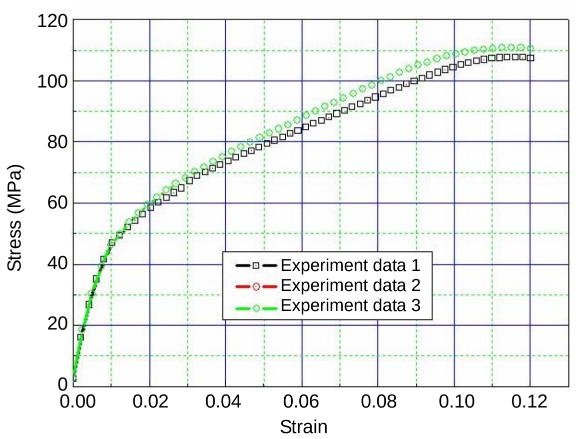

Fig. 4 In-plane shear stress-strain curves under monotonic tension

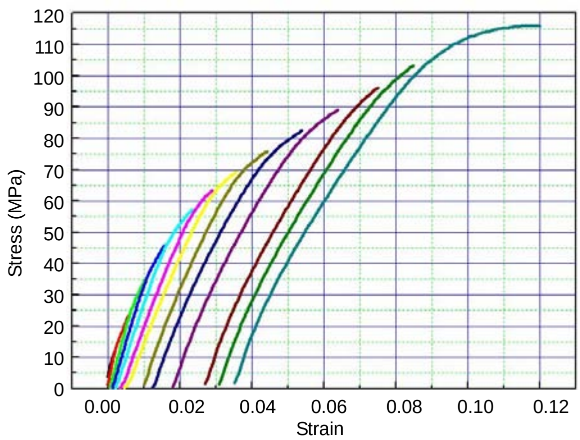

Fig. 5 Cyclic in-plane shear stress-strain curves

$$d_{12,i}=1-\frac{G_{12,i}}{G_{12}^{0}}, \tag{37a}$$

$$\begin{aligned}\underline{Y}_{12}(t_i)&=\max\left\{\sqrt{Y_{12}(t_i)}\right\}=\max\left\{\sqrt{\frac{1}{2}\frac{\sigma_{12,i}^{2}}{G_{12}^{0}(1-d_{12,i})^{2}}}\right\}\\&=\max\left\{\sqrt{\frac{1}{2}\frac{\left[G_{12}^{0}(1-d_{12,i})(2\varepsilon_{12,i}^{\mathrm{e}})\right]^{2}}{G_{12}^{0}(1-d_{12,i})^{2}}}\right\}\\&=\max\left\{\sqrt{\frac{1}{2}G_{12}^{0}(2\varepsilon_{12,i}^{\mathrm{e}})^{2}}\right\}.\end{aligned} \tag{37b}$$

The damage evolution relationship of d_{12} - $\underline{Y}_{12}$ from experiments according to Eq. (37) is plotted as the solid line in Fig. 6, which illustrates a nonlinear relationship between d_{12} and $\underline{Y}_{12}$. The fitted logarithm function of Eq. (23c) is shown as the dashed line in Fig. 6. The fitting constant α_{12} is 0.187, and $\underline{Y}_{120}$ is 71.01 $\mathrm{Pa}^{1/2}$.

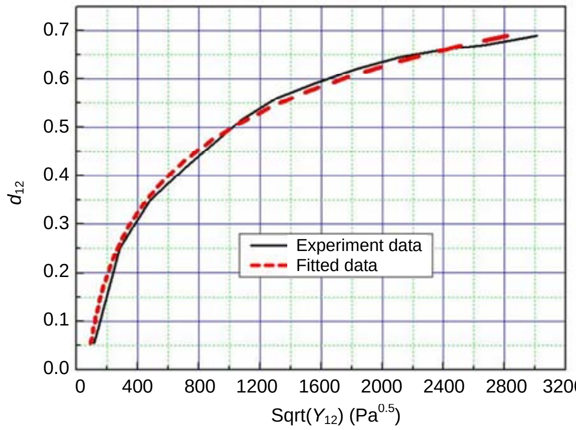

Fig. 6 Damage evolution in the in-plane shear direction

The hardening function of the accumulated inelastic strain can be determined from the yield stress $\sigma_{12,i}$ and the inelastic strain $2\varepsilon_{12,i}^{\mathrm{p}}$ of the ith cyclic stress-strain curve in Fig. 5:

$$R_i=\frac{\sigma_{12,i}}{1-d_{12,i}}-R_0, \tag{38a}$$

$$p_i=\int\dot{\varepsilon}_{12,i}^{\mathrm{p}}(1-d_{12,i}). \tag{38b}$$

The evolution of the inelastic behavior, p-R, in the in-plane shear direction from experiments according to Eq. (38) is plotted as the solid line in Fig. 7. The fitted power law hardening function of Eq. (15) is shown as the dashed line in Fig. 7.

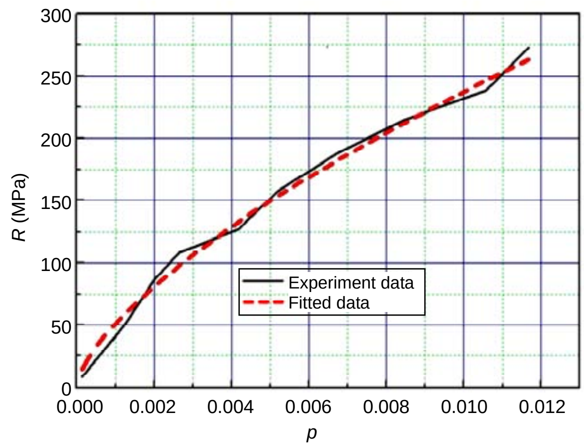

Fig. 7 Plastic hardening function in the in-plane shear direction

The strength and critical energy release rate of the interface delamination in mode I and mode II can be determined from the double cantilever beam (DCB) and end-notched flexure (ENF) experiments, respectively. The average values of P_{1c}, P_{2c}, G_{1c}, and G_{2c} with five replicates are listed in Table 1. P_{3c} and G_{3c} are supposed to equal P_{2c} and G_{2c}, respectively.

Table 1 Strength and critical energy release rate of interface delamination

P_{1c} (MPa)	P_{2c} (MPa)	P_{3c} (MPa)	G_{1c} (J/m^2)	G_{2c} (J/m^2)	G_{3c} (J/m^2)
6	20	20	462	1311	1311

4.2 Numerical results and experiment comparison

The preceding model and material parameters were implemented in the finite element package ABAQUS/Explicit to predict the dynamic response and damage of a 2D woven glass fibre/epoxy laminate $[0_3/45_3]_s$ under impact loading. The mass and velocity of the external impactor are 5.115 kg and 2 m/s, respectively. The nominal gauge length, width, and thickness of the specimen are 70 mm, 15 mm, and 2.4 mm, respectively. Two longitudinal ends of the specimen are clamped. Only half of the composite laminate and impactor were built due to the symmetry of geometry, boundary condition, and material in the finite element model (Fig. 8). Three layers of 2D woven composite sublaminates with the same orientation are represented by the shell element S4R with hourglass control. The CDM-based intraply damage constitutive model is adopted to describe the mechanical behavior of each sublaminate layer. Since the stack layout of this laminate has two potential delamination regions, the interface elements COH3D8 are placed there. The CDM-based interply delamination model is adopted to describe the mechanical behavior of these interfaces, and the quadratic coupled failure criterion for delamination is used in the numerical model. The impactor is represented by the brick element C3D8R with hourglass control. Surface-based tie constraints are introduced between the face of interface elements and the reference surface of shell elements for displacement continuity. A general contact algorithm is employed for the potential contact between the impactor and the laminate panel. Contact constraints are also introduced between two adjacent shell element layers in advance to simulate the potential contact between them in the case of interface elements deletion induced by the delamination damage.

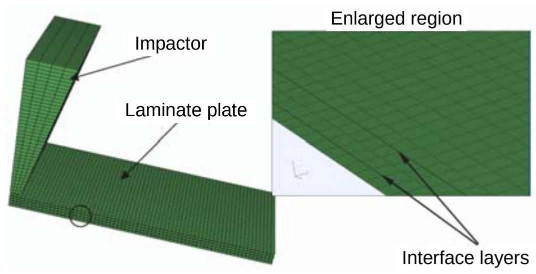

Fig. 8 Finite element model

The predicted damage regions in every 2D woven composite sublaminate layer are shown in Fig. 9. There are three damage regions in each layer, one at the center region where the impactor collides directly, the rest close to the clamped ends. The maximum damage degree is located at the impactor collision region. Moreover, fibre rupture is indicated in the center region of the outmost layer relative to the impacted surface (Fig. 9c).

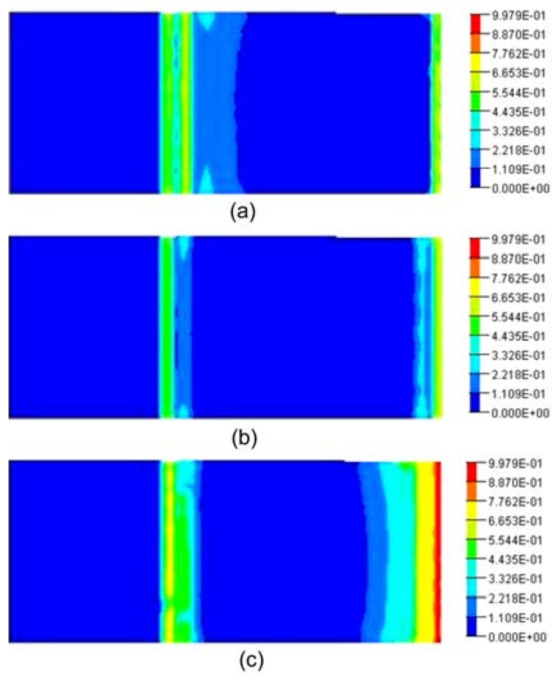

Fig. 9 Predicted damage in each ply
(a) Nearest ply; (b) Middle ply; (c) Outmost ply

The predicted delamination regions in two interfaces are located in the elliptical and rectangular areas in Fig. 10. There are two delamination regions in the upper interface relative to the impactor surface (Fig. 10a), which are both located at the clamped ends. There are three delamination regions in the lower interface (Fig. 10b), one at the center, and the rest at the clamped ends.

A drop weight experiment on the same material under the same conditions as the simulation was performed to verify the simulation results. The drop weight testing machine Instron Dynatup with a line nosed impactor is shown in Fig. 11. The experimental result is shown in Fig. 12. It is revealed that there are three damage regions in this kind of 2D woven composite laminate, one at the center, and the others close to the clamped ends. The size and arrangement of these damage regions are coincident with the simulation results. Fibre rupture is also observed in the outmost ply relative to the impacted surface, just as predicted by the simulation.

The contact force comparison is presented in Fig. 13. It is shown that the established numerical model can predict the dynamic response of a 2D woven fabric-reinforced composite laminate under a low velocity impact. Fig. 13 also shows that the contact force decreases rapidly after a peak value, which indicates an apparent damage phenomenon.

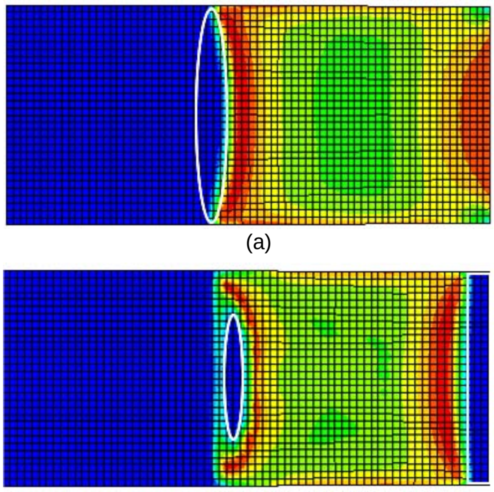

Fig. 10 Predicted delamination in each interface
(a) Upper interface; (b) Lower interface

Fig. 11 The impact experiment instrument

Fig. 12 Result of the impact experiment

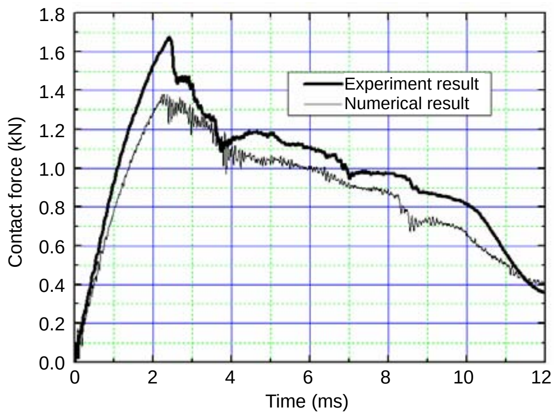

Fig. 13 Contact force between impactor and laminate

5 Conclusion

An explicit arithmetic model based on the continuum damage mechanics method has been developed for the progressive failure behavior of a 2D woven fabric-reinforced composite laminate under a transversely low velocity impact. Laminated structures are represented with stacked composite sublaminates glued by interface elements according to damage characteristics. An anisotropic elastic-plastic damage constitutive law for a single composite ply and an isotropic damage constitutive law with the concept of damage surface for the interface are derived, respectively. The experimental results of a drop weight impact verify the proposed progressive failure model. The model can effectively reveal main failure modes and dynamic response of laminated composite structures without initial defects in the low-velocity impact process.

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